Corrigé Série TD n° 03 / 2019/2020

Exercice n°2 :

1°/ Calcul la charge totale du système en fonction de $\boldsymbol{\sigma}$ et $\boldsymbol{R}$.

$Q_T = Q_0 + Q_1$ avec $Q_0 = \sigma\pi R^2$ et $\iint dQ_1 = Q_1 = \iint \sigma_1\, ds = \sigma_1 s = \frac{3}{4}\sigma\, 4\pi R^2$

$Q_T = \sigma\pi R^2 + \frac{3}{4}\sigma\, 4\pi R^2 = 4\sigma\, \pi R^2$

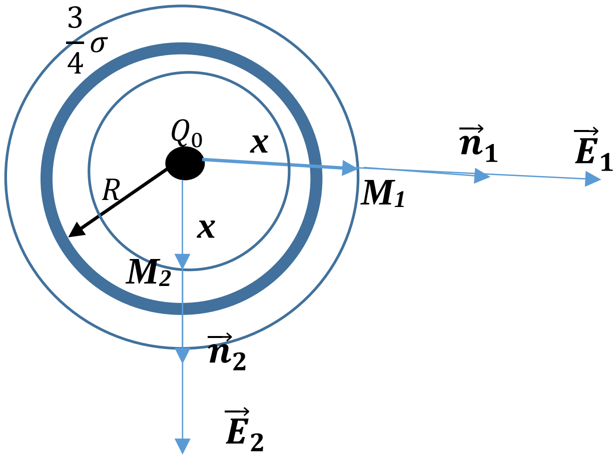
2°/ Calcul le champ électrique $\boldsymbol{E(x)}$

En utilisant le théorème de GAUSS : $\oiint \vec{E}.\vec{n}.ds_G = \frac{1}{\varepsilon_0}\sum_i Q_i$ **ou** $\oiint \vec{E}.d\vec{s}_G = \frac{1}{\varepsilon_0}\sum_i Q_i$

a. Au point M_1 $x \epsilon\, [R, +\infty[$

$$E_1 S_G = \frac{1}{\varepsilon_0} Q \rightarrow E_1\, 4\pi x^2 = \frac{1}{\varepsilon_0}\, 4\sigma\, \pi R^2$$

$$E_1 = \frac{\sigma R^2}{\varepsilon_0 x^2}$$



b. Au point M_2 $x \epsilon\, [0, R]$

$$E_2 S_G = \frac{1}{\varepsilon_0} Q \rightarrow E_2\, 4\pi x^2 = \frac{1}{\varepsilon_0}\, \sigma\, \pi R^2$$

$$E_2 = \frac{\sigma R^2}{4\varepsilon_0 x^2}$$

3°/ le potentiel électrique $\boldsymbol{V(x)}$ en tous points de l'espace

$$\vec{E} = -\overrightarrow{grad}V \rightarrow dV = -E dx$$

$$V_1 = -\int E_1\, dx = \frac{\sigma R^2}{\varepsilon_0 x} + C_1 \quad \text{et} \quad V_2 = -\int E_2\, dx = \frac{\sigma R^2}{4\varepsilon_0 x} + C_2$$

$$V_1(\infty) = 0 \rightarrow C_1 = 0$$

$$\lim_{x\to R} V_1 = \lim_{x\to R} V_2 \rightarrow \frac{\sigma R^2}{\varepsilon_0 R} = \frac{\sigma R^2}{4\varepsilon_0 R} + C_2 \rightarrow \frac{\sigma R}{\varepsilon_0} - \frac{\sigma R}{4\varepsilon_0} = C_2 \rightarrow C_2 = \frac{3\sigma R}{4\varepsilon_0}$$

$$V_1 = \frac{\sigma R^2}{\varepsilon_0 x} \quad \text{et} \quad V_2 = \frac{\sigma R^2}{4\varepsilon_0 x} + \frac{3\sigma R}{4\varepsilon_0}$$

Ex n°3 :

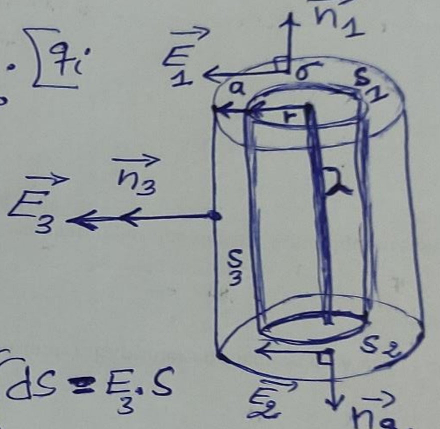
من أجل حساب مركبات الحقل الكهربائي، نختار سطح قوص عبارة عن أسطوانة نصف قطرها a وارتفاعها الخيط الكهربائي «ℓ»

pour Calculer la Composante du champ électrique, on choisit pour la surface de Gauss un cylindre de rayon "a" et qui a comme axe le fil électrique.

$$\oiint_S \vec{E} \cdot \vec{dS_G} = \frac{1}{\varepsilon_0} \sum q_i \quad / \quad \vec{dS} = dS \cdot \vec{n}$$

n: الشعاع العمودي على المساحة

1* $a \in [r, +\infty[$



$$\oiint_{S_1} \vec{E_1} \cdot \vec{dS_1} + \oiint_{S_2} \vec{E_2} \cdot \vec{dS_2} + \oiint_{S_3} \vec{E_3} \cdot \vec{dS_3} = \frac{1}{\varepsilon_0} \cdot \sum q_i$$

$$\vec{E_1} \perp \vec{S_1}, \quad \vec{E_2} \perp \vec{S_2} \quad \text{et} \quad \vec{E_3} \parallel \vec{S_3}$$

$$\Rightarrow \vec{E_1} \cdot \vec{dS_1} = \vec{E_2} \cdot \vec{dS_2} = 0 \quad /$$

$$\Rightarrow \vec{E_3} \cdot \vec{dS} = E_3 \cdot dS_3$$

$$\oiint_{S_3} \vec{E_3} \cdot \vec{dS} = \oiint_{S_3} \vec{E_3} \cdot \vec{dS_3} = E_3 \oiint_{S_3} dS = E_3 \cdot S$$

S: مساحة الاسطوانة الحقيقية $S = 2\pi r h$

ومنه

$$E_3 S = E_3 \cdot 2\pi r h$$

حساب الشحنة الكلية

$$q_i = q_\ell + q_s$$

$$dq_i = \lambda d\ell + \sigma dS \Rightarrow q_i = \int_\ell \lambda d\ell + \iint_c \sigma dS$$

$$\Rightarrow q_i = \lambda \ell + \sigma S$$

$$\Rightarrow q_i = \lambda \ell + \sigma 2\pi a h.$$

S: مساحة أسطوانة قوص، وكذلك $\ell = h$

$$\Rightarrow q_i = (\lambda + 2\pi a \sigma)\ell$$

SHOT ON POCO X3 NFC

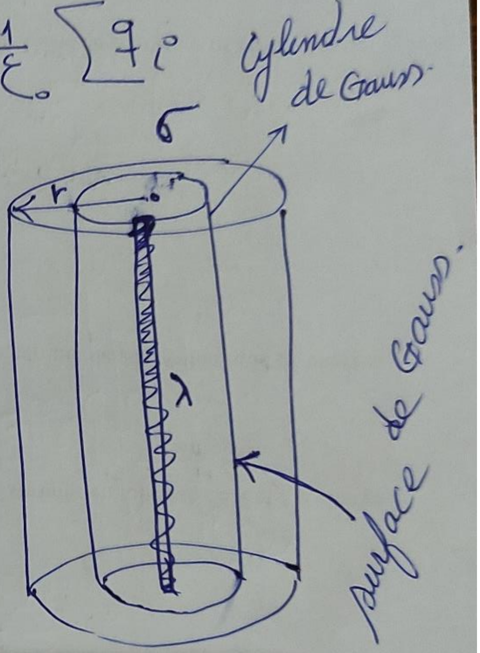
$$\Rightarrow 2\pi r h E_3 = \frac{1}{\varepsilon_0}(\lambda + 2\pi a \sigma) l \quad \text{avec } h = l$$

$$\Rightarrow E_3 = \frac{1}{\varepsilon_0}\frac{(\lambda + 2\pi a\sigma) l}{2\pi r l}$$

$$\Rightarrow \boxed{E_3 = E = \frac{\lambda + 2\pi a \sigma}{2\pi\varepsilon_0 r}}$$

2* $a \in [0, r]$

$$\oint_{S_1} \vec{E_1} d\vec{S_1} + \oint_{S_2} \vec{E_1} \cdot d\vec{S_2} + \oint_{S_3} \vec{E_3} d\vec{S_3} = \frac{1}{\varepsilon_0} \sum q_i$$



$$E_1 \cdot S_1 = E_2 \cdot S_2 = 0$$

$$\vec{E_3} \cdot \vec{S_3} = E_3 \cdot S_3.$$

$$\oint_{S_3} \vec{E_3} \cdot d\vec{S_3} = \oint_{S_3} \vec{E} \cdot d\vec{S} = E_3 \cdot S_3$$

$$E_3 \cdot S_3 = E_3 \cdot 2\pi r h \ / h = l.$$

$$q_i = q_l = \lambda l.$$

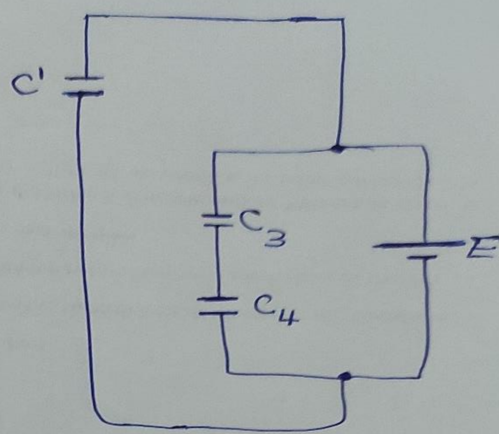
$$2\pi r l E' = \frac{\lambda l}{\varepsilon_0} \quad \text{avec } E' = E_3$$

$$\boxed{E' = \frac{\lambda}{2\pi\varepsilon_0 r}}$$

SHOT ON POCO X3 NFC

Ex n°5 : 1) calcul C_{eq}

C_1 et C_2 en série



$$C' = \frac{C_1 C_2}{C_1+C_2} = \frac{2}{3}\,\mu F$$

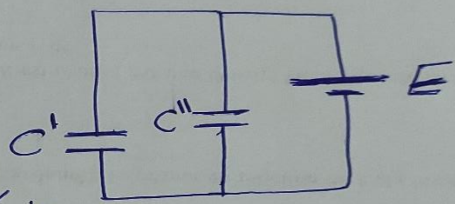
C_3 et C_4 en série

$$C'' = \frac{C_3 C_4}{C_3+C_4} = \frac{6}{5}\,\mu F$$

C' et C'' en parallèle

$$C_{eq} = C' + C'' = 2/3 + 6/5$$

$$C_{eq} = \frac{28}{15}\,\mu F$$



2/ calcul q_1, q_2, q_3 et q_4

$q_1 = q_2$ car C_1 et C_2 en série

$q_3 = q_4$ car C_3 et C_4 en série

$$E = V_1 + V_2 = \frac{q_1}{C_1} + \frac{q_2}{C_2}$$

$$E = V_3 + V_4 = \frac{q_3}{C_3} + \frac{q_4}{C_4}$$

$$\Rightarrow q_1 = \frac{10}{3}\,\mu C \;,\; q_2 = \frac{10}{3}\,\mu C$$

$$\Rightarrow q_3 = 6\,\mu C \text{ et } q_4 = 6\,\mu C$$

$$q_1 = q_2 = \frac{C_1 C_2}{C_1+C_2}\cdot V \text{ et } q_3 = q_4 = \frac{C_3 C_4}{C_3+C_4}\cdot V$$