Université Mohamed Boudiaf–M’sila
Faculté des Sciences

Socle commun : **SM**, Module : **Physique 2**
Corrigé TD n° 02, Année **2022/2023**

Exercice 1.

1. charge linéaire λ_0 distribuée uniformément sur le périmètre d’un carré de côté a.

$$dQ = 4\lambda_0 dl \longrightarrow Q = 4\lambda_0 \int_0^a dl \longrightarrow Q = 4\lambda_0 a$$

2. charge surfacique σ_0 distribuée uniformément sur une couronne comprise entre deux cercles concentriques de rayons a et b ($b > a$).

$$dQ = \sigma_0\, dS \longrightarrow Q = \sigma_0 \int_a^b r\, dr \int_0^{2\pi} d\varphi$$

$$Q = \sigma_0 (b^2 - a^2)\pi$$

3. charge volumique ρ_0 distribuée uniformément entre deux sphères de rayons a et b ($b > a$).

$$Q = \rho_0\, d\tau \longrightarrow Q = 4\pi\rho_0 \int_a^b r^2\, dr$$

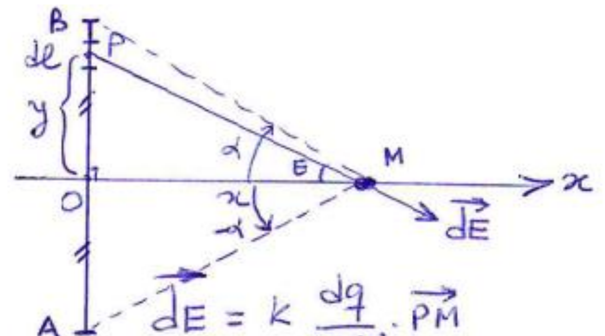
$$Q = \frac{4}{3}\rho_0 (b^3 - a^3)\pi$$

4. charge volumique ρ_0 distribuée uniformément entre deux cylindres coaxiaux de rayons a et b ($b > a$) et de hauteur h.

$$Q = \rho_0\, d\tau \longrightarrow Q = 2\pi\rho_0 h \int_a^b r dr$$

$$Q = \pi\rho_0 h (b^2 - a^2)$$

Exercice 2.



$$\vec{dE} = k \frac{dq}{\|\vec{PM}\|^3} \cdot \vec{PM}$$

1°/

$dq = \lambda\, dl = \lambda\, dy$.

$\|\vec{PM}\|^2 = y^2 + x^2$.

$Proj(\vec{PM}/x) = x$

$Proj(\vec{dE}/x) = dE_x$

$$dE_x = k \frac{\lambda\, dy}{(y^2+x^2)^{3/2}} \cdot x.$$

$$E_x = k\lambda x \int_{-a}^{a} \frac{dy}{(y^2+x^2)^{3/2}}$$

Faisons un changement de variable

$tg\,\theta = \frac{y}{x} \Rightarrow dy = \frac{x}{\cos^2\theta} d\theta$

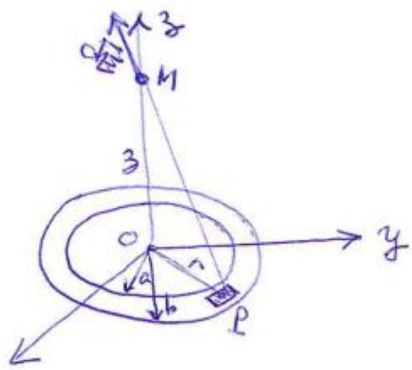
$y \in [-a, a] \Longrightarrow \theta \in [-\alpha, \alpha]$

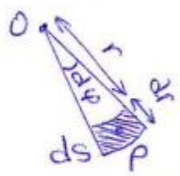
$$E_x = \frac{k\lambda}{x} \int_{-\alpha}^{\alpha} \cos\theta\, d\theta = \frac{2k\lambda}{x} \sin\alpha$$

$$\boxed{E_x = \frac{\lambda a}{2\pi\varepsilon_0 x\sqrt{x^2+a^2}}}$$

2°/ $E_\infty = \lim\limits_{a\to\infty} E \Rightarrow \boxed{E = \frac{\lambda}{2\pi\varepsilon_0 x}}$

Exercice 3.





$ds = dr \times r\, d\varphi$

1°/ $r \in [a, b)$ et $\varphi \in [0, 2\pi]$.

$$\vec{dE} = k \frac{dq}{\|\vec{PM}\|^3} \vec{PM}.$$

$\|\vec{PM}\|^2 = r^2 + z^2$.

$Proj(\vec{PM}/z) = z$, $Proj(\vec{dE}/z) = dE_z$

$$dE_z = \frac{k\sigma\, r\,dr\,d\varphi}{(r^2+z^2)^{3/2}} \cdot z$$

$$E_z = k\sigma z \int_a^b \frac{r\,dr}{(r^2+z^2)^{3/2}} \int_0^{2\pi} d\varphi$$

$$E_z = \frac{\sigma z}{2\varepsilon_0} \int_a^b \frac{r\,dr}{(r^2+z^2)^{3/2}}$$

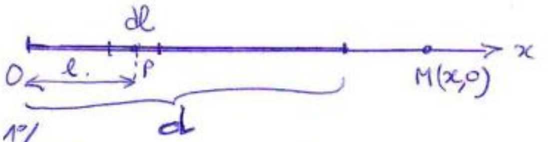
on obtient :

$$\boxed{E_z = \frac{\sigma z}{2\varepsilon_0}\left(\frac{1}{\sqrt{z^2+a^2}} - \frac{1}{\sqrt{z^2+b^2}}\right)}$$

2°/ On en déduit le champ créé par un ~~fil~~ plan infini, soit :

$$E_\infty = \lim_{\substack{a \to 0 \\ b \to \infty}} E_z \Rightarrow \boxed{E_\infty = \begin{cases} \dfrac{\sigma}{2\varepsilon_0} & z>0 \\ \dfrac{-\sigma}{2\varepsilon_0} & z<0 \end{cases}}$$

Exercice 4.



1°/ $\vec{dE} = k \dfrac{dq}{\|\vec{PM}\|^3} \cdot \vec{PM}$

$P(\ell,0)$, $M(x,0)$, $\vec{PM} = (x-\ell)\vec{i}$

$\mathrm{Proj}(\vec{dE}/x) = dE_x$, $\mathrm{proj}(\vec{PM}/x) = (x-\ell)$

$$dE_x = \frac{k\lambda\, d\ell}{(x-\ell)^2}.$$

On pose : $x-\ell = T \Rightarrow -d\ell = dT$

$$E_x = -k\lambda \int \frac{dT}{T^2} = -k\lambda \left[-\frac{1}{T}\right]$$

$$E_x = -k\lambda \left[\frac{1}{x-\ell}\right]_d^0$$

$$E_x = \frac{-\lambda}{4\pi\varepsilon_0}\left(\frac{1}{x} - \frac{1}{x-d}\right)$$

$$\boxed{E_x = \frac{\lambda d}{4\pi\varepsilon_0 (x-d)x}}$$

2°/ $dV = -E_x\,dx$.

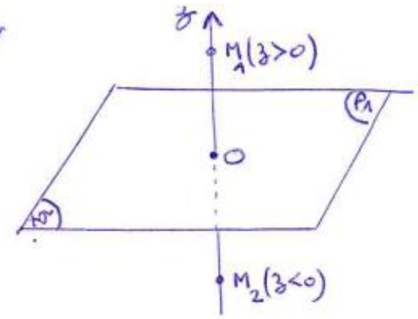
$$\boxed{V = \frac{\lambda}{4\pi\varepsilon_0} \ln\left(\frac{x}{x-d}\right)}$$

Sachant que $V(\infty) = 0$.

Exercice 5.

$$\vec{E}_M = \begin{cases} \dfrac{\sigma}{2\varepsilon_0}\vec{k} & z>0 \\ -\dfrac{\sigma}{2\varepsilon_0}\vec{k} & z<0 \end{cases}$$

1°/



$$dV_M = -E_M\,dz.$$

si $z>0$: $V_{M_1} = -\dfrac{\sigma}{2\varepsilon_0}\displaystyle\int dz + C_1$

$V_{M_1} = -\dfrac{\sigma z}{2\varepsilon_0} + C_1$, $V_{M_1}(0) = 0 \Rightarrow C_1 = 0$

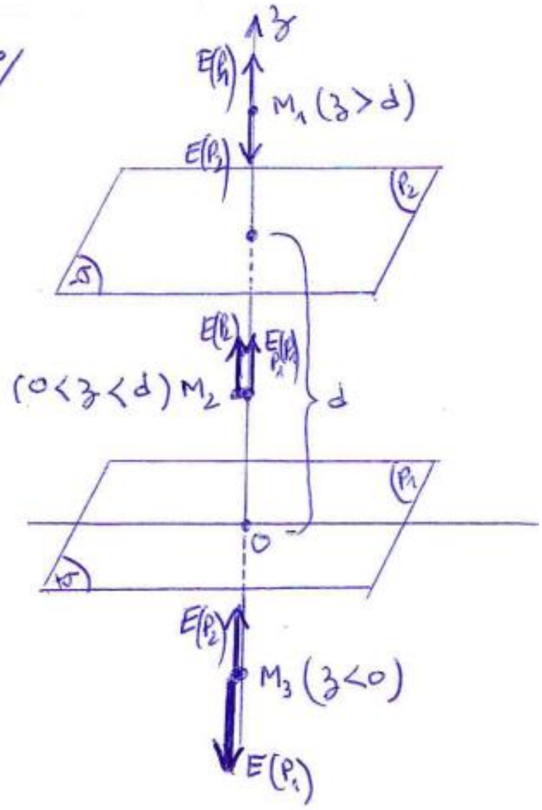
$$\boxed{V_{M_1} = -\frac{\sigma z}{2\varepsilon_0}}$$

si $z<0$: $V_{M_2} = \dfrac{\sigma}{2\varepsilon_0}\displaystyle\int dz + C_2$

$V_{M_2} = \dfrac{\sigma z}{2\varepsilon_0} + C_2$, $V_{M_2}(0) = 0 \Rightarrow C_2 = 0$

$$\boxed{V_{M_2} = \frac{\sigma z}{2\varepsilon_0}}$$

2°/



$\underline{z>d}$: $\vec{E}_{M_1} = \vec{E}_{M_1}(P_1) - \vec{E}_{M_1}(P_2)$

$\vec{E}_{M_1} = \frac{\sigma}{2\varepsilon_0}\vec{k} - \frac{\sigma}{2\varepsilon_0}\vec{k} \Rightarrow \boxed{\vec{E}_{M_1} = \vec{0}}$

$\underline{0<z<d}$: $\vec{E}_{M_2} = \vec{E}_{M_2}(P_1) + \vec{E}_{M_2}(P_2)$

$\vec{E}_{M_2} = \frac{\sigma}{2\varepsilon_0}\vec{k} + \frac{\sigma}{2\varepsilon_0}\vec{k} \Rightarrow \boxed{\vec{E}_{M_2} = \frac{\sigma}{\varepsilon_0}\vec{k}}$

$\underline{z<0}$: $\vec{E}_{M_3} = -\vec{E}_{M_3}(P_1) + \vec{E}_{M_3}(P_2)$

$\boxed{\vec{E}_{M_3} = \vec{0}}$

$dV_M = -E_M\,dz$

$\underline{z \geq d}$: $V_{M_1} = -\int 0\,dz + C_1$

$V_{M_1} = C_1$

$\underline{0 \leq z \leq d}$: $V_{M_2} = -\frac{\sigma}{\varepsilon_0}\int dz + C_2$

$V_{M_2} = -\frac{\sigma z}{\varepsilon_0} + C_2$

$\underline{z \leq 0}$: $V_{M_3} = -\int 0\,dz + C_3$

$V_{M_3} = C_3$

Calcul des Cts C_1, C_2 et C_3.

$V_M(0) = 0$.

$V_M(0 \leq z \leq d) = -\frac{\sigma z}{\varepsilon_0} + C_2 = V_{M_2}$

$V_M(0 \leq z \leq d) = -\frac{\sigma \times 0}{\varepsilon_0} + C_2 = 0$

Continuité du potentiel :

$V_{M_3}(0) = V_{M_2}(0)$.

$V_{M_1}(d) = V_{M_2}(d)$

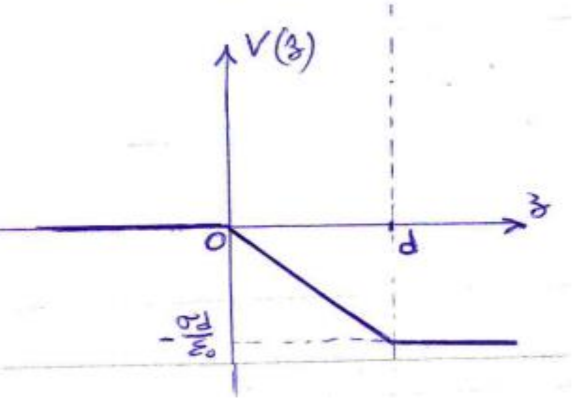
$-\frac{\sigma d}{\varepsilon_0} = C_1 \quad , \quad C_3 = 0.$

Conclusion :

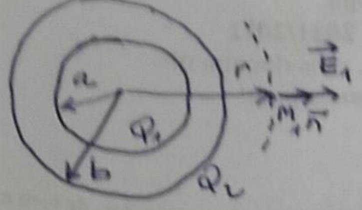
$$E_M = \begin{cases} \vec{0} & z>d \\ \frac{\sigma}{\varepsilon_0}\vec{k} & 0<z<d \\ \vec{0} & z<0 \end{cases}$$

$$V_M = \begin{cases} -\frac{\sigma d}{\varepsilon_0} & z \geq d \\ -\frac{\sigma z}{\varepsilon_0} & 0 \leq z \leq d \\ 0 & z \leq 0 \end{cases}$$

3°/



Exercice 6.



$$\oiint_{S_G} \vec{E}\cdot\vec{n}\,dS_G = \frac{1}{\varepsilon_0}\sum Q_{int}$$

$$\oiint_{S_G} \vec{E}\cdot\vec{n}\,dS_G = \frac{1}{\varepsilon_0}\sum Q_{int}$$

1°/ Champ $\vec{E}_M$?

a) Au pt M_1 : $r \in [b, +\infty[$

$$E_1 \times 4\pi r^2 = \frac{1}{\varepsilon_0}(Q_1 + Q_2)$$

$$Q_1 = \frac{4}{3}\pi a^3 \rho$$

$$Q_2 = 4\pi b^2 \sigma$$

$$\boxed{E_1 = \frac{1}{\varepsilon_0 r^2}\left(\frac{a^3\rho}{3} + b^2\sigma\right)}$$

b°/ Au pt M_2 : $r \in [a, b]$

$$E_2 \times 4\pi r^2 = \frac{Q_1}{\varepsilon_0}.$$

$$\boxed{E_2 = \frac{a^3\rho}{3\varepsilon_0 r^2}}$$

c/ Au pt M_3 : $r \in]0, a]$

$$E_3 \times 4\pi r^2 = \frac{Q'_1}{\varepsilon_0}$$

$$Q'_1 = \frac{4}{3}\pi r^3 \rho$$

$$\boxed{E_3 = \frac{\rho r}{3\varepsilon_0}}$$

2°/ Potentiel V_M ? $\quad dV = -\vec{E}\cdot\vec{dr}$

a) Au pt M_1 :

$$V_1 = -\int E_1\,dr = -\frac{1}{\varepsilon_0}\left(\frac{a^3\rho}{3} + b^2\sigma\right)\int \frac{dr}{r^2}$$

$$V_1 = +\frac{1}{\varepsilon_0 r}\left(\frac{a^3\rho}{3} + b^2\sigma\right) + C_1$$

$$V_1(\infty) = 0 \Rightarrow C_1 = 0.$$

$$\boxed{V_1 = +\frac{1}{\varepsilon_0 r}\left(\frac{a^3\rho}{3} + b^2\sigma\right)}$$

b) Au pt M_2 :

$$V_2 = -\int E_2\,dr = -\frac{a^3\rho}{3\varepsilon_0}\int \frac{dr}{r^2}$$

$$V_2 = \frac{a^3\rho}{3\varepsilon_0 r} + C_2$$

Pour Calculer C_2, on utilise la continuité du Potentiel.

$$\lim_{r\to b} V_2 = \lim_{r\to b} V_1$$

$$\Rightarrow C_2 = \frac{b\sigma}{\varepsilon_0}$$

d'où :

$$\boxed{V_2 = \frac{a^3\rho}{3\varepsilon_0 r} + \frac{b\sigma}{\varepsilon_0}}$$

c) Au pt $M_3 \in$

$$V_3 = -\int E_3\, dr = -\frac{\rho}{3\varepsilon_0}\int r\, dr$$

$$V_3 = -\frac{\rho r^2}{6\varepsilon_0} + C_3$$

$$\lim_{r \to a} V_3 = \lim_{r \to a} V_2$$

$$C_3 = \frac{\rho a^2}{2\varepsilon_0} + \frac{b\sigma}{\varepsilon_0}.$$

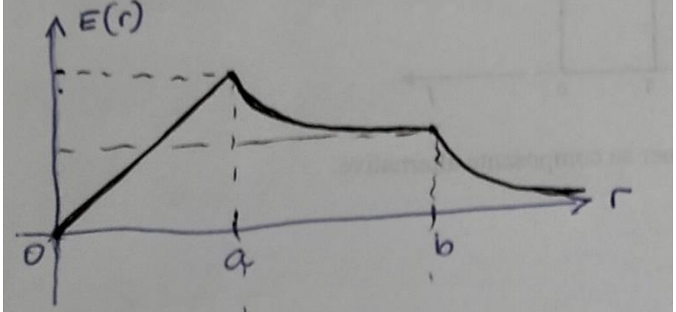
$$\boxed{V_3 = -\frac{\rho r^2}{6\varepsilon_0} + \frac{\rho a^2}{2\varepsilon_0} + \frac{b\sigma}{\varepsilon_0}}$$

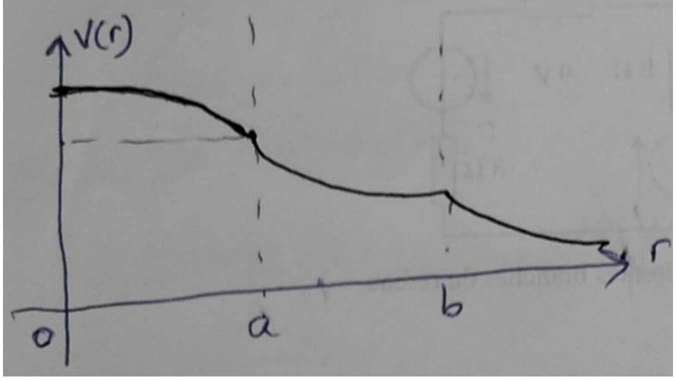
3/ $b = 2a$, $\rho = 3\sigma/a$

$$\boxed{E_1 = \frac{5a^2\sigma}{\varepsilon_0 r^2}} ; \quad \boxed{V_1 = \frac{5a^2\sigma}{\varepsilon_0 r}}$$

$$\boxed{E_2 = \frac{a^2\sigma}{\varepsilon_0 r^2}} ; \quad \boxed{V_2 = \frac{a^2\sigma}{\varepsilon_0 r} + \frac{2a\sigma}{\varepsilon_0}}$$

$$\boxed{E_3 = \frac{\sigma r}{\varepsilon_0 a}} ; \quad \boxed{V_3 = -\frac{\sigma r^2}{2\varepsilon_0 a} + \frac{7\sigma a}{2\varepsilon_0}}$$





Le Responsable de la matière : H. Latelli