

Formal Verification and Specification  
Lab Session (TP) 011. **RegulMach:**

Implement in Atelier B the Reglum machine presented in the course.

2. **Eclude:**

Define an abstract machine of a the Ecludian division

3. **PGCD:**

Define an abstract machine of a the PGCD problem and its refinement in Method B

4. **8-bit calculator:**

Define a an abstract machine and its implementation for 8-bit calculator. The calculator has two registers for storing values, one i principal ( $Rp$ ), and other secondary ( $Rs$ ). The calculator performs five(05) main operations:

- *Increment*: which increments the principle register
- *Decrement*: which decrements the principle register
- *storeRp*: stores a value into  $Rp$
- *storeRs*: stores a value into  $Rs$
- *comp*: which compares between  $Rs$  and  $Rp$

# Answers

## 1. RegulMach:

Implement in Atelier B the Reglum machine presented in the course.

**Answer:** See course.

## 2. Eclude:

Define an abstract machine of a the Ecludian division

```
MACHINE
Eclld
OPERATIONS
reste, quot <-- calculReste (divis , divid ) =
PRE
  divis : NAT  $\wedge$  divid : NAT  $\wedge$  divis > 0
   $\wedge$  divis <= divid
THEN
ANY vq, vr WHERE
  vq : NAT
   $\wedge$  vr : NAT
   $\wedge$  divid = vq*divis + vr
THEN
  quot := vq
  || reste := vr
END
END
END
```

Figure 1: Abstract machine Eclud

**Answer:**

## 3. PGCD:

Define an abstract machine of a the PGCD problem and its refinement in Method B

```
MACHINE
pgcd1 /* PGCD of two integers*/
/* pgcd(x,y) is d | x mod d = 0  $\wedge$  y mod d = 0
 $\wedge$   $\forall$  other divisors dx d > dx
 $\wedge$   $\forall$  other divisors dy d > dy */
OPERATIONS
rr <-- pgcd(xx,yy) = /* specification of pgcd */
PRE
  xx : INT  $\wedge$  xx >= 1  $\wedge$  xx < MAXINT
   $\wedge$  yy : INT  $\wedge$  yy >= 1  $\wedge$  yy < MAXINT
THEN
ANY dd WHERE
  dd : INT
   $\wedge$  (xx - (xx/dd)*dd) = 0 /* d is a divisor of x */
   $\wedge$  (yy - (yy/dd)*dd) = 0 /* d is a divisor of y */
  /* and the other common divisors are < d */
   $\wedge$  !dx.((dx : INT  $\wedge$  dx < MAXINT
   $\wedge$  (xx - (xx/dx)*dx) = 0  $\wedge$  (yy - (yy/dx)*dx)=0) => dx < dd)
THEN
END
END
END
```

Figure 2: Abstract machine PGCD

```

IMPLEMENTATION pgcd_i
REFINES pgcd1

OPERATIONS
rr <-- pgcd (xx, yy) = /* operation refined */
BEGIN
VAR cd, rx, ry, cr IN
cd := 1;
WHILE ( cd < xx & cd < yy) DO
  rx := xx - (xx/cd)*cd ; ry := yy - (yy/cd)*cd;
  IF (rx = 0 & ry = 0)
    THEN /* cd divise x et y, possible PGCD */
      cr := cd /* possible rr */
    END;
  cd := cd + 1 ; /* look for bigger */
INVARIANT
xx : INT & yy : INT & rx : INT & rx < MAXINT
& ry : INT & ry < MAXINT & cd < MAXINT
& xx = cr*(xx/cr) + rx & yy = cr*(y/cr) + ry
VARIANT
  xx - cd
END
END
END
END

```

Figure 3: Implementation PGCD

**Answer:**

#### 4. 8-bit calculator:

Define an abstract machine and its implementation for 8-bit calculator. The calculator has two registers for storing values, one principal ( $R_p$ ), and other secondary ( $R_s$ ). The calculator performs five(05) main operations:

- *Increment*: which increments the principle register
- *Decrement*: which decrements the principle register
- *storeRp*: stores a value into  $R_p$
- *storeRs*: stores a value into  $R_s$
- *comp*: which compares between  $R_s$  and  $R_p$

```

MACHINE Calculette8
VARIABLES rp, /* registre principal */ rs /* registre secondaire */
INVARIANT rp : NAT & rs : NAT
  & 0 <= rp & rp <= 255 /* 8 bits */
  & 0 <= rs & rs <= 255 /* 8 bits */
INITIALISATION rp :=0 || rs := 0
OPERATIONS
incl = /* incrementer le registre principal de 1 */
PRE rp +1 <= 255 THEN
  rp := rp + 1
END;
deci = /* decrements le registre principal de 1 */
PRE rp - 1 >= 0 THEN
  rp := rp - 1
END;
storeRP(vv) =
  PRE vv : NAT & 0<= vv & vv <= 255 THEN
    rp := vv
END;
storeRS(val) =
  PRE val : NAT & val <= 0 & val <= 255 THEN
    rs := val
END;
res <-- cmp = /* comparer les contenus de RP et RS */
res := bool(rs = rp);
res <-- getRP = /* recuperer la valeur de RP */
res := rp ;
res <-- getRS = /* recuperer la valeur de RS */
res := rs
END

```

Figure 4: Abstract machine Claculette\_8

```

IMPLEMENTATION Calculette8_i REFINES Calculette8
CONCRETE_VARIABLES rp , rs
INITIALISATION rp := 0 ; rs := 0
OPERATIONS
incl = BEGIN rp := rp + 1 END;
decl = BEGIN rp := rp - 1 END;
storeRP ( vv ) = BEGIN rp := vv END;
storeRS ( val ) = BEGIN rs := val END;
res <-- cmp = res := bool ( rs = rp );
res <-- getRP = res := rp;
res <-- getRS = res := rs
END

```

Figure 5: Abstract machine Claculette\_8

**Answer:**