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Gauge theory of electroweak interactions

Théorie de jauge des interactions électrofaibles

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The standard Model

I-The electroweak mode

The electroweak mode is the second successfully unified model after the unification between electrical and magnetic interaction in the mode of Maxwell. Glashow, Weinberg and Salam released this model in 1967-1971. The electroweak mode is a local gauge theory for the gauge-unified group $G_{EW} \equiv SU(2)_L \otimes U(1)_Y$, where the notation L denote to left polarization while hypercharge $Y = B - L$ the deference between tow quantum numbers. The electric charge Q is linked to the weak isospin I and the hypercharge Y through the Gell-Mann –Nishijima relation:

$$Q = I_z + \frac{Y}{2} \tag{1}$$

For the weak interactions, the fermion field can represent by isospin doublet as follows:

$$\Psi_1 = \begin{pmatrix} \Psi_{\nu_e} \\ \Psi_e \end{pmatrix}_L \equiv \begin{pmatrix} \frac{1}{2}(1-\gamma_5)\Psi_{\nu_e} \\ \frac{1}{2}(1-\gamma_5)\Psi_e \end{pmatrix} = \begin{pmatrix} \nu_{eL} \\ e_L^- \end{pmatrix}, \tag{2}$$

$$\Psi_2 = \begin{pmatrix} \Psi_{\nu_\mu} \\ \Psi_\mu \end{pmatrix}_L \equiv \begin{pmatrix} \frac{1}{2}(1-\gamma_5)\Psi_{\nu_\mu} \\ \frac{1}{2}(1-\gamma_5)\Psi_\mu \end{pmatrix} = \begin{pmatrix} \nu_{\mu L} \\ \mu_L^- \end{pmatrix} \tag{3}$$

And

$$\Psi_3 = \begin{pmatrix} \Psi_{\nu_\tau} \\ \Psi_\tau \end{pmatrix}_L \equiv \begin{pmatrix} \frac{1}{2}(1-\gamma_5)\Psi_{\nu_\tau} \\ \frac{1}{2}(1-\gamma_5)\Psi_\tau \end{pmatrix} = \begin{pmatrix} \nu_{\tau L} \\ \tau_L^- \end{pmatrix} \tag{4}$$

The Fermi model, which is a current-current interaction, can describe the weak interaction using the Following Lagrangian density:

$$L_F = \frac{G_F}{\sqrt{2}} \sum_{\beta=1}^{3} j_{\beta\lambda}(x) j^{\beta\lambda}(x) \tag{5}$$

Where

$$j^{1\lambda}(x) = \overline{\Psi}_{\nu_e} \gamma^\lambda (1-\gamma_5) \Psi_{\nu_e} \tag{6}$$

$$j^{1\lambda}(x) = \overline{\Psi}_{\mu} \gamma^\lambda (1-\gamma_5) \Psi_{\nu_\mu} \tag{7}$$

And

$$j^{1\lambda}(x) = \overline{\Psi}_{\nu_\tau} \gamma^\lambda (1-\gamma_5) \Psi_{\tau} \tag{8}$$

For the doublet left $Y_L = -1$. For the electromagnetic interactions, the fermion field can represent by singlet fields as follows:

$$E_1 = \Psi_{eR} \equiv \frac{1}{2}(1+\gamma_5)\Psi_e, \tag{9}$$

$$E_2 = \Psi_{\mu R} \equiv \frac{1}{2}(1+\gamma_5)\Psi_\mu \tag{10}$$

And

$$E_3 = \Psi_{\tau R} \equiv \frac{1}{2}(1+\gamma_5)\Psi_\tau \tag{11}$$

In addition, the hypercharge right $Y_R = -2$. The gauge fields for the weak interactions $SU(2)_L$ is linked by $A_\mu^a(x)$, ($a = 1,2,3$) (we have three gauge fields) with coupling g while the gauge fields for the electromagnetic interactions $U(1)_Y$ is linked by $B_\mu(x)$ (only one gauge field) with coupling g'.

For scalar fields, we use the following complex doublet:

$$\varphi = \begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix} \tag{12}$$

With, the hypercharge $Y = +1$. The Lagrangian density terms for electroweak model can be represent by the following principle terms:

$$L_{GT} = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu}_a - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \tag{13}$$

$$L_{ST} = (D_\mu \varphi)^+ D^\mu \varphi - m^2 \varphi^+ \varphi - \lambda (\varphi^+ \varphi)^2 \tag{14}$$

$$L_{FP} = \sum_{i=1}^{3} i \overline{\Psi}_i \gamma^\mu D_\mu \Psi_i + \sum_{i=1}^{3} i \overline{E}_i \gamma^\mu D_\mu E_i \tag{15}$$

and

$$L_{Yu} = \left(\sum_{i,j}^{3} f_{ij} \overline{\Psi}_i E_j \varphi + h.c. \right) \tag{15}$$

The hyper-complex term (h. c.) added to the Lagrangian density to ensure that is a scalar quantity. With $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g \varepsilon^a_{bc} A^b_\mu A^c_\nu$ and $F_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$. The total Lagrangian density L_{EW} for electroweak model can be represent by:

$$L_{EW} = L_{GT} + L_{ST} + L_{TP} + L_{Yu} \tag{16}$$

Where $D_\mu = \partial_\mu - ig \frac{\tau^a}{2} A^a_\mu + ig' \frac{Y}{2} B_\mu$ is the covariant derivative operator, we have three-cases:

$$\begin{aligned} D_\mu \Psi_i &= \left(\partial_\mu - ig \frac{\tau^a}{2} A^a_\mu + i \frac{g'}{2} B_\mu \right) \Psi_i && \text{for} \quad Y = +1 \\ D_\mu E_i &= (\partial_\mu + ig' B_\mu) E_i && \text{for} \quad Y = 0 \\ D^\mu \varphi &= \left(\partial_\mu - ig \frac{\tau^a}{2} A^a_\mu - i \frac{g'}{2} B_\mu \right) \varphi && \text{for} \quad Y = -1 \end{aligned} \tag{17}$$

Where τ^a are the three usual Pauli matrices. Now, the generalization of Higgs mechanism to theories with non-abelian gauge symmetry allows writing:

$$D^{\mu}\varphi = \left(\partial_{\mu} - ig\frac{\tau^{a}}{2}A_{\mu}^{a}\right)\varphi$$
$$F_{\mu\nu}^{a} = \partial_{\mu}A_{\nu}^{a} - \partial_{\nu}A_{\mu}^{a} + g\varepsilon_{bc}^{a}A_{\mu}^{b}A_{\nu}^{c} \tag{18}$$
$$V(\varphi) = (D_{\mu}\varphi)^{+}D^{\mu}\varphi - m^{2}\varphi^{+}\varphi - \lambda(\varphi^{+}\varphi)^{2}$$

The physical vacuum corresponding to the expectation value having the following form:

$$\langle\varphi\rangle_{0} = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v\end{pmatrix} \qquad \text{with} \qquad v = \left(-\frac{m^{2}}{\lambda}\right)^{1/2} \tag{19}$$

Thus m^2 will be negative. If we define:

$$\langle\varphi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}\chi^{1}(x) + i\chi^{2}(x)\\ v + h(x) + i\chi^{3}(x)\end{pmatrix} \equiv \begin{pmatrix}\varphi_{1}^{'}(x)\\ \varphi_{2}^{'}(x)\end{pmatrix} \tag{20}$$

The covariant derivative term will generate a mass for the vector boson fields for A_{μ}^{a} as follows:

$$M = \frac{gv}{2} \tag{21}$$

Now, we generalize this procedure to the case of $G_{EW} \equiv SU(2)_{L} \otimes U(1)_{Y}$, thus we have for the physical vacuum corresponding to the expectation value having the following form:

$$\langle\varphi\rangle_{0} = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ \rho(x)\end{pmatrix} \tag{22}$$

The first step:

$$D^{\mu}\varphi = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ \partial_{\mu}\rho(x)\end{pmatrix} + \left\{-ig\frac{\tau^{a}}{2}A_{\mu}^{a} - i\frac{g'}{2}B_{\mu}\right\}\begin{pmatrix}0\\ \frac{\rho(x)}{\sqrt{2}}\end{pmatrix} \tag{23}$$

After straightforward calculations, we obtain:

$$D^{\mu}\varphi = \frac{1}{\sqrt{2}}\begin{pmatrix}-\frac{i}{2\sqrt{2}}g(A_{\mu}^{1} - iA_{\mu}^{2})\rho(x)\\ \frac{\partial_{\mu}\rho(x)}{\sqrt{2}} - \frac{i}{2\sqrt{2}}(-gA_{\mu}^{3} + g'B_{\mu})\rho(x)\end{pmatrix} \tag{24}$$

Which allows us to gives the following results:

$$\left(D_\mu\varphi\right)^+ D^\mu\varphi = \frac{1}{2}\left(\partial_\mu\rho(x)\right)\left(\partial^\mu\rho(x)\right) + \frac{1}{8}g^2\left(A^1_\mu - iA^2_\mu\right)^2\rho(x)^2 + \frac{1}{8}g^2\left(gA^3_\mu - g'B_\mu\right)^2\rho(x)^2 \tag{25}$$

We have observe one scalar field ($\rho(x) = h(x) + v$) and three-boson fields $A^a_\mu(x)$, ($a = 1,2,3$) and one boson $B_\mu(x)$. We are interest now for the following part of the quadratic Lagrangian density L^{quad}_{GT}:

$$L^{quad}_{GT} = -\frac{1}{4}\sum_{a=1}^{3}\left(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu\right)^2 - \frac{1}{4}\left(\partial_\mu B_\nu - \partial_\nu B_\mu\right)\left(\partial^\mu B^\nu - \partial^\nu B^\mu\right) + \frac{1}{8}g^2v^2\left(A^1_\mu - iA^2_\mu\right)^2\rho^2 + \frac{1}{8}v^2\left(gA^3_\mu - g'B_\mu\right)^2\rho(x)^2 \tag{26}$$

We introduce the following notations of the bosons fields as follows:

$$\begin{aligned} W_\mu &= \frac{1}{\sqrt{2}}\left(A^1_\mu - iA^2_\mu\right) \\ W^+_\mu &= \frac{1}{\sqrt{2}}\left(A^1_\mu + iA^2_\mu\right) \end{aligned} \tag{27}$$

We can expressed the term $\left(gA^3_\mu - g'B_\mu\right)^2$ as follows:

$$\left(gA^3_\mu - g'B_\mu\right)^2 = \begin{pmatrix} A^3_\mu & B_\mu \end{pmatrix}\begin{pmatrix} g^2 & -gg' \\ -gg' & g'^2 \end{pmatrix}\begin{pmatrix} A^3_\mu \\ B_\mu \end{pmatrix} \tag{28}$$

The orthogonal matrix and the Weinberg angle are defined as:

$$O = \begin{pmatrix} \frac{g}{\sqrt{g^2 + g'^2}} & -\frac{g'}{\sqrt{g^2 + g'^2}} \\ \frac{g'}{\sqrt{g^2 + g'^2}} & \frac{g}{\sqrt{g^2 + g'^2}} \end{pmatrix} \equiv \begin{pmatrix} \cos(\Theta_W) & -\sin(\Theta_W) \\ \sin s(\Theta_W) & \cos(\Theta_W) \end{pmatrix} \tag{29}$$

We also introduce two bosons fields as:

$$\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} \cos(\Theta_W) & -\sin(\Theta_W) \\ \sin(\Theta_W) & \cos(\Theta_W) \end{pmatrix}\begin{pmatrix} A^3_\mu \\ B_\mu \end{pmatrix} \tag{30}$$

Substituting Eqs. (30), (29), (28), (27) in (26) gives

$$L_{GT}^{quad} = -\frac{1}{2}\left(\partial_\mu h(x)\right)\left(\partial^\mu h(x)\right) - \frac{1}{2} W_{\mu\nu} W^{+\mu\nu} - \frac{1}{2} A_{\mu\nu} A^{\mu\nu} - \frac{1}{2} Z_{\mu\nu} Z^{\mu\nu} + \frac{1}{2}\mathbf{m}_h^2 h(x)^2 + \frac{1}{2}\mathbf{m}_W^2 W_\mu W^{+\mu} + \frac{1}{2}\mathbf{m}_Z^2 Z_\mu Z^\mu \tag{31}$$

Where

$$\begin{aligned} W_{\mu\nu} &= \partial_\mu W_\nu - \partial_\nu W_\mu \\ Z_{\mu\nu} &= \partial_\mu Z_\nu - \partial_\nu Z_\mu \\ A_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu \\ \mathbf{m}_h^2 &= -2m^2 \\ \mathbf{m}_Z^2 &= \frac{v^2}{4}\left(g^2 + g'^2\right) \\ \mathbf{m}_W^2 &= \frac{v^2}{4} g^2 \\ \mathbf{m}_A^2 &= 0 \end{aligned} \tag{32}$$

Thus, the quadratic Lagrangian density L_{GT}^{quad} describe the following particles:

1- One scalar real field known by the Higgs boson with spin-0 and mass $\mathbf{m}_h = \sqrt{-2m^2}$

2- One vectoriel real field known by Z_μ with spin-1 and mass $\mathbf{m}_z = \sqrt{\frac{v^2}{4}\left(g^2 + g'^2\right)}$ and null charge

3- One vectoriel real field known by the Higgs boson with spin-1 and mass $\mathbf{m}_A = 0$

4- One vectoriel complex field with mass $\mathbf{m}_w = \sqrt{\frac{v^2}{4} g^2} = vg/2$ and anti-vectoriel complex field with same mass

It should be noting that the Weinberg angle satisfies the relation $g'\cos(\Theta_W) = g\sin(\Theta_W)$. The fermionic part of the Lagrangian density gives:

$$f_{ij}\,\overline{\Psi}_i E_j \varphi \to \left(\overline{\Psi}_{\nu_e} \quad \overline{\Psi}_e\right)_L \begin{pmatrix} 0 \\ \frac{\rho(x)}{\sqrt{2}} \end{pmatrix} \Psi_{eR} = f_{ij}\,\overline{\Psi}_e \Psi_{eR} \frac{\rho(x)}{\sqrt{2}} \to f_{ij}\,\overline{\Psi}_e \Psi_{eR} \frac{v}{\sqrt{2}} + f_{ij}\,\overline{\Psi}_e \Psi_{eR} \frac{h(x)}{\sqrt{2}}$$

$$f_{ij}\,\overline{\Psi}_i E_j \varphi \to \left(\overline{\Psi}_{\nu_\mu} \quad \overline{\Psi}_\mu\right)_L \begin{pmatrix} 0 \\ \frac{\rho(x)}{\sqrt{2}} \end{pmatrix} \Psi_{\mu R} = f_{ij}\,\overline{\Psi}_\mu \Psi_{\mu R} \frac{\rho(x)}{\sqrt{2}} \to f_{ij}\,\overline{\Psi}_\mu \Psi_{\mu R} \frac{v}{\sqrt{2}} + f_{ij}\,\overline{\Psi}_\mu \Psi_{\mu R} \frac{h(x)}{\sqrt{2}}, \qquad (33)$$

$$f_{ij}\,\overline{\Psi}_i E_j \varphi \to \left(\overline{\Psi}_{\nu_\tau} \quad \overline{\Psi}_\tau\right)_L \begin{pmatrix} 0 \\ \frac{\rho(x)}{\sqrt{2}} \end{pmatrix} \Psi_{\tau R} = f_{ij}\,\overline{\Psi}_\tau \Psi_{\tau R} \frac{\rho(x)}{\sqrt{2}} \to f_{ij}\,\overline{\Psi}_\tau \Psi_{\tau R} \frac{v}{\sqrt{2}} + f_{ij}\,\overline{\Psi}_\tau \Psi_{\tau R} \frac{h(x)}{\sqrt{2}}$$

After the diagonalization of the matrix of masses, we obtain the fermionic masses as follows:

$$\mathbf{m}_e = \frac{v}{\sqrt{2}} F_1$$
$$\mathbf{m}_\mu = \frac{v}{\sqrt{2}} F_2 \qquad (34)$$
$$\mathbf{m}_\tau = \frac{v}{\sqrt{2}} F_3$$

And

$$\mathbf{m}_{\nu_e} = \mathbf{m}_{\nu_\mu} = \mathbf{m}_{\nu_\tau} = 0 \qquad (35)$$

For study the interaction between electromagnetic field and the fermionic system, we rewrite the covariant derivative as:

$$\begin{aligned} D_\mu &= \partial_\mu - ig\frac{\tau^a}{2} A_\mu^a + ig'\frac{Y}{2} B_\mu \\ &= \partial_\mu - ig\frac{\tau^a}{2}\left(\cos(\Theta_W) Z_\mu + \sin(\Theta_W) A_\mu\right) + ig'\frac{Y}{2}\left(\cos(\Theta_W) A_\mu - \sin(\Theta_W) Z_\mu\right) + f\left(A_\mu^1, A_\mu^1\right) \\ &= \partial_\mu - ieA_\mu\left(\frac{\tau^3}{2} + \frac{Y}{2}\right) + f\left(A_\mu^1, A_\mu^1, Z_\mu\right) \end{aligned} \qquad (36)$$

Where $f\left(A_\mu^1, A_\mu^1, Z_\mu\right)$ is different terms, which not interest with the electromagnetic interaction with fermionic fields. Here $e\left(\frac{\tau^3}{2} + \frac{Y}{2}\right)$ is just the charge values, which cratered this interaction. Now, the interactions electromagnetic with bosons can be observe in studying of the gauge term:

$$-\frac{1}{4}F^{a}_{\mu\nu}F^{\mu\nu}_{a} = -\frac{1}{4}\left(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g\varepsilon^a_{bc}A^b_\mu A^c_\nu\right)\left(\partial^\mu A^\nu_a - \partial^\nu A^\mu_a + g\varepsilon^{b'c'}_a A^\mu_{b'}A^\nu_{c'}\right)$$

$$= -\frac{1}{2}\left[\left(\partial_\mu - ieA_\mu\right)W^+_\nu - \left(\partial_\nu - ieA_\nu\right)W_\mu + ig\cos\left(\Theta_W\right)\left(Z_\mu W^+_\nu - Z_\nu W_\mu\right)\right] \quad (37)$$

$$\left[\left(\partial_\mu - ieA_\mu\right)W_\nu - \left(\partial_\nu - ieA_\nu\right)W^+_\mu + ig\cos\left(\Theta_W\right)\left(Z_\mu W^+_\nu - Z_\nu W\mu\right)\right] - \frac{1}{4}F^3_{\mu\nu}F^{\mu\nu}_3$$

II- The standard Model

The standard model include both the electroweak mode and the quantum chromodynamics. The local gauge theory for the electroweak mode $G_{EW} \equiv SU(2)_L \otimes U(1)_Y$ will be extend to become as follows:

$$G_{EW} \equiv SU(2)_L \otimes U(1)_Y \rightarrow G_{SM} \equiv SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \quad (38)$$

Where $SU(3)_C$ is non-abelian lie group with dimension equal 8, thus for this group we have eight generators or eight gauge boson fields known by gluons. As in $SU(2)_L$, in $SU(3)_C$, we have the following fermionic fields:

$$Q_1 = \begin{pmatrix} u_i \\ d_i \end{pmatrix}_L \equiv \begin{pmatrix} \frac{1}{2}\left(1-\gamma_5\right)\Psi_{u_i} \\ \frac{1}{2}\left(1-\gamma_5\right)\Psi_{d_i} \end{pmatrix}, \quad (39)$$

$$Q_2 = \begin{pmatrix} c_i \\ s \end{pmatrix}_L \equiv \begin{pmatrix} \frac{1}{2}\left(1-\gamma_5\right)\Psi_{c_i} \\ \frac{1}{2}\left(1-\gamma_5\right)\Psi_{s_i} \end{pmatrix} \quad (40)$$

And

$$Q_3 = \begin{pmatrix} t_i \\ b_i \end{pmatrix}_L \equiv \begin{pmatrix} \frac{1}{2}\left(1-\gamma_5\right)\Psi_{t_i} \\ \frac{1}{2}\left(1-\gamma_5\right)\Psi_{b_i} \end{pmatrix} \quad (41)$$

With $Y_{Q_i} = 1/3$. In addition to the following singlet fields:

$$\begin{cases} U_1 = \Psi_{u_i R} \equiv \frac{1}{2}(1+\gamma_5)\Psi_{u_i} \\ U_2 = \Psi_{c_i R} \equiv \frac{1}{2}(1+\gamma_5)\Psi_{c_i} \quad \text{with} \quad \mathrm{Y_U} = 4/3, \\ U_3 = \Psi_{t_i R} \equiv \frac{1}{2}(1+\gamma_5)\Psi_{t_i} \end{cases} \tag{42}$$

And

$$\begin{cases} D_1 = \Psi_{d_i R} \equiv \frac{1}{2}(1+\gamma_5)\Psi_{u_i} \\ D_2 = \Psi_{s_i R} \equiv \frac{1}{2}(1+\gamma_5)\Psi_{c_i} \qquad \text{with} \quad \mathrm{Y_D} = -2/3, \\ D_3 = \Psi_{b_i R} \equiv \frac{1}{2}(1+\gamma_5)\Psi_{t_i} \end{cases} \tag{43}$$

The indices $\mathrm{i} = 1,2,3$ is linked to the quarks colors. The covariant derivative operator in the electroweak model $D_\mu = \partial_\mu - ig\frac{\tau^a}{2}A_\mu^a + ig'\frac{Y}{2}B_\mu$ will be extends to become in the standard model as follows:

$$D_\mu = \partial_\mu - ig\frac{\tau^a}{2}A_\mu^a + ig'\frac{Y}{2}B_\mu \rightarrow D_\mu = \partial_\mu - ig_s\frac{\lambda^a}{2}G_\mu^a - ig\frac{\tau^a}{2}A_\mu^a + ig'\frac{Y}{2}B_\mu \tag{44}$$

Here G_μ^a are the eight gluons and g_s is the strongly coupling constant. The Lagrangian density terms for electroweak model:

$$L_{GT} = -\frac{1}{4}F_{\mu\nu}^a F_a^{\mu\nu} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \tag{13}$$

$$L_{ST} = (D_\mu\varphi)^+ D^\mu\varphi - m^2\varphi^+\varphi - \lambda(\varphi^+\varphi)^2 \tag{14}$$

$$L_{FP} = \sum_{i=1}^{3} i\overline{\Psi}_i\gamma^\mu D_\mu\Psi_i + \sum_{i=1}^{3} i\overline{E}_i\gamma^\mu D_\mu E_i \tag{15}$$

And

$$L_{Yu} = \left(\sum_{i,j}^{3} f_{ij}\,\overline{\Psi}_i E_j\varphi + h.c.\right) \tag{15}$$

Will be in the standard model as follows:

$$L_{GT}=-\frac{1}{4}G^a_{\mu\nu}G_a^{\mu\nu}-\frac{1}{4}F^a_{\mu\nu}F_a^{\mu\nu}-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} \quad (45)$$

$$L_{ST}=(D_\mu\varphi)^+D^\mu\varphi-m^2\varphi^+\varphi-\lambda(\varphi^+\varphi)^2 \quad (46)$$

$$L_{FP}=\sum_{i=1}^{3}i\overline{Q}_{Li}\gamma^\mu D_\mu Q_{Li}+\sum_{i=1}^{3}i\overline{U}_{iR}\gamma^\mu D_\mu U_{iR}+\sum_{i=1}^{3}i\overline{D}_{iR}\gamma^\mu D_\mu D_{iR}+\sum_{i=1}^{3}i\overline{\Psi}_i\gamma^\mu D_\mu\Psi_i+\sum_{i=1}^{3}i\overline{E}_i\gamma^\mu D_\mu E_i \quad (47)$$

And

$$L_{Yu}=\left(\overline{\Psi}_L M_L E_R\varphi+\overline{Q}_L M_D D_R\varphi+\overline{Q}_L M_U U_R\varphi+h.c.\right) \quad (48)$$

Where $G^a_{\mu\nu}=\partial_\mu G^a_\nu-\partial_\nu G^a_\mu+g_s f^a_{bc}G^b_\mu G^c_\nu$. The Lagrangian density contains new gauge term ($-\frac{1}{4}G^a_{\mu\nu}G_a^{\mu\nu}$) which represent the dynamic of gluons and self-interactions. In addition to three fermionic terms $\sum_{i=1}^{3}i\overline{Q}_{Li}\gamma^\mu D_\mu Q_{Li}$, $\sum_{i=1}^{3}i\overline{U}_{iR}\gamma^\mu D_\mu U_{iR}$ and $\sum_{i=1}^{3}i\overline{D}_{iR}\gamma^\mu D_\mu D_{iR}$, furthermore, we note that the new Yukawa term will be complicate term when compared with corresponding term in the electroweak model.

References

1- Glashow, S. L. (1961). Partial-symmetries of weak interactions. Nuclear Physics, 22(4), 579–588. doi : 10.1016/0029-5582(61)90469-2

2- Steven Weinberger (1967). A Model of Leptons. Physical Review Letters, 19(21), 1264–1266. doi:10.1103/physrevlett.19.1264

3- A. Salam and J. Strathdee, A renormalizable gauge model of lepton interactions, Nuovo Cim. 11A (1972).

4- Sven Bachmann, Champs Quantiques relativistes, Ecole Polytechnique Fédérale de Lausanne (2005).

5- D. Bailin and A. love, Introduction to gauge field theory, IOP Publishing Limited 1993, ISBN 0-65274-817-5, London.

6- Paul H. Frampton, Gauge field theory, second edition, A WILEY-INTERSCIENCE PUBLICATION (1999).

7- Rabindra N. Mohapatra, Unification and Supersymmetry: The frontiers of Quark-Lepton Physics, Third edition, Springer, USA (2002).

8- P. J. Mulders, Introduction to quantum field theory, vrije universities Amsterdam (2005)