

### Terminologie et definitions:

Elasticity is an elegant and fascinating subject that deals with determination of the stress, strain, and displacement distribution in an elastic solid under the influence of external forces.

Elasticity theory establishes a mathematical model of the deformation problem, and this requires mathematical knowledge to understand the formulation and solution procedures.

The concept of the elastic force-deformation relation was first proposed by Robert Hooke in 1678. However, the major formulation of the mathematical theory of elasticity was not developed until the 19th century. In 1821 Navier presented his investigations on the general equations of equilibrium, and this was quickly followed by Cauchy who studied the basic elasticity equations and developed the notation of stress at a point.

#### Scalar, Vector, Matrix, and Tensor Definitions

Elasticity theory is formulated in terms of many different types of variables that are either specified or sought at spatial points in the body under study. Some of these variables are scalar quantities, representing a single magnitude at each point in space. Common examples include the material density  $\rho$  and material moduli such as Young's modulus  $E$ , Poisson's ratio  $\nu$ , or the shear modulus  $\mu$ . Other variables of interest are vector quantities that are expressible in terms of components in a two- or three-dimensional coordinate system. Examples of vector variables are the displacement and rotation of material points in the elastic continuum.

Formulations within the theory also require the need for matrix variables, which commonly require more than three components to quantify. Examples of such variables include stress and strain. As shown in subsequent chapters, a three-dimensional formulation requires nine components (only six are independent) to quantify the stress or strain at a point. For this case, the variable is normally expressed in a matrix format with three rows and three columns.

To summarize this discussion, in a three-dimensional Cartesian coordinate system, scalar, vector, and matrix variables can thus be written as follows:

$$\begin{aligned} \text{mass density scalar} &= \rho \\ \text{displacement vector} & \mathbf{u} = u\mathbf{e}_1 + v\mathbf{e}_2 + w\mathbf{e}_3 \\ \text{stress matrix} &= [\boldsymbol{\sigma}] = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{bmatrix} \end{aligned}$$

where  $\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3$  are the usual unit basis vectors in the coordinate directions. Thus, scalars, vectors, and matrices are specified by one, three, and nine components, respectively.

The formulation of elasticity problems not only involves these types of variables, but also incorporates additional quantities that require even more components to characterize. Because of this, most field theories such as elasticity make use of a tensor formalism using index notation.

This enables efficient representation of all variables and governing equations using a single standardized scheme. The tensor concept is defined more precisely in a later section, but for now we can simply say that scalars, vectors, matrices, and other higher-order variables can all be represented by tensors of various orders. We now proceed to a discussion on the notational rules of order for the tensor formalism. Additional information on tensors and index notation can be found in many texts such as Goodbody (1982) or Chandrasekharaiah and Debnath (1994).

Elasticity: élasticité: مرونة	Torsion: torsion: ثني
Matrix: matrice: مصفوفة	Complex :complexe: مركبة
Vector: vecteur: شعاع	Variable: variable: متغير
Scalar: scalaire: سلمى	Methods: méthode : طريقة
Stress: contrainte: اجهاد	Boundary Conditions: conditions aux limites: الشروط الحدية
Strain: déformation: تشوه	
Displacement: déplacement: انتقال	
Tensile: traction: شد	
Flexure or Bending: flexion: انحناء	
Fatigue: fatigue: كلل	
Strength: ténacité: صلادة	
Hardness: dureté: صلابة	
Yield stress: limite d'élasticité: حد المرونة	
Ultimate stress: résistance mécanique: حد الاجهاد الأعظمي	
Young's modulus E: module de Young: معامل يونغ	
Shear modulus: module de cisaillement: معامل القص	
Poisson's ratio $\nu$ : coefficient de poisson: معامل بواسون	
Equilibrium Equations: équations d'équilibre: معادلات التوازن	
Linear Elastic Materials—Hooke's Law: المرونة الخطية	

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