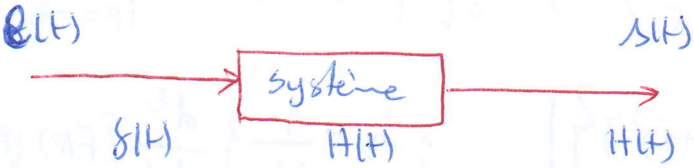
CHAPITRE 04 : produit de convolution

4-1 reponse impulsionnelle :



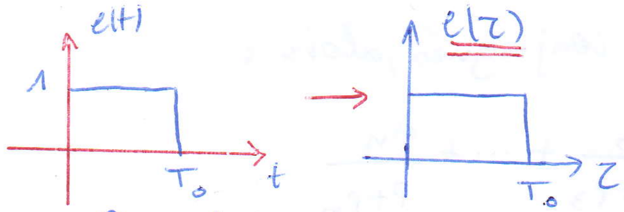
si e(t) est une entrée qlq : la sortie s(t) est donnée par :

$$s(t) = e(t) * h(t) = \int_{-\infty}^{+\infty} e(\tau)\, h(t-\tau)\, d\tau$$

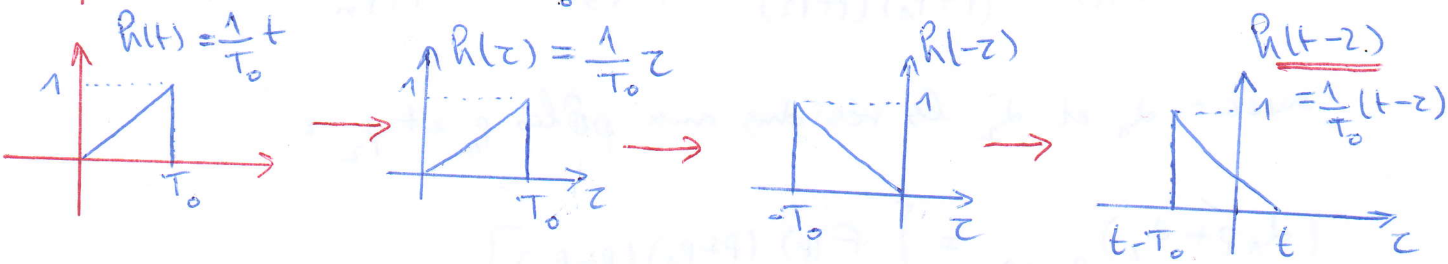
$*$: Produit de convolution

Exemple : calculer la sortie s(t) :

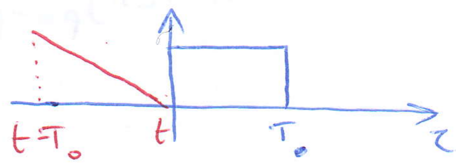
$t - T_0 \leq \tau \leq t$



$$\begin{cases} e(\tau) = 1 & 0 \leq \tau \leq T_0 \\ e(\tau) = 0 & \text{si non} \end{cases} \qquad \begin{cases} h(t-\tau) = \frac{1}{T_0}(t-\tau) \\ h(t-\tau) = 0 & \text{sinon} \end{cases}$$

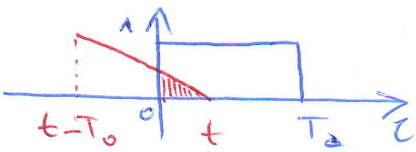


① 1ère cas $t \leq 0$



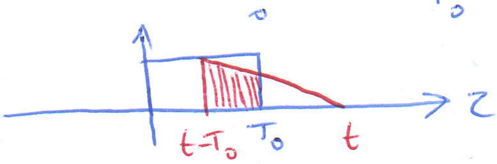
$\Rightarrow s(t) = 0$

② 2ème cas $0 < t \leq T_0$



$$s(t) = \int_0^t e(\tau) \cdot h(t-\tau)\, d\tau = \int_0^t 1 \cdot \frac{1}{T_0}(t-\tau)\, d\tau = \frac{t^2}{2T_0}$$

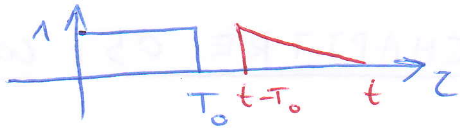
③ 3ème cas



$0 < t - T_0 \leq T_0$

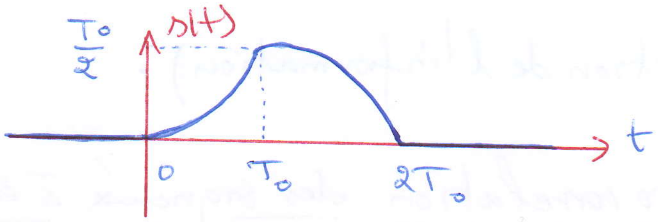
$$\Rightarrow T_0 < t \leq 2T_0 \Rightarrow s(t) = \int_{t-T_0}^{T_0} 1 \cdot \frac{1}{T_0}(t-\tau)\, d\tau = \frac{-t^2}{2T_0} + t$$

④ $4^{\text{ème}}$ cas



$$t - T_0 > T_0 \Rightarrow t > 2T_0 \Rightarrow s(t) = 0$$

Recap :



$$s(t) = \begin{cases} 0 & t < 0 \\ \dfrac{t^2}{2T_0} & 0 < t \leq T_0 \\ \dfrac{-t^2}{2T_0} + t & T_0 < t \leq 2T_0 \\ 0 & t > 2T_0 \end{cases}$$

4-2 propriétés :

① commutativité : $e(t) * h(t) = h(t) * e(t)$

② associativité : $s_1(t) * (s_2(t) * s_3(t)) = (s_1(t) * s_2(t)) * s_3(t)$

③ distributivité : $s_1(t) * (s_2(t) + s_3(t)) = s_1(t) * s_2(t) + s_1(t) * s_3(t)$

④ Derivatiation de la convolution :

$$\frac{d}{dt}[s_1(t) * s_2(t)] = s_1(t) * \frac{ds_2(t)}{dt} + \frac{ds_1(t)}{dt} * s_2(t)$$

4-3 théorème de la convolution :

soit à calculer $TF\{s(t) = e(t) * h(t)\}$

$$S(f) = \int_{-\infty}^{+\infty} s(t)\, e^{-j2\pi f t}\, dt = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} e(\tau)\, h(t-\tau)\, d\tau \cdot e^{-j2\pi f t}\, dt$$

$$S(f) = \int_{-\infty}^{+\infty} e(\tau) \underbrace{\int_{-\infty}^{+\infty} h(t-\tau)\, e^{-j2\pi f t}\, dt}_{H(f)\, e^{-j2\pi f \tau}}\, d\tau = \int_{-\infty}^{+\infty} e(\tau)\, H(f)\, e^{-j2\pi f \tau}\, d\tau$$

$$= H(f) \underbrace{\int_{-\infty}^{+\infty} e(\tau)\, e^{-j2\pi f \tau}\, d\tau}_{E(f)} = E(f) \cdot H(f)$$

$$\boxed{S(f) = E(f) \cdot H(f) \Longleftrightarrow s(t) = e(t) * h(t)}$$