

TD N°3 : ALGÈBRE DE BOOLE

Exercice 1.

Soient les expressions booléennes suivantes.

$$F(A, B, C) = (A + \bar{B}).(\bar{A} + B).(\bar{A} + C)$$

$$F(A, B, C) = (A.B + \bar{A}.C).(\bar{A}.\bar{B} + A.\bar{C})$$

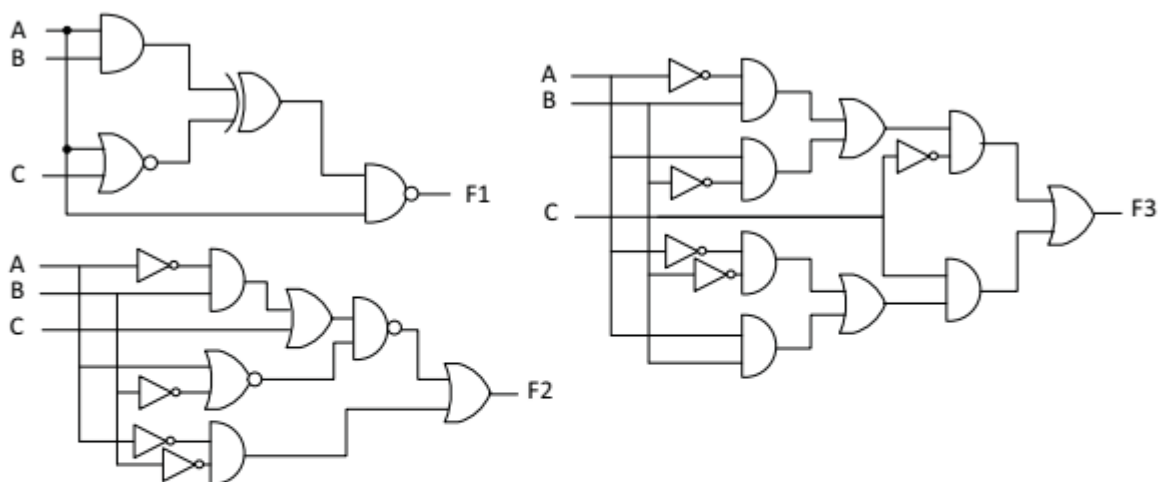
$$F(A, B, C) = \overline{\overline{(A + \bar{B}.C)}.(\bar{B} + \bar{C})} + \bar{A} + \bar{B}.\bar{C}$$

Pour chacune de ces expressions.

1. Tracer la table de vérité correspondante.
2. Tracer le logigramme correspondant.
3. Déterminer la 1^{ière} et la 2^{ème} forme canonique (F.C) à partir de la table de vérité.
4. Déterminer la 1^{ière} et la 2^{ème} F.C en utilisant les règles de l'algèbre de Boole.

Exercice 2.

1. Déterminer les expressions logiques correspondant aux logigrammes suivants.



2. En se basant sur les règles de l'algèbre de Boole. Montrer que l'on a :

$$\bar{A}.B + A.\bar{B} = (A + B).(\bar{A} + \bar{B})$$

$$A.B + \bar{A}.C + B.C = A.B + \bar{A}.C$$

$$A.B + B.C + A.C + A.\bar{B}.\bar{C} + \bar{A}.B.\bar{C} + \bar{A}.\bar{B}.C = A + B + C$$

Exercice 3.

1. Simplifier les expressions suivantes en utilisant les règles de l'algèbre de Boole.

$$F(A, B, C) = (A + B + C).(\bar{A} + B + C) + A.B + B.C$$

$$F(A, B, C) = \overline{\overline{A + B} + \overline{A + C} + \overline{A + C}}$$

$$F(A, B, C, D) = (\bar{A}.B + A.B + A.\bar{B}).(C.\bar{D} + \bar{C}.\bar{D}) + \bar{C}.D.(\bar{A}.B + A.B)$$

2. Simplifier les expressions suivantes en utilisant la méthode de karnaugh.

$$F(A, B, C) = (A + \bar{B} + C).(A + B + \bar{C}).(\bar{A} + B + \bar{C})$$

$$F(A, B, C) = \bar{A} + A.B + A.\bar{B}.C + A.\bar{B}.C.D$$

$$F(A, B, C, D) = \bar{A}.\bar{B}.\bar{D} + \bar{A}.\bar{C}.\bar{D} + \bar{A}.B.C.\bar{D} + A.B.D + \bar{B}.\bar{C}.\bar{D} + A.\bar{B}.C.\bar{D}$$

Exercice 4.

1. Simplifier les expressions suivantes en utilisant les tables de karnaugh à 5 variables.

$$F(A, B, C, D, E) = \sum(0, 1, 2, 4, 8, 10, 12, 14, 16, 17, 18, 20, 24, 26, 27, 30, 31)$$

$$F(A, B, C, D, E) = \sum(0, 1, 4, 5, 8, 9, 10, 16, 17, 20, 21, 24, 26, 28, 29, 30, 31)$$

$$F(A, B, C, D, E) = \sum(0, 1, 2, 3, 4, 5, 8, 9, 10, 12, 14, 16, 17, 18, 19, 24, 25, 26, 28, 29, 30)$$

2. Simplifier les expressions suivantes en utilisant les tables de karnaugh à 6 variables.

$$F(A, B, C, D, E, F) = \sum(2, 3, 9, 13, 16, 18, 24, 25, 29, 34, 37, 41, 45, 48, 50, 53, 56, 57, 61)$$

$$F(A, B, C, D, E, F) = \sum(0, 1, 2, 3, 9, 11, 13, 16, 17, 18, 24, 25, 29, 32, 33, 34, 36, 37, 41, 45, 48, 49, 50, 51, 53, 55, 57, 61)$$

$$F(A, B, C, D, E, F) = \sum(2, 3, 6, 7, 8, 12, 13, 14, 17, 19, 21, 23, 25, 27, 28, 29, 30, 32, 33, 34, 35, 40, 44, 46, 49, 51, 53, 55, 57, 59, 61, 62, 63)$$

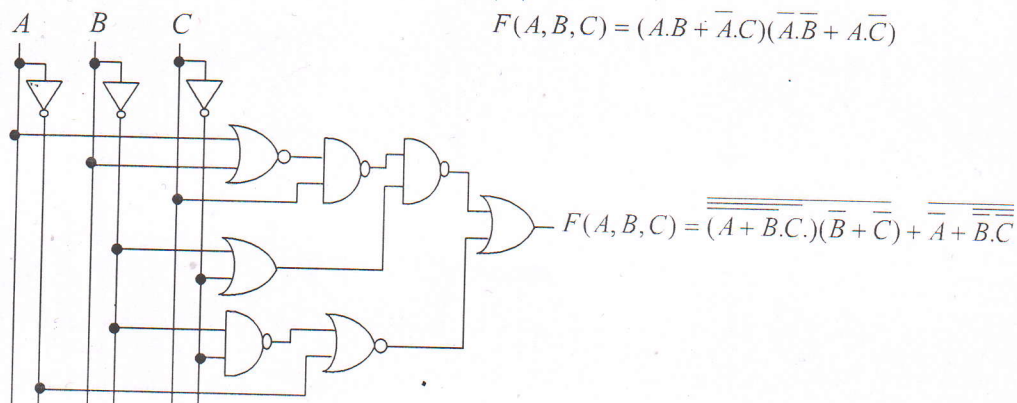
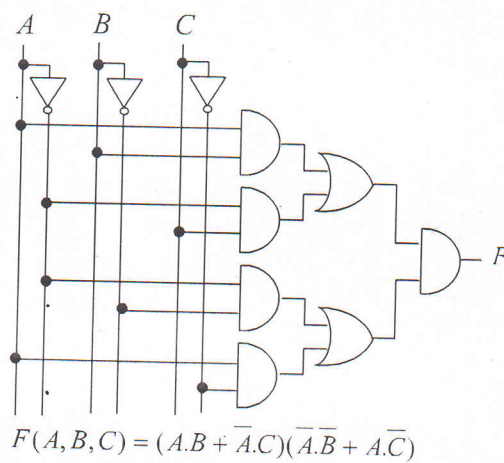
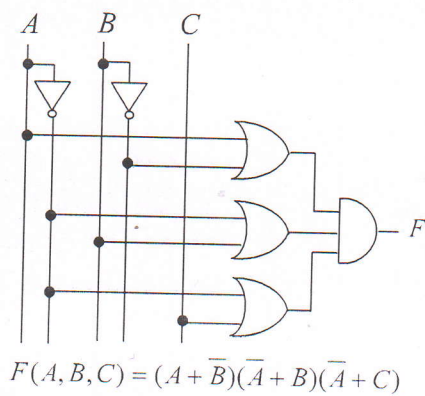
CORRIGÉ DE LA SERIE DE TD N° 3

Exercice 1.

1. Tracer la table de vérité correspondante.

A	B	C	$(A + \bar{B}) \cdot (\bar{A} + B) \cdot (\bar{A} + C)$	$(A \cdot B + \bar{A} \cdot C) \cdot (\bar{A} \cdot \bar{B} + A \cdot \bar{C})$	$\overline{\overline{(A + \bar{B} \cdot C)} \cdot (\bar{B} + \bar{C})} + \overline{\bar{A} + \bar{B} \cdot \bar{C}}$
0	0	0	1	0	0
0	0	1	1	1	1
0	1	0	0	0	0
0	1	1	0	0	1
1	0	0	0	0	1
1	0	1	0	0	0
1	1	0	0	1	0
1	1	1	1	0	1

2. Tracer le logigramme correspondant.



3. Déterminer la 1^{ière} et la 2^{ième} forme canonique pour chacune de ces expressions.

$$(1) F(A, B, C) = (A + \overline{B}).(\overline{A} + B).(\overline{A} + C)$$

$$- 1^{\text{ière}} \text{ F.C : } F(A, B, C) = \overline{A}.\overline{B}.\overline{C} + \overline{A}.\overline{B}.C + A.B.C$$

$$- 2^{\text{ième}} \text{ F.C : } F(A, B, C) = (A + \overline{B} + C).(A + \overline{B} + \overline{C}).(\overline{A} + B + C).(\overline{A} + B + \overline{C}).(\overline{A} + \overline{B} + C)$$

$$(2) F(A, B, C) = (A.B + \overline{A}.C).(\overline{A}.\overline{B} + A.\overline{C})$$

$$- 1^{\text{ière}} \text{ F.C : } F(A, B, C) = \overline{A}.\overline{B}.C + A.B.\overline{C}$$

$$- 2^{\text{ième}} \text{ F.C : } F(A, B, C) = (A + B + C).(A + \overline{B} + C).(A + \overline{B} + \overline{C}).(\overline{A} + B + C).(\overline{A} + B + \overline{C}).(\overline{A} + \overline{B} + \overline{C})$$

$$(3) F(A, B, C) = \overline{\overline{A + B.C}}.(\overline{B + C}) + \overline{\overline{A + B.C}}$$

$$- 1^{\text{ière}} \text{ F.C : } F(A, B, C) = \overline{A}.\overline{B}.C + \overline{A}.B.C + A.\overline{B}.\overline{C} + A.B.C$$

$$- 2^{\text{ième}} \text{ F.C : } F(A, B, C) = (A + B + C).(A + \overline{B} + C).(\overline{A} + B + \overline{C}).(\overline{A} + \overline{B} + C)$$

Exercice 2.

1. Déterminer les expressions logiques correspondant aux logigrammes suivants.

$$F1 = \overline{(A.B \oplus A + B)}.A$$

$$F2 = \overline{(\overline{A}.B + C)}.(\overline{A + B}) + \overline{A}.\overline{B}$$

$$F3 = (\overline{A}.B + A.\overline{B}).\overline{C} + (\overline{A}.\overline{B} + A.B).C$$

2. En se basant sur les règles de l'algèbre de Boole. Montrer que l'on a :

$$\overline{A}.B + A.\overline{B} = (\overline{A} + A).(\overline{A} + \overline{B}).(A + B).(B + \overline{B}) = (A + B).(\overline{A} + \overline{B})$$

$$\begin{aligned} A.B + \overline{A}.C + B.C &= A.B + \overline{A}.C + B.C.(A + \overline{A}) = A.B + \overline{A}.C + A.B.C + \overline{A}.B.C \\ &= A.B.(1 + C) + \overline{A}.C.(1 + B) = A.B + \overline{A}.C \end{aligned}$$

$$\begin{aligned} A.B + B.C + A.C + A.\overline{B}.\overline{C} + \overline{A}.B.\overline{C} + \overline{A}.\overline{B}.C &= A.(B + \overline{B}.\overline{C}) + B.(C + \overline{C}.\overline{A}) + \\ C.(A + \overline{A}.\overline{B}) &= A.B + A.\overline{C} + B.C + B.\overline{A} + A.C + \overline{B}.C \\ &= B.(A + \overline{A}) + A.(C + \overline{C}) + C.(B + \overline{B}) = A + B + C \end{aligned}$$

Exercice 3.

1. Simplifier les expressions suivantes en utilisant les règles de l'algèbre de Boole.

$$\begin{aligned} F(A, B, C) &= (A + B + C).(\overline{A} + B + C) + A.B + B.C = [A.\overline{A} + (B + C)] + A.B + B.C \\ &= B + C + A.B + B.C = B.(1 + A) + C.(1 + B) = B + C \end{aligned}$$

$$\begin{aligned} F(A, B, C) &= \overline{\overline{A + B} + \overline{\overline{A} + \overline{C}}} + \overline{A + C} = (A + B).(\overline{A} + \overline{C}) + \overline{A}.\overline{C} \\ &= A.\overline{A} + A.\overline{C} + \overline{A}.B + B.\overline{C} + \overline{A}.\overline{C} = \overline{C}.(A + \overline{A} + B) + \overline{A}.B = \overline{A}.B + \overline{C} \end{aligned}$$

$$\begin{aligned}
 F(A, B, C, D) &= (\bar{A}.B + A.B + A.\bar{B}).(C.\bar{D} + \bar{C}.\bar{D}) + \bar{C}.D.(A.B + A.B) \\
 &= [B.(A + \bar{A}) + A.(B + \bar{B})].[\bar{D}.(C + \bar{C})] + \bar{C}.D.[B.(A + \bar{A})] \\
 &= (A + B).\bar{D} + B.\bar{C}.D = A.\bar{D} + B.(\bar{D} + D.\bar{C}) = A.\bar{D} + B.\bar{D} + B.\bar{C}
 \end{aligned}$$

2. Simplifier les expressions suivantes en utilisant la méthode de karnaugh.

$$\begin{aligned}
 F(A, B, C) &= (A + \bar{B} + C).(A + B + \bar{C}).(\bar{A} + B + \bar{C}) = B.C + \bar{B}.\bar{C} + A.\bar{C} \\
 &= B.C + \bar{B}.\bar{C} + A.B
 \end{aligned}$$

$$F(A, B, C) = \bar{A} + A.B + A.\bar{B}.C + A.\bar{B}.C.D = \bar{A} + B + C$$

$$\begin{aligned}
 F(A, B, C, D) &= \bar{A}.\bar{B}.\bar{D} + \bar{A}.\bar{C}.\bar{D} + \bar{A}.B.C.\bar{D} + A.B.D + \bar{B}.\bar{C}.\bar{D} + A.\bar{B}.C.\bar{D} \\
 &= \bar{A}.\bar{D} + \bar{B}.\bar{D} + A.B.D
 \end{aligned}$$

Exercice 4.

1. Simplifier les expressions suivantes en utilisant les tables de karnaugh à 5 variables.

$$\begin{aligned}
 F(A, B, C, D, E) &= \sum(0, 1, 2, 4, 8, 10, 12, 14, 16, 17, 18, 20, 24, 26, 27, 30, 31) \\
 &= \bar{C}.\bar{E} + A.B.D + \bar{B}.\bar{D}.\bar{E} + \bar{B}.\bar{C}.\bar{D} + \bar{A}.B.\bar{E}
 \end{aligned}$$

$$\begin{aligned}
 F(A, B, C, D, E) &= \sum(0, 1, 4, 5, 8, 9, 10, 16, 17, 20, 21, 24, 26, 28, 29, 30, 31) \\
 &= \bar{B}.\bar{D} + \bar{A}.\bar{C}.\bar{D} + B.\bar{C}.\bar{E} + A.B.C
 \end{aligned}$$

$$\begin{aligned}
 F(A, B, C, D, E) &= \sum(0, 1, 2, 3, 4, 5, 8, 9, 10, 12, 14, 16, 17, 18, 19, 24, 25, 26, 28, 29, 30) \\
 &= \bar{C}.\bar{D} + B.\bar{E} + \bar{B}.\bar{C} + \bar{A}.\bar{B}.\bar{D} + A.B.\bar{D}
 \end{aligned}$$

2. Simplifier les expressions suivantes en utilisant les tables de karnaugh à 6 variables.

$$\begin{aligned}
 F(A, B, C, D, E, F) &= \sum(2, 3, 9, 13, 16, 18, 24, 25, 29, 34, 37, 41, 45, 48, 50, 53, 56, 57, 61) \\
 &= C.\bar{E}.F + B.\bar{D}.\bar{E}.\bar{F} + A.D.\bar{E}.F + \bar{C}.\bar{D}.E.\bar{F} + \bar{A}.\bar{B}.\bar{C}.\bar{D}.E
 \end{aligned}$$

$$\begin{aligned}
 F(A, B, C, D, E, F) &= \sum(0, 1, 2, 3, 9, 11, 13, 16, 17, 18, 24, 25, 29, 32, 33, 34, 36, 37, 41, \\
 &\quad 45, 48, 49, 50, 51, 53, 55, 57, 61) \\
 &= \bar{C}.\bar{D}.\bar{F} + C.\bar{E}.F + A.B.\bar{C}.F + \bar{A}.\bar{B}.\bar{D}.F + A.\bar{B}.\bar{C}.\bar{E} + \bar{A}.B.\bar{D}.\bar{E}
 \end{aligned}$$

$$\begin{aligned}
 F(A, B, C, D, E, F) &= \sum(2, 3, 6, 7, 8, 12, 13, 14, 17, 19, 21, 23, 25, 27, 28, 29, 30, 32, 33, \\
 &\quad 34, 35, 40, 44, 46, 49, 51, 53, 55, 57, 59, 61, 62, 63) \\
 &= B.\bar{C}.F + A.B.F + B.\bar{D}.F + C.D.E.\bar{F} + \bar{A}.C.D.\bar{E} + \bar{B}.C.\bar{E}.\bar{F} + \\
 &\quad A.\bar{B}.\bar{C}.\bar{D} + \bar{A}.\bar{B}.\bar{C}.E
 \end{aligned}$$