Solution TD N°3
TRANSFORMEE DE FOURIER

Exercice 1 :

1. Sachant que : $\begin{cases} \text{TF}\{x(t)\ e^{j2\pi f_0 t}\} = x(f - f_0) \\ TF\{1\} = \delta(f) \end{cases}$

Donc : $\mathbf{TF}\{\ \boldsymbol{e^{j2\pi f_0 t}}\} = \boldsymbol{\delta(f - f_0)}$

2. $x(t) = cos(2\pi f_0 t) = \frac{e^{j2\pi f_0 t} + e^{-j2\pi f_0 t}}{2}$

$$\text{TF}\{\ x(t)\ \}$$
$$= \text{TF}\left\{\frac{1}{2}\left[e^{j2\pi f_0 t} + e^{-j2\pi f_0 t}\right]\right\}$$
$$= \frac{1}{2}\left[\text{TF}\{\ e^{j2\pi f_0 t}\ \} + \text{TF}\{\ e^{-j2\pi f_0 t}\ \}\right]$$
$$= \boldsymbol{\frac{1}{2}}\left[\boldsymbol{\delta(f - f_0) + \delta(f + f_0)}\right]$$

Exercice 2 :

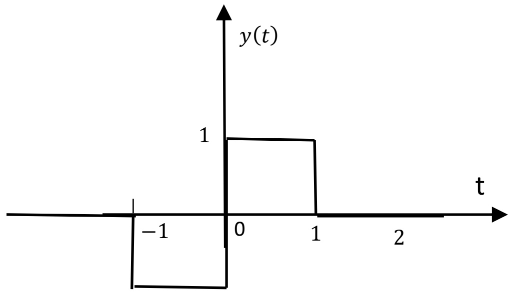
Calculer la TF des signaux suivants :

- $x(t) = e^{at}u(-t)$, a> 0

$$X(f) = \int_{-\infty}^{+\infty} x(t)\ e^{-j2\pi f t}dt = \int_{-\infty}^{+\infty} e^{at}u(-t)\ e^{-j2\pi f t}dt$$

$$= \int_{-\infty}^{0} e^{at}\ e^{-j2\pi f t}dt = \int_{-\infty}^{0} e^{(a-j2\pi f)t}\ dt = \left[\frac{e^{(a-j2\pi f)t}}{a - j2\pi f}\right]_{-\infty}^{0}$$
$$= \frac{1 - 0}{a - j2\pi f}$$
$$\Rightarrow \boldsymbol{X(f) = \frac{1}{a - j2\pi f}}$$

- $y(t) = -u(t + 1) + 2u(t) - u(t - 1)$



$$Y(f) = \mathrm{TF}\{\, y(t) \,\}$$

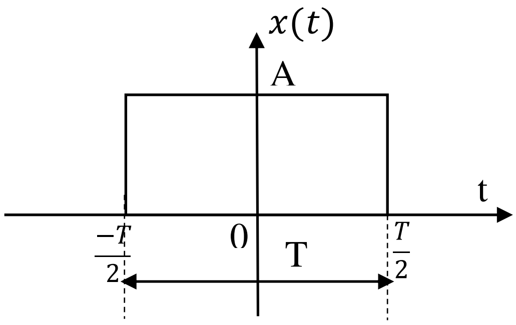
$$= \int_{-\infty}^{+\infty} y(t)\, e^{-j2\pi f t} dt = \int_{-1}^{0} (-1)\, e^{-j2\pi f t} dt + \int_{0}^{1} (1)\, e^{-j2\pi f t} dt$$

$$= \left[\frac{-e^{(-j2\pi f t)}}{-j2\pi f}\right]_{-1}^{0} + \left[\frac{e^{(-j2\pi f t)}}{-j2\pi f}\right]_{0}^{1}$$

$$= \left[\frac{1 - e^{(+j2\pi f)} - e^{(-j2\pi f)} + 1}{j2\pi f}\right] = \left[\frac{2 - 2\cos(2\pi f)}{j2\pi f}\right] = \boldsymbol{\frac{1 - 1\cos(2\pi f)}{j\pi f}}$$

Exercice 3

$x(t) = Arect(\frac{t}{T})$



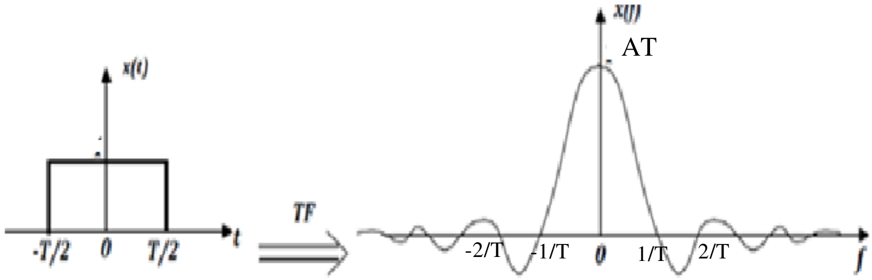
1. La TF

$$X(f) = \int_{-\infty}^{+\infty} x(t)\, e^{-j2\pi f t} dt = X(f) = \int_{-\frac{T}{2}}^{+\frac{T}{2}} Ae^{-j2\pi f t} dt$$

$$= \frac{-A}{j2\pi f}\left[e^{-j2\pi f t}\right]_{-\frac{T}{2}}^{\frac{T}{2}} = \frac{A}{\pi f}\left(\frac{e^{j\pi f T} - e^{-j\pi f T}}{2j}\right)$$

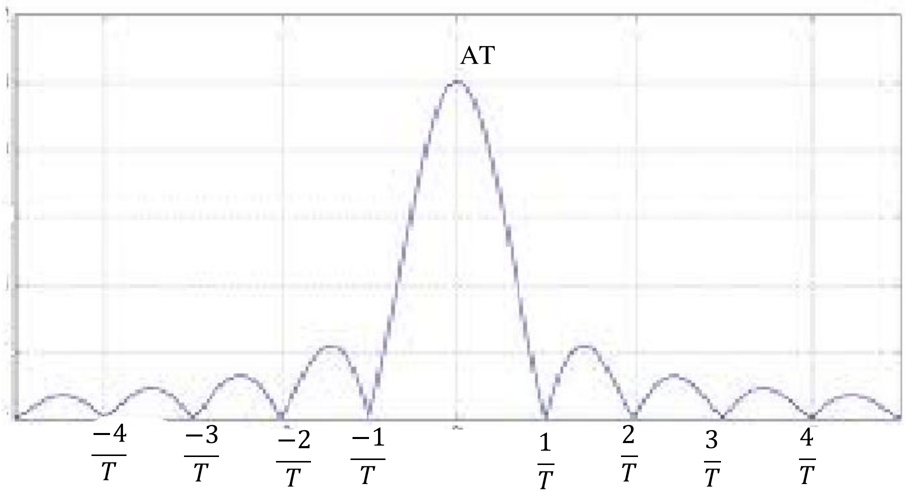
$$= \frac{A}{\pi f}\sin(\pi f T) = AT\,\frac{\sin(\pi f T)}{\pi f T} = \boldsymbol{ATsinc(fT)}$$

- **Représentation de $X(f)$:**



2. **Spectre d'amplitude :**

$$|\boldsymbol{X(f)}| = |ATsinc(fT)|$$



3. $\boldsymbol{E} = \int_{-\infty}^{+\infty} |\boldsymbol{x(t)}|^2 \boldsymbol{dt}$ **(temps)**

   $\boldsymbol{E} = \int_{-\infty}^{+\infty} |\boldsymbol{X(f)}|^2 \boldsymbol{df}$ **(Fréquence)**

$$E = \int_{-\infty}^{+\infty} |x(t)|^2 dt = \int_{-\frac{T}{2}}^{+\frac{T}{2}} A^2 dt = \boldsymbol{A^2T}$$

$$E = \int_{-\infty}^{+\infty} |X(f)|^2 df = \int_{-\infty}^{+\infty} (AT)^2 sinc^2(fT) df$$

$$= A^2T \int_{-\infty}^{+\infty} Tsinc^2(fT) df = \boldsymbol{A^2T}$$

Avec ; $\int_{-\infty}^{+\infty} Tsinc^2(fT) df = 1$ (ptopriété)

Donc le théorème de **Parseval** est vérifié :

$$E = \int_{-\infty}^{+\infty} |x(t)|^2 dt = \int_{-\infty}^{+\infty} |X(f)|^2 df$$