solution de Série 05 ;

Exercice 01 :

1/ $\Omega = \{PPP; PPf; PfP; Pff; fff; ffP; fPf; fPP\}$

2/ Loi de Probabilité de X :

X	0	1	2	3
P	1/8	3/8	3/8	1/8

L'espérance : $E(X) = \sum_{i=1}^{4} p_i x_i = 1,5$.

La variance : $V(X) = \sum_{i=1}^{n} p_i x_i^2 - (E(X))^2 = 0,75$

L'ecart type : $\sigma(X) = \sqrt{0,75}$.

3/ $P(X \leq 2) = \frac{1}{8} + \frac{3}{8} + \frac{3}{8} = \frac{7}{8}$

$P(X > 2) = \frac{1}{8}$.

4/ On a : $\begin{cases} E(b) = b \\ E(aX + bY) = aE(X) + bE(Y). \\ V(aX + bY) = a^2 V(X) + b^2 V(Y). \end{cases}$

$Z = 2X - 1$; donc :

$E(Z) = 2E(X) - 1 = \boxed{2}$

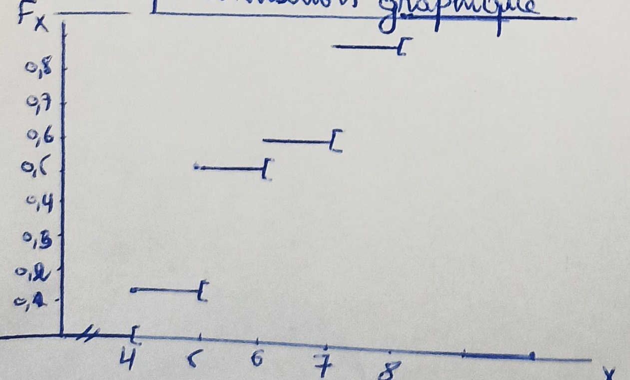
$V(Z) = V(2X - 1) = 4V(X) = \boxed{3}$

Exercice 02 :

1/ On a : $F_X(x) = P(X \leq x)$ donc :

X	4	5	6	7	8
P	0,15	0,35	0,10	0,25	0,15
F_X	0,15	0,5	0,6	0,85	1

La représentation graphique



$$F_X(x) = \begin{cases} 0 & \text{si } x < 4 \\ 0,15 & \text{si } 4 \leq x < 5 \\ 0,50 & \text{si } 5 \leq x < 6 \\ 0,60 & \text{si } 6 \leq x < 7 \\ 0,85 & \text{si } 7 \leq x < 8 \\ 1 & \text{si } x \geq 8 \end{cases}$$

2/ $P(X \leq 7,5) = P(X=4) + P(X=5) + P(X=6) + P(X=7)$
$= 0,85$

ou bien

$P(X \leq 7,5) = P(X \leq 7) = F_X(7) = 0,85$.

- $P(X > 8) = 0$
- $P(4 \leq X \leq 6,5) = P(4) + P(X=5) + P(X=6)$
$= \boxed{0,60}$

3/ $E(X) = 5,9$; $V(X) = 1,79$; $\sigma(X) = \sqrt{1,79}$.

Exercice 03 :

1/ On a $\int_{-\infty}^{+\infty} f(x)\,dx = 1 \iff K\int_0^{+\infty} e^{-2x}\,dx = 1$

$\iff K\left(-\frac{1}{2}e^{-2x}\right)_0^{+\infty} = 1$

$\iff \frac{K}{2} = 1$

$\iff \boxed{K = }$

- $F_X(t) = P(X \leq t) = \int_{-\infty}^{t} f(x)\,dx$ $\quad \overbrace{}^{0} \overbrace{}^{Ke^{-2x}}$

$-\infty \quad t \quad 0 \quad t. \quad +\infty$

si $t < 0$: $F_X(t) = 0$

si $t \geq 0$: $F_X(t) = \int_0^t e^{-2x}\,dx = \left[-e^{-2x}\right]_0^t = 1 - e^{-2t}$

donc :

$$\boxed{F_X(t) = \begin{cases} 0 & ; t < 0 \\ 1 - e^{-2t} & ; t \geq 0 \end{cases}}$$

2/ $P(X > 2) = 1 - P(X \leq 2) = 1 - F(2) = \boxed{e^{-4}}$

$P(0,5 < X < 1,5) = F(1,5) - F(0,5) = \boxed{e^{-1} - e^{-3}}$

Exercice 04

1/ $P(1 \leq X \leq 3) = F_X(3) - F_X(1) = \left(\frac{1}{16}(3)^2\right) - \left(\frac{1}{16} \times 1^2\right)$
$= 0,5 = 50\%$.

2/ $f(x) = F_X'(x) = \left(\frac{1}{16}x^2\right)'$; $0 \leq x \leq 4$
$= \frac{x}{8}$

donc :

$$f(x) = \begin{cases} \frac{x}{8} & \text{si } 0 \leq x \leq 4 \\ 0 & \text{sinon.} \end{cases}$$

3/ $E(X) = \int_{-\infty}^{+\infty} x f(x)\,dx = \left[\frac{x^3}{24}\right]_0^4 = \frac{8}{3}$

$V(X) = E(X^2) - (E(X))^2$
$= \left[\frac{x^4}{32}\right]_0^4 - \left(\frac{8}{3}\right)^2 = \frac{8}{9}$.

$\sigma(X) = \boxed{\frac{2\sqrt{2}}{3}}$

Exercice 5 :

1/ On a $\int_{e^{-1}}^{e} f(x)\,dx = 1 \iff K\int_{e^{-1}}^{e} \frac{1}{x}\,dx = 1$

$\iff K \ln x \Big|_{e^{-1}}^{e} = 1 \iff \boxed{K = \frac{1}{2}}$

2/ $P(1 \leqslant X \leqslant e) = \int_{1}^{e} f(x)\,dx = \frac{1}{2}$.

3/ $P_{(X>1)}(X<2) = \frac{P(X>1 \cap X<2)}{P(X>1)}$

$= \frac{P(1<X<2)}{P(X>1)}$

$= \boxed{\ln 2}$

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