

Exercice 1.

1°/

$$U_{\text{eff}} = 220V, \quad f = 50\text{Hz},$$

$$I_{\text{eff}} = 150\text{mA}, \quad P = 20W.$$

$$P = U_{\text{eff}} I_{\text{eff}} \cos\varphi \rightarrow f_p = \cos\varphi = \frac{P}{U_{\text{eff}} I_{\text{eff}}}$$

$$f_p = \cos\varphi = \frac{20}{220 \times 0.15} = \mathbf{0.61}$$

2°/

Ampoule basse consommation:

$$W_{bc} = 3 \times 6 \times 365 \times 0.02 = \mathbf{131.4 KWh}$$

3°/

Ampoule classique :

$$W_{class} = 3 \times 6 \times 365 \times 0.1 = 657.0 KWh$$

La différence de consommation est :

$$W_{class} - W_{bc} = 525.6 KWh$$

Soit :

$$525.6 \times 4 = \mathbf{2102.4 DA}$$

Exercice 2.

1°/

$$P = U_{\text{eff}} I_{\text{eff}} \cos\varphi = I_{\text{eff}}^2 R$$

$$Q = U_{\text{eff}} I_{\text{eff}} \sin\varphi = I_{\text{eff}}^2 L\omega$$

2°/

$$S = \sqrt{P^2 + Q^2} = I_{\text{eff}}^2 \sqrt{R^2 + L^2 \omega^2}$$

$$f_p = \cos\varphi = \frac{R}{\sqrt{R^2 + L^2 \omega^2}}$$

3°/

$$S = U_{\text{eff}} I_{\text{eff}} \rightarrow U_{\text{eff}} = \frac{S}{I_{\text{eff}}} = I_{\text{eff}} \sqrt{R^2 + L^2 \omega^2}$$

$$f_p = \cos\varphi \rightarrow \varphi = \mathbf{arc(cos f_p)}$$

A.N :

$$P = \mathbf{129.6 W} \quad , \quad Q = \mathbf{813.89 VAR}$$

$$S = \mathbf{824.14 VA} \quad , \quad f_p = \mathbf{0.16}$$

$$U = \mathbf{228.93 V} \quad , \quad \varphi = \mathbf{81^\circ}.$$

Exercice 3.

1. Expressions des puissances : active (P), réactive (Q) et apparente (S) :

$$P = U_{\text{eff}} I_{\text{eff}} \cos\varphi$$

$$Q = U_{\text{eff}} I_{\text{eff}} \sin\varphi$$

$$S = \sqrt{P^2 + Q^2} = U_{\text{eff}} I_{\text{eff}}$$

2. Calcul de P et Q pour chaque élément

Pour les trois lampes : $f_p = \cos\varphi = 1 \rightarrow \varphi = 0$

$$P_1 = 3 \times 75 = \mathbf{225 W}$$

$$Q_1 = 3 \times U_{\text{eff}} I_{\text{eff}} \sin\varphi = \mathbf{0 VAR}$$

Pour le radiateur : $f_p = \cos\varphi = 1 \rightarrow \varphi = 0$

$$P_2 = \mathbf{2000 W}$$

$$Q_2 = U_{\text{eff}} I_{\text{eff}} \sin\varphi = U_{\text{eff}} I_{\text{eff}} \times 0 = \mathbf{0 VAR}$$

Pour le moteur : $f_p = \cos\varphi = 0.80$

$$\rightarrow \varphi = 36.89^\circ$$

$$P_3 = \mathbf{1500 W}$$

$$\frac{Q_3}{P_3} = \frac{U_{\text{eff}} I_{\text{eff}} \sin\varphi}{U_{\text{eff}} I_{\text{eff}} \cos\varphi} = \tan\varphi$$

$$Q_3 = P_3 \tan\varphi = 1500 \times 0.75 = \mathbf{1125 VAR}$$

3. Calcul de P et Q pour toute l'installation

$$P_T = \sum_{i=1}^3 P_i = P_1 + P_2 + P_3 = \mathbf{3725 W}$$

$$Q_T = \sum_{i=1}^3 Q_i = Q_1 + Q_2 + Q_3 = \mathbf{1125 VAR}$$

4. Calcul de f_p pour toute l'installation

$$S = \sqrt{P_T^2 + Q_T^2} = U_{\text{eff}} I_{\text{eff}} = \frac{P_T}{\cos\varphi} = \frac{P_T}{f_p}$$

$$f_p = \frac{P_T}{\sqrt{P_T^2 + Q_T^2}} = \frac{3725}{\sqrt{3725^2 + 1125^2}}$$

$$f_p = \mathbf{0.96}$$

5. Calcul de l'intensité du courant totale :

$$S = U_{\text{eff}} I_{\text{eff}} \rightarrow I_{\text{eff}} = \frac{S}{U_{\text{eff}}}$$

$$I_{\text{eff}} = \frac{P_T}{f_p U_{\text{eff}}} = \frac{3725}{0.96 \times 220} = \mathbf{17.64 A}$$

Exercice 4.

1.

$$Z^* = Z_R^* + Z_L^* + Z_C^* = R + jL\omega - \frac{1}{C\omega}j$$

$$Z^* = R + j\left(L\omega - \frac{1}{C\omega}\right)$$

2. La réactance représente la partie imaginaire de Z^* , soit :

$$X = \text{Im}\{Z^*\} = L\omega - \frac{1}{C\omega}$$

3.

$$P = \text{Re}\{Z^*\} I^2 = R I^2$$

$$Q = \text{Im}\{Z^*\} I^2 = \left(L\omega - \frac{1}{C\omega}\right) I^2$$

$$S = \sqrt{P^2 + Q^2} = I^2 \sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2}$$

4.

$u(t)$ et $i(t)$ sont en phase si $X = 0$.

D'où :

$$L\omega - \frac{1}{C\omega} = 0 \rightarrow Q = 0$$

$$L\omega = \frac{1}{C\omega} \rightarrow LC\omega^2 = 1 \rightarrow \omega = \frac{1}{\sqrt{LC}}$$

5.

$$\omega = 2\pi f \rightarrow f = \frac{1}{2\pi\sqrt{LC}}$$

Exercice 5.

1.

$$\varphi = 0 \rightarrow P_R = V_{\text{eff}} I_{\text{eff}} = I_{\text{eff}}^2 R = \frac{V_{\text{eff}}^2}{R}$$

$$P_R = \frac{230^2}{1600} = 33 \text{ W}$$

2.

$$Q_L = \frac{V_{\text{eff}}^2}{L\omega} = \frac{230^2}{1.25 \times 2\pi \times 50} = 134.7 \text{ VAR}$$

3.

$$S = \sqrt{P_R^2 + Q_L^2} = V_{\text{eff}}^2 \sqrt{\frac{1}{R^2} + \frac{1}{L^2\omega^2}}$$

$$S = 230^2 \sqrt{\frac{1}{1600^2} + \frac{1}{1.25^2 \times 2\pi^2 \times 50^2}}$$

$$S = 138.7 \text{ VA}$$

4.

$$S = \sqrt{P_R^2 + Q_L^2} = V_{\text{eff}} I_{\text{eff}}$$

$$I_{\text{eff}} = \frac{S}{V_{\text{eff}}} = \frac{138.7}{230} = 0.60 \text{ A}$$

$$f_p = \cos\varphi = \frac{P_R}{S} = \frac{33}{138.7} = 0.24$$

5.

$$\cos\varphi = 0.24 \rightarrow \varphi = 76.1^\circ$$

6. Nous avons obtenu :

$$S = \sqrt{P_R^2 + Q_L^2} = V_{\text{eff}}^2 \sqrt{\frac{1}{R^2} + \frac{1}{L^2\omega^2}}$$

et sachant que :

$$S = \sqrt{P_R^2 + Q_L^2} = V_{\text{eff}} I_{\text{eff}}$$

$$Z = |Z^*| = \frac{V_{\text{eff}}}{I_{\text{eff}}} = \frac{V_{\text{eff}}^2}{S} = \frac{1}{\sqrt{\frac{1}{R^2} + \frac{1}{L^2\omega^2}}}$$

En posant : $X = L\omega$, on obtient :

$$Z = \frac{1}{\sqrt{\frac{1}{R^2} + \frac{1}{X^2}}}$$

$$\cos\varphi_{v/i} = \frac{P_R}{S} = \frac{\frac{V_{\text{eff}}^2}{R}}{V_{\text{eff}}^2 \sqrt{\frac{1}{R^2} + \frac{1}{L^2\omega^2}}}$$

$$\cos\varphi_{v/i} = \frac{1}{R \sqrt{\frac{1}{R^2} + \frac{1}{X^2}}}$$