Exercice N°3 :

$$\bar{\bar{L}} = \overline{\overline{\text{grad}}}\,\vec{V} = \left(\frac{\partial V_i}{\partial x_j}\right)_{i,j} = \begin{pmatrix} \frac{\partial V_1}{\partial x} & \frac{\partial V_1}{\partial y} & \frac{\partial V_1}{\partial z} \\ \frac{\partial V_2}{\partial x} & \frac{\partial V_2}{\partial y} & \frac{\partial V_3}{\partial z} \\ \frac{\partial V_3}{\partial x} & \frac{\partial V_3}{\partial y} & \frac{\partial V_3}{\partial z} \end{pmatrix} \quad 1$$

1

$$= \frac{f(x,z)}{x^2+z^2} \cdot \begin{pmatrix} 2xz & 0 & z^2-x^2 \\ 0 & 0 & 0 \\ z^2-x^2 & 0 & -2xz \end{pmatrix} \quad 1,5$$

$\bar{\bar{L}}$ est symétrique alors $\bar{\bar{\Omega}} = \bar{\bar{0}}$, $\bar{\bar{D}} = \bar{\bar{L}}$

0,5 0,5 0,5

Exercice N°4 :

$$\bar{\bar{T}} = \frac{V.V^T}{V^T.V} = \frac{\begin{pmatrix}V_1\\V_2\\V_3\end{pmatrix}\cdot(V_1\ V_2\ V_3)}{(V_1\ V_2\ V_3)\cdot\begin{pmatrix}V_1\\V_2\\V_3\end{pmatrix}} = \frac{1}{V_1^2+V_2^2\ V_3^2} \cdot \begin{pmatrix} V_1V_1 & V_1V_2 & V_1V_3 \\ V_2V_1 & V_2V_1 & V_2V_3 \\ V_3V_1 & V_3V_2 & V_3V_3 \end{pmatrix}$$

1

$$\bar{\bar{T}} = (S_{ij})_{ij} \ / \ S_{ij} = T_{il}T_{lj} = \frac{V_iV_l}{V_kV_k}\,\frac{V_lV_j}{V_mV_m} = \frac{V_iV_j}{V_kV_k}\,\frac{V_lV_l}{V_kV_k}$$

$$= \frac{V_iV_j}{V_kV_k} = T_{ij} \quad 1,5$$

alors $\bar{\bar{T}}^2 = \bar{\bar{T}}$.

de même $\bar{\bar{T}}^3 = \bar{\bar{T}}^2 . \bar{\bar{T}} = \bar{\bar{T}} . \bar{\bar{T}} = \bar{\bar{T}}$ 1

alors $\bar{\bar{T}}^n = \bar{\bar{T}}$ 1

$$\bar{\bar{T}} = \bar{\bar{T}}^2 = \frac{1}{45}\begin{pmatrix} 25 & 20 & 10 \\ 20 & 16 & 8 \\ 10 & 8 & 4 \end{pmatrix}$$

1,5

Correction d'Examen

Exercice N°1 :

S est symetrique alors $S_{ij} = S_{ji}$

A est antisymetrique alors $A_{ij} = -A_{ji}$

$$\bar{\bar{S}} : \bar{\bar{A}} = S_{ij} A_{ji} = S_{11}A_{11} + S_{12}A_{21} + S_{13}A_{13} + S_{21}A_{12} + S_{22}A_{22}$$
$$+ S_{23}A_{32} + S_{31}A_{13} + S_{32}A_{23} + S_{33}A_{33}$$

05

et comme $S_{21} = S_{12}$, $S_{31} = S_{13}$, $S_{32} = S_{23}$,

$A_{11} = A_{22} = A_{33} = 0$

$A_{21} = -A_{12}$, $A_{31} = -A_{13}$, $A_{32} = -A_{23}$

On deduit $\bar{\bar{S}} : \bar{\bar{A}} = 0$.

Exercice N°2 :

1)
$$\begin{cases} V_1 = \frac{Dx_1}{Dt} = \omega(-X_1 \sin\omega t - X_2 \cos\omega t) \\ V_2 = \frac{Dx_2}{Dt} = \omega(X_1 \cos\omega t - X_2 \sin\omega t) \\ V_3 = \frac{Dx_3}{Dt} = 0 \end{cases}$$ 0,5

$$\begin{cases} \gamma_1 = \frac{DV_1}{Dt} = \omega^2(-X_1 \cos\omega t + X_2 \sin\omega t) \\ \gamma_2 = \frac{DV_2}{Dt} = \omega^2(-X_1 \sin\omega t - X_2 \sin\omega t) \\ \gamma_3 = 0 \end{cases}$$ 0,5

2) On a :
$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} \cos\omega t & -\sin\omega t & 0 \\ \sin\omega t & \cos\omega t & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix} \Rightarrow \begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix} = \begin{pmatrix} \cos\omega t & -\sin\omega t & 0 \\ \sin\omega t & \cos\omega t & 0 \\ 0 & 0 & 1 \end{pmatrix}^{-T} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

$$= \frac{1}{\cos^2\omega t + \sin^2\omega t} \begin{pmatrix} \cos\omega t & \sin\omega t & 0 \\ -\sin\omega t & \cos\omega t & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix}$$

$$\Rightarrow \begin{cases} X_1 = x_1 \cos\omega t + x_2 \sin\omega t \\ X_2 = -x_1 \sin\omega t + x_2 \cos\omega t \\ X_3 = x_3 \end{cases}$$ 1

0,5

D'après 1 :
$$\begin{cases} V_1 = \omega\left(-(x_1 \cos\omega t + x_2 \sin\omega t)\sin\omega t\right) - \omega\left(-x_1 \sin\omega t + x_2 \cos\omega t\right)\cos\omega t \\ V_2 = \omega(x_1 \cos\omega t + x_2 \sin\omega t)\cos\omega t + \omega\left((-x_1 \sin\omega t + x_2 \cos\omega t)\sin\omega t\right) \\ V_3 = 0 \end{cases} \Rightarrow \begin{cases} V_1 = -\omega x_2 \\ V_2 = \omega x_1 \\ V_3 = 0 \end{cases}$$ 0,5

$$\begin{cases} \gamma_1 = -\omega^2 x_1 \\ \gamma_2 = -\omega^2 x_2 \\ \gamma_3 = 0 \end{cases}$$ 0,5

3)
$$\bar{\bar{L}} = \overline{\overline{\text{grad}}}\,\vec{V} = \begin{bmatrix} 0 & -\omega & 0 \\ \omega & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$, $\bar{\bar{L}}$ est symetrique alors 0,5

$$\bar{\bar{D}} = \frac{1}{2}(\bar{\bar{L}} + \bar{\bar{L}}^T) = \bar{\bar{0}}$$ 0,5 et $$\bar{\bar{\Omega}} = \frac{1}{2}(\bar{\bar{L}} - \bar{\bar{L}}^T) = \bar{\bar{L}}$$ 0,5