Master 1 : STLC

Practice #1: Z transform and inverse Z transform

1. Purposes of the TP:

- Calculation the Z and IZ transforms using Matlab codes.
- Calculation the poles/zeros of transfer functions using Matlab codes.
- Calculation the frequency response (magnitude and phase) of a discrete system.
- Results observations.

2. Overview of ZT and IZT:

The Z transform (ZT) is a mathematical tool used for numerical signals and systems analysis. Particularly, it is used in signal processing domain to design IIR and FIR filters and in automatic domain to model dynamic discrete systems. The ZT is basically obtained from Laplace transform by approximating its integral. The ZT of a causal discrete signal or a specific sequence, $x(n)$ for $n = 0, \ldots, +\infty$ is given by

$$X(z) = \sum_{0}^{+\infty} x(n) z^{-n} \quad (1)$$

Generally, the inverse ZT is calculated based on four methods; Table method, partial-fraction expansion method, power series method (Euclidian or polynomial division) and residues method. Concerning the last method the IZT is:

$$\begin{aligned} x(n) &= \frac{1}{2\pi j} \oint_C X(Z) Z^{n-1} dZ \\ &= \sum_{Z_k = \text{pôles de } Z^{n-1} X(Z)} \operatorname{Res}\left\{Z^{n-1} X(Z)\right\}_{Z=Z_k} \end{aligned} \quad (2)$$

The residue associated to a pole $z = a$ of order q of the function $Z^{n-1} X(Z)$ is:

$$\operatorname{Res}_a^q = \lim_{z \to a} \frac{1}{(q-1)!} \frac{d^{q-1}}{dz^{q-1}} \left[X(Z) Z^{n-1} (z-a)^q \right] \quad (3)$$

4. Manipulations:

In this part, we will use the Matlab functions for the calculation of ZI of some sequences, the IZT of some functions, the poles/zeros of the transfer functions and finally their frequency responses (magnitude and phase).

Manip # 1: Calculation of ZT of *x*(*n*)

- Determine the ZT of the following sequences using the Matlab codes « syms » and « ztrans ».

$x(n) = n$

$x(n) = n + 2$

$x(n) = n - 3$

$x(n) = n^2$

$x(n) = \frac{1}{4^n}$

$x(n) = 2(2)^n + 4(1/2)^n$

```
%TZ:
syms z n;
ztrans(n),
```

Manip # 2 : Calculation the IZT of *X*(*z*)

- Calculate the IZT of the following functions using the Matlab codes « syms » and « iztrans ».

$X(z) = \frac{2z}{2z-1}$,

$X(z) = \frac{(1-0.13)z}{(z-1)(z-0.13)}$

$X(z) = \frac{6-9z^{-1}}{1-2.5z^{-1}+z^{-2}}$

```
%TZI:
syms z n;
iztrans(2*z/(2*z-1)),
```

- Calculate the partial-fraction of $X(z)$:

$X(z) = \frac{1+2z^{-1}+z^{-2}}{1-z^{-1}+0.356z^{-2}}$

The Matlab function « residuez » is used to find the coefficients and poles of partial-fraction of the function $X(z)$ given by

$$X(z) = \frac{b_0 + b_1 z^{-1} + ... + b_n z^{-n}}{a_0 + a_1 z^{-1} + ... + a_n z^{-m}} = \frac{r_0}{1-p_1 z^{-1}} + ... + \frac{r_n}{1-p_n z^{-1}} + k_1 + k_2 z^{-1} + ... + k_{m-n+1} z^{-(m-n)}$$

for $n < m$, the coefficients k_i are null. Otherwise, the coefficients number depends on the index, m-n+1.

The corresponding Matlab code is given by :

>> [r,p,k] = residuez(b,a);

>> [r,p,k] = residuez([1,2,1],[1,-1,0.3561])

Manip # 3 : Calculation the poles/zeros, magnitude and phase of *X*(*z*)

- Sketch the diagram of poles/zeros of the following transfer function (TF) in *Z* plan using the Matlab code «zplane» :

$$X(z) = \frac{1 - 1.618^{-1} + z^{-2}}{1 - 1.5161z^{-1} + 0.87z^{-2}}$$

```
%Pôles et zéros:
b= [1 -1.618 1] ;
a=[1 -1.5161 0.878] ;
roots(a),
roots(b),
zplane(b,a),
```

- Sketch the magnitude and phase of *X*(*z*) by a substitution of z = e^{iw}.

```
b= [1 -1.618 1] ;
a=[1 -1.5161 0.878] ;
freqz(b,a)
```