2

Target Detection

Chapter outline:

II. 1 Introduction

In target detection systems (e.g. radar), the effects of interfering signals (clutter) from the environment are usually partially unknown and/or varying in terms of their statistical properties. In such instances, where the performance of the optimal detector deteriorates significantly, CFAR detectors can be used since they are insensitive to changes in the underlying statistics of the clutter. Statistical decision theory used in such fields as radar, sonar, digital communication and ultrasonic imaging, attempts to discriminate between information bearing signals and noise or interference [4]. In recent years, general compound and compound Gaussian distributions have been utilized extensively to fit sea and land clutter. In target detection context, the main objective of radar researchers is to select the best statistical clutter model with consistent estimation of its parameters and robust CFAR detectors.

The present Chapter is organized as follows. Section II. 2 summarizes classical binary decision rules based on Bayes, minimax and Neyman-Pearson criteria. Then, Section II. 3 presents firstly the principle of CFAR detector describes using using its general architecture. After that, some popular CFAR detectors used for target detection in homogeneous and heterogeneous environments are presented with the evaluation of false alarm and detection probabilities. Finally, a conclusion is drawn in Section II. 4.

II. 2 Classical detection

In engineering, when there is a radar signal detection problem, the returned signal is observed and a decision is made as to whether a target is present or absent. In a digital communication system, a string of zeros and ones may be transmitted over some medium. At the receiver, the received signals representing the zeros and ones are corrupted in the medium by some additive noise and by the receiver noise. The receiver does not know which signal represents a zero and which signal represents a one, but must make a decision as to whether the received signals represent zeros or ones. The process that the receiver undertakes in selecting a decision rule falls under the theory of signal detection [41].

The situation above may be described by a source emitting two possible outputs at various instants of time. The outputs are referred to as hypotheses. The null hypothesis H_0 represents a zero (target not present) while the alternate hypothesis H_1 represents a one (target present), as shown in Figure II.1. (a). Each hypothesis corresponds to one or more observations that are represented by random variables. Based on the observation values of these random variables, the receiver decides which hypothesis (H_0 or H_1) is true. Assume that the receiver is to make a decision based on a single observation of the received signal. The range of values that the random variable Y takes constitutes the observation space Z. The observation space is partitioned into two regions Z_0 and Z_1, such that if Y lies in Z_0 the receiver decides in favor of H_0, while if Y lies in Z_1 the receiver decides in favor of H_1, as shown in Figure II. 1. (b). The observation space Z is the union of Z_0 and Z_1; that is, $Z = Z_0 \cup Z_1$

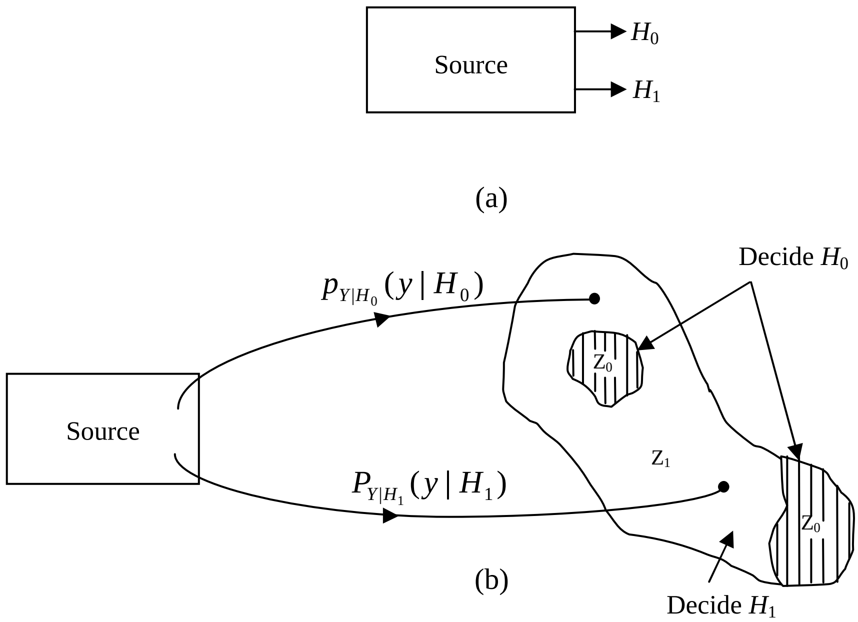


Figure II. 1: (a) Source for binary hypothesis.
(b) Decision regions.

The PDF of Y corresponding to each hypothesis are $P_{Y|H_0}(y\,|\,H_0)$ and $P_{Y|H_1}(y\,|\,H_1)$, where y is a particular value of the random variable Y.

Each time, a decision is made, based on some criteria, for this binary hypothesis testing problem, four possible cases can occur:

(1). Decide H_0 when H_0 is true.

(2). Decide H_0 when H_1 is true.

(3). Decide H_1 when H_0 is true.

(4). Decide H_1 when H_1 is true.

Observe that for cases (1) and (4), the receiver makes a correct decision, while for cases (2) and (3), the receiver makes an error. From radar nomenclature, case (2) is called miss, case (3) a false alarm, and case (4) detection.

In the next sections, we study some of the criteria that are used in decision theory, and the conditions under which these criteria are useful [41].

II. 1. 1 Bayes criterion

In using Bayes criterion, two assumptions are made. First, the probability of occurrence of the two source outputs is known. They are the *a priori* probabilities $P(H_0)$ and $P(H_1)$. $P(H_0)$ is the probability of occurrence of hypothesis H_0, while $P(H_1)$ is the probability of occurrence of hypothesis H_1. Denoting the *a priori* probabilities $P(H_0)$ and $P(H_1)$ by P_0 and P_1 respectively. If we let D_i, $i = 0,1$, where D_0 denotes "decide H_0" and D_1 denotes "decide H_1," we can define C_{ij}, $i, j = 0,1$, as the cost associated with the decision D_i, given that the true hypothesis is H_j. Given $P(D_i, H_j)$, the joint probability that we decide D_i, and that the hypothesis H_j is true, the average cost is

$$\Re = E[C] = C_{00}P(D_0, H_0) + C_{01}P(D_0, H_1) + C_{10}P(D_1, H_0) + C_{11}P(D_1, H_1) \tag{II.1}$$

From Bayes′ rule, we have

$$P(D_i, H_j) = P(D_i \mid H_j)P(H_j) \tag{II.2}$$

where

$$\begin{cases} P(D_0 \mid H_0) = \int_{Z_0} P_{Y|H_0}(y \mid H_0)dy \\ P(D_0 \mid H_1) = \int_{Z_0} P_{Y|H_1}(y \mid H_1)dy \\ P(D_1 \mid H_0) = \int_{Z_1} P_{Y|H_0}(y \mid H_0)dy \\ P(D_1 \mid H_1) = \int_{Z_1} P_{Y|H_1}(y \mid H_1)dy \end{cases} \tag{II.3}$$

The probabilities $P(D_0 \mid H_1)$, $P(D_1 \mid H_0)$, and $P(D_1, H_1)$ represent the probability of miss, P_M, the probability of false alarm, P_{FA}, and the probability of detection, P_D, respectively ($P_M = 1 - P_D$ and $P(D_0 \mid H_0) = 1 - P_F$). The average cost is given by

$$\Re = E[C] = C_{00}(1 - P_F)p_0 + C_{01}(1 - P_D)p_1 + C_{10}P_F p_0 + C_{11}P_D p_1 \tag{II.4}$$

In terms of the decision regions, the average cost is expressed as

$$\begin{aligned} \Re = P_0C_{00}\int_{Z_0} P_{Y|H_0}(y \mid H_0)dy + P_1C_{01}\int_{Z_0} P_{Y|H_1}(y \mid H_1)dy \\ + P_0C_{10}\int_{Z_1} P_{Y|H_0}(y \mid H_0)dy + P_1C_{11}\int_{Z_1} P_{Y|H_1}(y \mid H_1)dy \end{aligned} \tag{II.5}$$

The fact that $\int_Z P_{Y|H_0}(y \mid H_0)dy = \int_Z P_{Y|H_1}(y \mid H_1)dy = 1$, we can write

$$\int_{Z_1} P_{Y|H_j}(y \mid H_j)dy = 1 - \int_{Z_0} P_{Y|H_j}(y \mid H_j)dy, \quad j = 0,1 \tag{II.6}$$

Consequently, the risk is minimized by selecting the decision region Z_0 to include only those points of Y for which the second term is larger, and hence the integrand is negative. Specifically, we assign to the region Z_0 those points for which

$$P_1(C_{01} - C_{11})P_{Y|H_1}(y \mid H_1) < P_0(C_{10} - C_{00})P_{Y|H_0}(y \mid H_0) \tag{II.7}$$

All values for which the second term is greater will be excluded from Z_0 and assigned to Z_1. The values for which the two terms are equal do not affect the risk, and can be assigned to either Z_0 or Z_1. Consequently, we say if

$$P_1(C_{01} - C_{11})P_{Y|H_1}(y \mid H_1) > P_0(C_{10} - C_{00})P_{Y|H_0}(y \mid H_0) \tag{II.8}$$

then we decide H_1. Otherwise, we decide H_0. Hence, the decision rule resulting from the Bayes criterion is [41].

$$\Lambda(y) = \frac{P_{Y|H_1}(y \mid H_1)}{P_{Y|H_0}(y \mid H_0)} \underset{H_0}{\overset{H_1}{\gtrless}} \frac{P_0(C_{10} - C_{00})}{P_1(C_{01} - C_{11})} \tag{II.9}$$

The ratio of $P_{Y|H_1}(y \mid H_1)$ over $P_{Y|H_0}(y \mid H_0)$ is called the likelihood ratio, and is denoted $\Lambda(y)$. We note that if we select the cost of an error to be one and the cost of a correct decision to be zero, that is, $C_{01} = C_{10} = 1 \text{ and } C_{00} = C_{11} = 0$, then the risk function of (II.9) reduces to

$$\Re = P_M P_1 + P_F P_0 = P(\varepsilon) \tag{II.10}$$

where $P(\varepsilon)$ is the error probability. Thus, in this case, minimizing the average cost is equivalent to minimizing the probability of error. Receivers for such cost assignment are called minimum probability of error receivers. The threshold reduces to, $\eta = \frac{P_0}{P_1}$.

II. 1. 2 Minimax criterion

The Bayes criterion assigns costs to decisions and assumes knowledge of the *a priori* probabilities. In many situations, we may not have enough information about the *a priori* probabilities and consequently, the Bayes criterion cannot be used. One approach would be to select a value of P_1, the *a priori* probability of H_1, for which the risk is maximum, and then minimize that risk function. This principle of minimizing the maximum average cost for the selected P_1 is referred to as minimax criterion. Seting $P_0 = 1 - P_1$, the risk function in terms of P_1 is given by

$$\Re = C_{00}(1-P_F) + C_{10}P_F + P_1[(C_{11}-C_{00}) + (C_{01}-C_{11})P_M - (C_{10}-C_{00})P_F] \quad \text{(II.11)}$$

Assuming a fixed value of P_1, $P_1 \in [0,1]$, we can design a Bayes′ test. These decision regions are then determined, as are the P_{FA}, and miss, P_M. The test results in

$$\Lambda(y) \underset{H_0}{\overset{H_1}{\gtrless}} \frac{(1-P_1)(C_{10}-C_{00})}{P_1(C_{01}-C_{11})} \quad \text{(II.12)}$$

The minimax equation is given by

$$(C_{11}-C_{00}) + (C_{01}-C_{11})P_M - (C_{10}-C_{00})P_F = 0 \quad \text{(II.13)}$$

If the cost of a correct decision is zero $(C_{00} = C_{11} = 0)$, then the minimax equation for $P_1 = P_1^*$ such that $P_1^* \in (0,1)$ reduces to [41]

$$C_{01}P_M = C_{10}P_F \quad \text{(II.14)}$$

II. 1. 3 Neyman-Pearson criterion

In the previous sections, we have seen that for the Bayes criterion we require knowledge of the *a priori* probabilities and cost assignments for each possible decision. Then, we have studied the minimax criterion, which is useful in situations where knowledge of the a priori probabilities is not possible. In many other physical situations, such as radar detection, it is very difficult to assign realistic costs and *a priori* probabilities. To overcome this difficulty, we use the conditional P_{FA}, and detection P_D. The Neyman-Pearson test requires that P_{FA}, be fixed to some value α while P_D is maximized. Since $P_M = 1 - P_D$, maximizing P_D is equivalent to minimizing P_M.

In order to minimize P_M (maximize P_D) subject to the constraint that $P_{FA} = \alpha$, we use the calculus of extrema, and form the objective function J to be

$$J = P_M + \lambda(P_{FA} - \alpha) \quad \text{(II.15)}$$

where $\lambda(\lambda \geq 0)$ is the Lagrange multiplier. We note that given the observation space Z, there are many decision regions Z_1 for which $P_{FA} = \alpha$. The question is to determine those decision regions for which P_M is minimum. Consequently, we rewrite the objective function J in terms of the decision region to obtain

$$J = \int_{Z_0} P_{Y|H_1}(y \mid H_1)dy + \lambda\left[\int_{Z_1} P_{Y|H_0}(y \mid H_0)dy - \alpha\right] \qquad \text{(II.16)}$$

using $Z = Z_0 \cup Z_1$, (I.49) can be rewritten as

$$\begin{aligned} J &= \int_{Z_0} P_{Y|H_1}(y \mid H_1)dy + \lambda\left[\int_{Z_0} P_{Y|H_0}(y \mid H_0)dy - \alpha\right] \\ &= \lambda(1-\alpha) + \int_{Z_0} [P_{Y|H_1}(y \mid H_1) - \lambda P_{Y|H_0}(y \mid H_0)]dy \end{aligned} \qquad \text{(II.17)}$$

Hence, J is minimized when values for which $f_{Y|H_1}(y \mid H_1) > f_{Y|H_0}(y \mid H_0)$ are assigned to the decision region Z_1. The decision rule is, therefore,

$$\Lambda(y) = \frac{P_{Y|H_1}(y \mid H_1)}{P_{Y|H_0}(y \mid H_0)} \underset{H_0}{\overset{H_1}{\gtrless}} \lambda \qquad \text{(II.18)}$$

The threshold η derived from the Bayes 'criterion is equivalent to λ, the Lagrange multiplier in the Neyman-Pearson (N-P) test for which the P_{FA} is fixed to the value α. If we define the conditional density of Λ given that H_0 is true as $P_{\Lambda|H_0}(\lambda \mid H_0)$, then $P_{FA} = \alpha$ may be rewritten as [41]

$$P_F = \int_{Z_1} P_{Y|H_0}(y \mid H_0)dy = \int_{\lambda}^{\infty} P_{\Lambda(y)|H_0}[\lambda(y) \mid H_0]d\lambda \qquad \text{(II.19)}$$

The test is called most powerful of level α if its probability of rejecting H_0 is α.

II. 3 Automatic CFAR detection

In practical radar signal detection systems, the problem is to automatically detect a target in thermal noise plus clutter [4, 41]. The input signal at the radar receiver, when a target is present, is an attenuated randomly phase-shifted version of the transmitted pulse in noise. A typical radar processor for a single-range cell sums the K samples of the matched filter output and compares the sum to a fixed threshold, as shown in Figure II. 2. In this case, a small increase in noise power causes the P_{FA}, to increase intolerably.

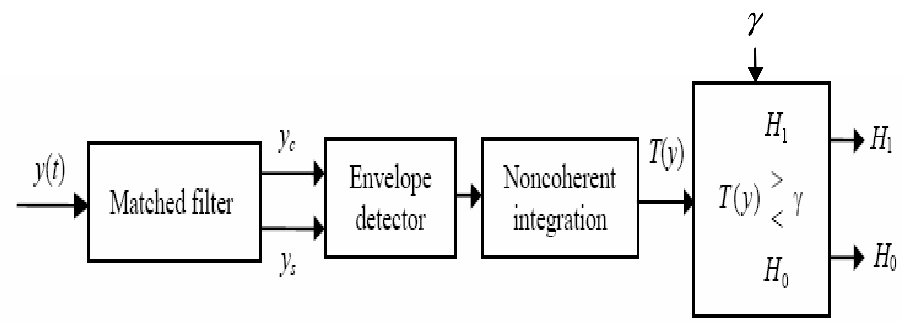


Figure II. 2: Conventional radar detector with fixed threshold.

The role of the CFAR circuitry is therefore to determine the power threshold which any return can be considered to probably originate from a target. If this threshold is too low, then more targets will be detected at the expense of increased numbers of false alarms. Conversely, if the threshold is too high, then fewer targets will be detected, but the number of false alarms will also be low [4, 41]. In most radar detectors, the threshold is set in order to achieve a required P_{FA} (or equivalently, false alarm rate or time between false alarms). If the background against which targets are to be detected is constant with time and space, then a fixed threshold level can be chosen that provides a specified P_{FA}, governed by the PDF of the noise, which is usually assumed to be Gaussian. The P_D is then a function of the SNR of the target return. However, in most fielded systems, unwanted clutter and interference sources mean that the noise level changes both spatially and temporally. In this case, a changing threshold can be used, where the threshold level is raised and lowered to maintain a constant P_{FA}. This is known as CFAR detection. Hence, when the noise variance is not known, and in order to regulate the P_{FA}, numerous CFAR procedures have been developed in the open literature in order to adaptively select a threshold level by taking a rigorous account of the statistics of the background in which targets are to be detected. In most simple CFAR detection schemes, the threshold level is calculated by estimating the level of the noise floor around the CUT. This can be found by taking a block of cells around the CUT and calculating the average power level. On the other hand, some procedures calculate separate averages for the cells to the left and right of the CUT, and then use the greatest-of or smallest-of these two power levels to define the local power level. Other related approaches estimate the background level after ordering the samples in the window. These are referred to as cell-averaging CFAR (CA-CFAR), greatest-of CFAR (GO-CFAR), smallest-of CFAR (SO-CFAR), order-statistics CFAR (OS-CFAR), censored-mean-level CFAR

(CMLD-CFAR),…etc. In the following, we give the description of some of these CFAR algorithms.

There are three main approaches to the CFAR problem: the adaptive threshold processor, the nonparametric processor, and the nonlinear receiver approach. The adaptive threshold processor is the one most commonly used, because it provides the lowest CFAR loss when the actual environment closely matches the design environment. Of the hundreds of papers published in this field, we shall mention only a few to give a sketch of the advance of this rich field up to the actual interest when using high-resolution radars. A real environment in which a radar operates cannot be described by a single clutter model. We refer to homogeneous clutter in situations where the outputs of the range cells are iid. In a non homogeneous background, the adaptive threshold setting is seriously affected, resulting in a degradation of the performance [4, 26, 42, 43].

(i) CA-CFAR detector: Finn and Johnson [26] proposed the use of a reference channel, from which an estimate of the noise environment can be obtained, and upon which the decision threshold is adapted. The radar uses the range cells surrounding the CUT as reference cells, as shown in Figure II. 3. The detector proposed in [26] is the CA-CFAR, where the adaptive threshold is obtained from the arithmetic mean or the sum of the reference cells, $Q = \sum_{i=1}^{M} x_i$.

For a homogeneous background noise, and iid reference cells outputs, the arithmetic is the MLE. This means that the detection threshold is designed to adapt changes in the environment (Gaussian clutter). If we consider a Swerling 1 fluctuating target, the PDF of received signal for each hypothesis H_0 and H_1 is given by

$$\begin{cases} H_0 : p_X(x|H_0) = \frac{1}{b} exp\left(-\frac{x}{b}\right) \\ H_1 : p_X(x|H_1) = \frac{1}{b+a} exp\left(-\frac{x}{b+a}\right) \end{cases} \tag{II.20}$$

where a and $b = 2\sigma^2$ represent the power of signal and the power of clutter respectively. Conventionally, P_{FA} and P_D are computed using the following integrals:

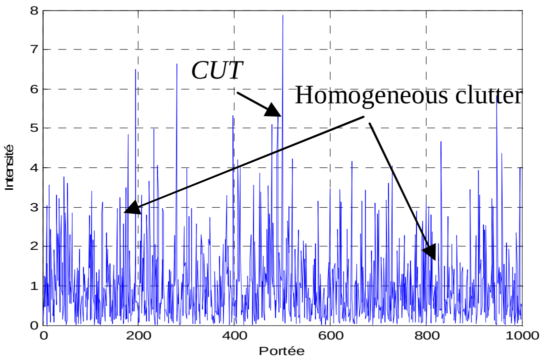
$$\begin{cases} P_{FA} = \int_0^\infty Pr(CUT > \alpha q | H_0) p_Q(q) dq \\ P_D = \int_0^\infty Pr(CUT > \alpha q | H_1) p_Q(q) dq \end{cases} \tag{II.20}$$

where α is a scale factor, and *Pr*(.) denotes probability with

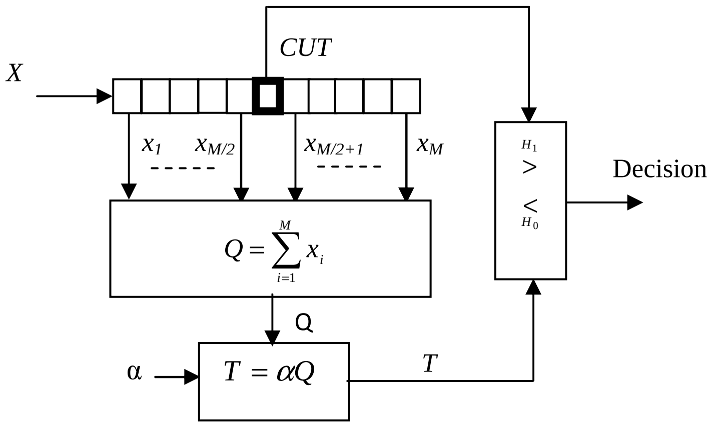
$$Pr(CUT > \alpha q | H_0) = exp\left(-\frac{\alpha q}{b} \right) \tag{II.21}$$

and

$$\Pr(CUT > \alpha q | H_1) = \exp\left(-\frac{\alpha q}{b(1+SNR)} \right) \tag{II.22}$$



(a)



(b)

Figure II. 3: CA-CFAR detector for homogeneous background
(a) Homogeneous clutter situation with $2\sigma^2 = 1$ and *SNR* = 5dB
(b) Arithmetic mean for ML estimate of clutter power

where *SNR=a/b*. The PDF of *Q* is found to be gamma distributed

$$p_Q(q) = \frac{q^{M-1}}{b^M \Gamma(M)} \exp\left(-\frac{q}{b}\right) \tag{II.23}$$

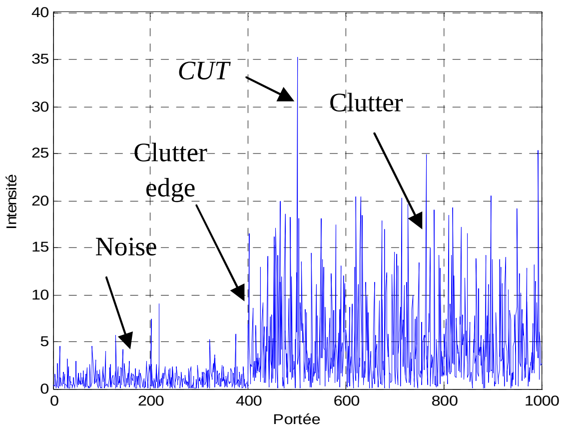
Substituting (II.21), (II.22) and (II.23) into (II.20), (II.20) becomes

$$\begin{cases} P_{FA} = (1+\alpha)^{-M} \\ P_D = \left(1 + \dfrac{\alpha}{1+SNR}\right)^{-M} \end{cases} \tag{II.24}$$

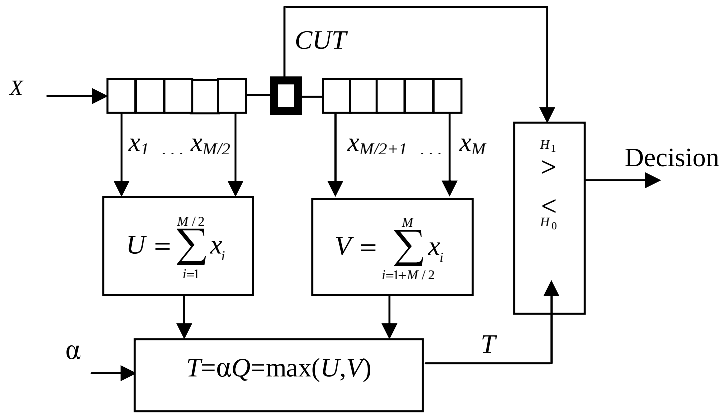
It is worth noting that the P_{FA} is independent of the clutter power *b*, which means that the CA-CFAR algorithm has a CFAR property in presence of Gaussian clutter.

(ii) GO-CFAR detector: In the case of clutter edge situations where there is a transition in the clutter power distribution, Hansen and Sawyers [44] proposed the greatest-of-selection logic in CA-CFAR detector (GO-CFAR) to control the increase in the P_{FA}. In the GO-CFAR detector, the estimate of the noise level in the CUT is selected to be the maximum of *U* and *V*, *Q* = max (*U*, *V*), where *U* and *V* are the sums of the outputs of the leading and the lagging cells, respectively (see Figures II. 4 (a) and II.4 (b)). The clutter-to-clutter ratio, *CCR* = 5dB is taken. The random variables *U* and *V* have analog PDFs given by

$$p_U(q) = p_v(q) = \frac{q^{M/2-1}}{b^{M/2}\Gamma(M/2)} \exp\left(-\frac{q}{b}\right) \tag{II.25}$$



(a)



(b)

Figure II. 4: GO-CFAR detector for hyterogeneous clutter
(a) clutter edge situation for $2\sigma^2 = 1$, *CCR* = 5dB and *SNR* = 10dB)
(b) ML estimation of *Q* from the maximum of leading and lagging windows

The Cumulative Distributed Function (CDF) of U or V is given by

$$P_U(q) = Pv(q) = \int_0^q \frac{q^{M/2-1}}{b^{M/2}\Gamma(M/2)} \exp\left(-\frac{q}{b}\right) dq \qquad \text{(II.26)}$$

From [4], the PDF of *Q* is

$$p_Q(q) = \frac{2q^{M/2-1}}{\Gamma(N/2)} exp(-q/b)\left[1 - exp(-q/b)\sum_{kr=0}^{M/2-1}\frac{(q/b)^k}{k!}\right] \qquad \text{(II.27)}$$

Substituting (II.27) into (II.20), the P_{FA} and the P_D have the following forms

$$\begin{cases} P_{FA} = 2(1+\alpha)^{-M/2} - 2\sum_{i=0}^{M/2-1}\binom{M/2+i-1}{i}(2+\alpha)^{-(M/2+i)} \\ P_D = 2\left(1+\frac{\alpha}{1+SNR}\right)^{-M/2} - 2\sum_{i=0}^{M/2-1}\binom{M/2+i-1}{i}\left(2+\frac{\alpha}{1+SNR}\right)^{-(M/2+i)} \end{cases} \qquad \text{(II.28)}$$

where $\binom{i}{j} = \frac{i!}{j!(i-j)!}$, is the binomial combination.

(iii) SO-CFAR detector: If one or more interfering targets are present, Weiss [45] has shown that the GO-CFAR detector performs poorly, and suggested the use of the smallest-of-selection logic in cell averaging constant false-alarm rate detector (SO-CFAR). In the SO-CFAR detector, the minimum of *U* and *V*, Q = min (*U*, *V*) , is selected to represent the noise level estimate in the cell under test. The SO-CFAR detector was first proposed by Trunk [46] while studying the target resolution of some adaptive threshold detectors. We can intuitively see that the SO-CFAR detector performs well for the case shown in Figure II. 5 (a). From [4], the PDF of Q is

$$\begin{aligned} p_Q(q) &= p_U(q)[1 - P_V(q)] + p_V(q)[1 - P_U(q)] \\ &= p_U(q) + p_V(q) - [p_U(q)P_V(q) + p_V(q)P_U(q)] \\ &= p_U(q) + p_V(q) - p_Q^{GO}(q) \end{aligned} \tag{II.29}$$

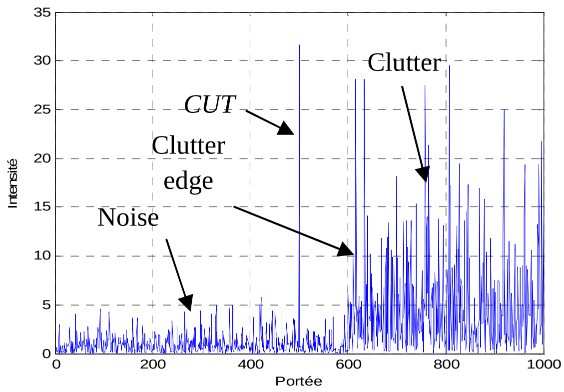
Substituting (I.60) into (I.63), (I.63) will be

$$p_Q(q) = \frac{2q^{M/2-1}}{\Gamma(N/2)} exp(-2q)) \sum_{kr=0}^{M/2-1} \frac{(q)^k}{k!} \tag{II.30}$$

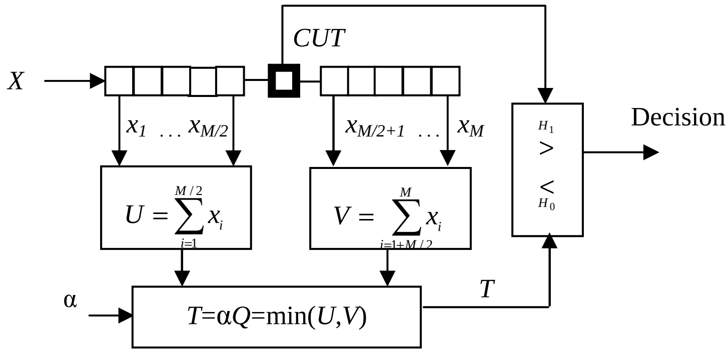
Replacing (II.30) into (II.20), the P_{FA} and the P_D are

$$\begin{cases} P_{FA} = 2(2+\alpha)^{-M/2} \sum_{i=0}^{M/2-1} \binom{M/2+i-1}{i} (2+\alpha)^{-i} \\ P_D = 2\left(2 + \frac{\alpha}{1+SNR}\right)^{-M/2} \sum_{i=0}^{M/2-1} \binom{M/2+i-1}{i} \left(2 + \frac{\alpha}{1+SNR}\right)^{-i} \end{cases} \tag{II.31}$$

(iv) OS-CFAR detector: By studying the homogeneity of the reference cells, it has been shown that targets can be detected by the SO-CFAR detector, especially in the case where secondary targets are in a single window and are not present in the other window [47, 48]. If interfering targets are present in both the leading and lagging windows, neither the GO-CFAR detector nor the SO-CFAR detector solves the problem of the capture effect. To remedy this limitation, [48] introduced the OS-CFAR detector, that is, the OS-CFAR as shown in Figure II. 6 with interfering-to-target ration, *ICR* = 5dB. Here, the samples of the reference window are sorted in ascending order and one ordered sample between them is chosen to represent the noise level estimate in the CUT. The k^{th} ordered sample value, *X* (*k*), selected as the test statistic *Q*, is multiplied by the scale factor α to achieve the desired P_{FA}, and then a decision is made by comparing the output of the *CUT* with the adaptive threshold, $T = \alpha Q$. The value suggested in [48] to represent a good background estimate for typical radar applications in Gaussian noise is $k = 3N / 4$. The calculations of P_D and P_{FA} require the formulation of the PDF of the k^{th} ranked sample, $Q = X_{(k)}$.



(a)



(b)

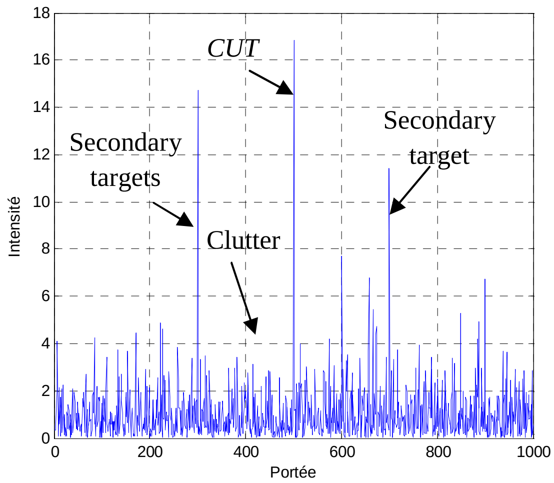
Figure II. 5: SO-CFAR detector for hyterogeneous clutter
(a) clutter edge situation for $2\sigma^2 = 1$, CCR = 5dB and SNR = 10dB)
(b) ML estimation of Q from the minimum of leading and lagging windows

In the case of Gaussian homogeneous background, it is shown in [4, 48] that

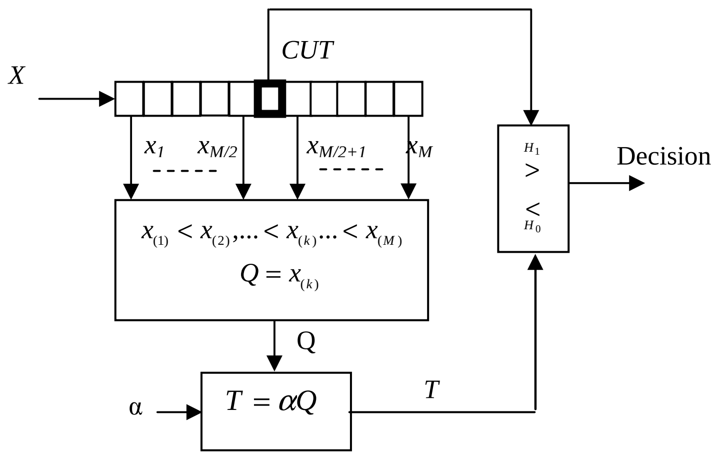
$$p_Q(q) = k\binom{M}{k}[P(q)]^{k-1}\,[1-P(q)]^{M-k}\,p(q)$$
$$= \frac{k}{b}\binom{M}{k}\left(1-exp\left(-\frac{q}{b}\right)\right)^{k-1} exp\left(-\frac{q}{b}\right)^{M-k+1} \tag{II.32}$$

Substituting (I.32) into (II.20), expressions of P_{FA} and P_D are

$$\begin{cases} P_{FA} = \int_0^\infty exp\left(-\frac{\alpha q}{b}\right)\frac{k}{b}\binom{M}{k} exp\left(-\frac{q}{b}\right)^{M-k+1}\left(1-exp\left(-\frac{q}{b}\right)\right)^{k-1} dq \\ P_D = \int_0^\infty exp\left(-\frac{\alpha q}{b(1+SNR)}\right)\frac{k}{b}\binom{M}{k} exp\left(-\frac{q}{b}\right)^{M-k+1}\left(1-exp\left(-\frac{q}{b}\right)\right)^{k-1} dq \end{cases} \tag{II.33}$$



(a)



(b)

Figure II. 6: OS-CFAR detector used for interfering targets situations
(a)Situation of two secondary targets ($2\sigma^2 = 1$, ICR = 5dB and *SNR* = 10dB).
(b) Estimation of clutter level from a selected ranked cell

If we set $y = q/b$, (II.34) is simplified to

$$\begin{cases} P_{FA} = k\binom{M}{k}\int_0^\infty exp(-(\alpha + M + 1 - k)y)(1 - exp(-y))^{k-1}dq \\ P_D = k\binom{M}{k}\int_0^\infty exp(-(\frac{\alpha}{1+SNR} + M + 1 - k)y)(1 - exp(-y))^{k-1}dq \end{cases} \quad \text{(II.34)}$$

Finally, solutions of (II.34) give

$$\begin{cases} P_{FA} = \dfrac{M!}{(M-k)!}\dfrac{\Gamma(M-k+\alpha+1)}{\Gamma(M+\alpha+1)} \\ P_D = \dfrac{M!}{(M-k)!}\dfrac{\Gamma(M-k+1+\alpha/(1+SNR))}{\Gamma(M+1+\alpha/(1+SNR))} \end{cases} \quad \text{(II.35)}$$

II. 4 Conclusion

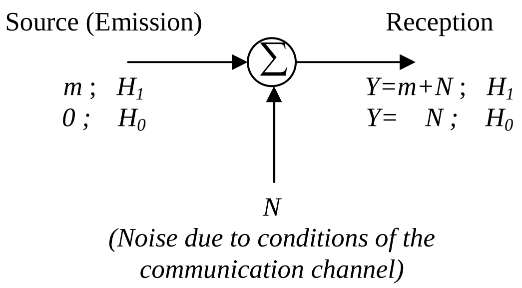
In this chapter, we presented some basic concepts of the radar system. Radar components are described firstly, as well as its classification, radar cross-section and radar equation. As the modeling of radar clutter plays an important role in CFAR detection, we presented some statistical models for high resolution radars. It is shown that radar echoes can be scattered from sea or land surface with different grazing angles. Targets models are also given using Rayleigh and other distributions. Decision theory is introduced by giving three decision rules. Finally, some CFAR detectors used in homogeneous and heterogeneous Gaussian clutter are also described where mathematical stages for computing probabilities of false alarm and detection are given.

Exercises 2

Exercise 1:

In the digital communication system, we consider a voltage source with a constant output of value m under the hypothesis H_1 and an output under the hypothesis H_0 of value 0. At reception, the received signal is contaminated by a white Gaussian noise, with zero mean and a variance σ^2. The probability density (fdp) function of the noise is given by:

$$f_Q(q)=\frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{q^2}{2\sigma^2}\right)$$



a- Write the density functions for each hypothesis.

b- Formulate the likelihood ratio and identify the decision regions using 'Bayes' criterion.

c- Give the false alarm probability and the detection probability expressions.

Exercise 2:

We come back to the exercise 1 whose the priori probabilities p_0 and p_1 are unknown.

a- Apply the 'minimax' criterion to calculate the probability of minimum error, $p(\varepsilon) = p_0 P_{FA} + p_1 P_M$ with $C_{00}=C_{11}=0$, $C_{01}=C_{10}=1$.

Exercise 3:

For the detection of radar targets embedded in an atmospheric noise, the density functions of the received echo for each hypothesis are given by:

$$\begin{cases} f_{Y/H_0}(y/H_0)=\dfrac{1}{\sqrt{2\pi}\sigma_0}\exp\left(-\dfrac{y^2}{2\sigma_0^2}\right) \\ f_{Y/H_1}(y/H_1)=\dfrac{1}{\sqrt{2\pi}\sigma_1}\exp\left(-\dfrac{y^2}{2\sigma_1^2}\right) \end{cases}$$

where $\sigma_1^2 > \sigma_0^2$

a- Give the likelihood ratio test.

b- If $\sigma_1^2 = 2$ and $\sigma_0^2 = 1$, calculate the probability of detection if the false alarm probability is fixed at 0.1.