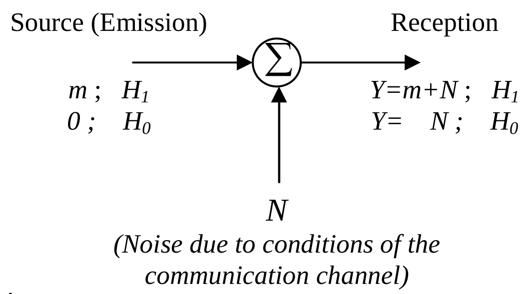
Tutorial # 1

Exercise 1:

In the digital communication system, we consider a voltage source with a constant output of value *m* under the hypothesis H_1 and an output under the hypothesis H_0 of value 0. At reception, the received signal is contaminated by a white Gaussian noise, with zero mean and a variance σ^2. The probability density (fdp) function of the noise is given by:

$$f_Q(q) = \frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{q^2}{2\sigma^2}\right)$$



a- Write the density functions for each hypothesis.

b- Formulate the likelihood ratio and identify the decision regions using ‘Bayes’ criterion.

c- Give the false alarm probability and the detection probability expressions.

Exercise 2:

We come back to the exercise 1 whose the priori probabilities p_0 and p_1 are unknown.

a- Apply the ‘minimax’ criterion to calculate the probability of minimum error, $p(\varepsilon) = p_0 P_{FA} + p_1 P_M$ with $C_{00}=C_{11}=0$, $C_{01}=C_{10}=1$.

Exercise 3:

For the detection of radar targets embedded in an atmospheric noise, the density functions of the received echo for each hypothesis are given by:

$$\begin{cases} f_{Y/H_0}(y/H_0) = \dfrac{1}{\sqrt{2\pi}\sigma_0}\exp\left(-\dfrac{y^2}{2\sigma_0^2}\right) \\ f_{Y/H_1}(y/H_1) = \dfrac{1}{\sqrt{2\pi}\sigma_1}\exp\left(-\dfrac{y^2}{2\sigma_1^2}\right) \end{cases}$$

where $\sigma_1^2 > \sigma_0^2$

a- Give the likelihood ratio test.

b- If $\sigma_1^2 = 2$ and $\sigma_0^2 = 1$, calculate the probability of detection if the false alarm probability is fixed at 0.1.