Master 1 : STLC

Practice #2: Simulation of discrete IIR filter

1. Objectives:

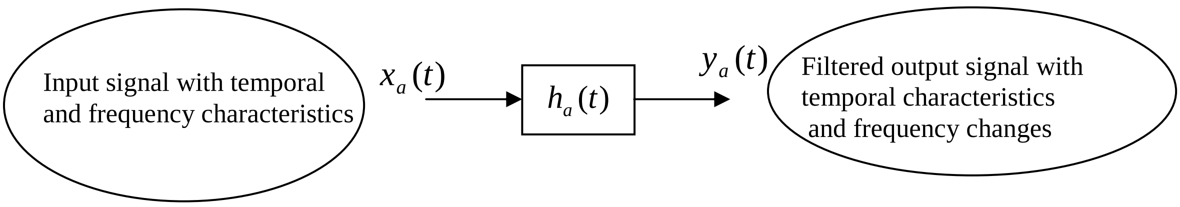
- Understand Matlab codes for IIR filter design via Butterworth and Chebyshev templates.
- Calculation of frequency responses of SIR filters.
- Observation of filtering results.

2. Theoretical review

A digital filter is a Discrete Linear System invariant in time and modifies the temporal and frequency representation of signals. The following analog system function is considered:

$$H_a(s) = \frac{\sum_{k=0}^{M} d_k s^k}{\sum_{k=0}^{N} c_k s^k} = \frac{Y_a(s)}{X_a(s)} \quad (1)$$

where $x_a(t)$ is the input signal and $X_a(s)$ its Laplace transform. $y_a(t)$ is the output signal and $Y_a(s)$ its Laplace transform.



Thus, $h_a(t)$ is the impulse response of the analog system (filter). Alternatively, an analog system with a transfer function $H_a(s)$ can be described by the following differential equation:

$$\sum_{k=0}^{N} c_k \frac{d^k y_a(t)}{d^k t} = \sum_{k=0}^{M} d_k \frac{d^k x_a(t)}{d^k t} \quad (2)$$

The rational function of the corresponding system for a digital filter has the form

$$H(z) = \frac{\sum_{k=0}^{M} b_k z^{-k}}{\sum_{k=0}^{N} a_k z^{-k}} = \frac{Y(z)}{X(z)} \quad (3)$$

Taking the TZI of (3), we obtain

$$\sum_{k=0}^{N} a_k y(n-k) = \sum_{k=0}^{M} b_k x(n-k) \qquad (4)$$

The input and output are connected by the following convolution product

$$y(n) = \sum_{k=-\infty}^{+\infty} x(k)h(n-k) \qquad (5)$$

3. Design methods of IIR filters

There are three methods for designing IIR digital filters; the invariant impulse method, the finite differences method and the bilinear transformation method.

- Invariant impulse method : $H(z) = \sum_{k=1}^{N} \frac{A_k}{1-e^{s_k T} z^{-1}}$ where s_k are obtained from the TF of the analog filter, $H_a(s) = \sum_{k=1}^{N} \frac{A_k}{s - s_k}$ and T is the sampling theorem.

- Finit differences method : $H(z) = H_a(s)\big|_{\frac{1-z^{-1}}{T}}$

- Bilinear transformation method: $H(z) = H_a(s)\big|_{s=\frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}}}$

4. Manipulations :

Manip # 1: (Conception via Butterworth approach)

In this manip, we want to design a digital low-pass filter using a Butterworth analog filter given by:

$$\left|H_a(j\Omega)\right|^2 = \frac{1}{1+\left(\frac{j\Omega}{j\Omega_c}\right)^{2N}} \qquad (6)$$

The desired specifications are:

- Attenuation in the pass-band is 1dB at 0.2π.
- Attenuation in the stop-band is 15dB at 0.3π.

The filter order, N and the cut-off frequency Ω_c represent unknown parameters. If we consider the method of invariant impulse method, we can write

$$\begin{cases} 20\log_{10}\left(H_a(j\Omega)\right) \geq -1 \\ 20\log_{10}\left(H_a(j\Omega)\right) \leq -15 \end{cases} \quad \text{or} \quad \begin{cases} 1+\left(\frac{0.2\pi}{\Omega_c}\right)^{2N} = 10^{0.1} \\ 1+\left(\frac{0.3\pi}{\Omega_c}\right)^{2N} = 10^{1.5} \end{cases}$$

Solution of these equations gives N = 6 and $\Omega_c = 0.7032$.

- Build an RII filter using the following Matlab commands:

```
clear all;clc;
 % Filtre de Butterworth
N=6
wc=0.7032
[b1 a1]=butter(N,wc/pi);
figure(1);
freqz(b1,a1)
axis([0 1 -20 0])
 %Exemple de filtrage des signaux:
t=0.001:0.001:1;
x=sin(2*pi*20*t)+cos(2*pi*500*t);
dataIn=x'+0.1*randn(1000,1);
dataOut=filter(b1,a1,dataIn);
figure(2);
subplot(2,1,1);plot(1:1000,dataIn);
legend('Signal bruité');grid;
subplot(2,1,2);plot(1:1000,dataOut,'r');
legend('Signal filtré');grid;
```

- Check and compare the specifications obtained from the resulting IIR filter.

Manip 2: (Conception via Chebyshev approach)

In this manip, we want to design a digital low-pass filter using the analog Chebyshev type 1 filter given by:

$$|H_a(j\omega)|^2 = \frac{1}{1+\varepsilon^2 V_N^2(\omega/\omega_c)} \quad (7)$$

Based on the above specifications, we obtain:

N = 4, ε = 0.50885 and $\omega_c = 0.6283$.

- Build an RII filter using the following Matlab commands:

```
clear all;clc;
 % Filtre de Chepyshev
N=4;
Ep=0.50885 ;
wc=0.6283;
```

```
[b2 a2]=cheby1(N,Ep,wc/pi);
figure(1);
freqz(b2,a2)
axis([0 1 -20 0])
% Exemple de filtrage des signaux:
t=0.001:0.001:1;
x=sin(2*pi*20*t)+cos(2*pi*500*t);
dataIn=x'+0.1*randn(1000,1);
dataOut=filter(b2,a2,dataIn);
figure(4);
subplot(2,1,1);plot(1:1000,dataIn);
legend('Signal bruité');grid;
subplot(2,1,2);plot(1:1000,dataOut,'r');
legend('Signal filtré');grid;
```

- Check and compare digital filter specifications.
- Use the following Matlab code to sketch and compare the two responses of the above IIR filters.

```
%Comparaison entre les deux filtres
figure(5);
freqz(b1,a1);
hold on;
freqz(b2,a2);axis([0 1 -20 0])
legend('Réponse de Butterworth','Réponse de Chebyshev');
```

- Provide conclusions on the results obtained in this practice.