Chapter 4

Discret Fourier Transform (TFD)

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4. 1 Introduction

We have already seen in the previous chapter the representation of sequences in terms of the Z transform. In the particular case where the sequence is finite, it is possible to develop an alternative Fourier representation as the TFD (Discrete Fourier Transform). This representation is based on the relationship between finite sequences and periodic sequences.

4. 2 Representation of periodic sequences

We consider a periodic sequence $\tilde{x}(n)$ with a period *N*. i.e., $\tilde{x}(n+kN)=\tilde{x}(n)$ for k integer. This sequence cannot be represented by its ZT because there is no value of z for which the sequence converges. However, it is possible to represent $\tilde{x}(n)$ in terms of Fourier series (a sum of sine and cosine or exponential with frequencies that are multiple integers of the fundamental frequency $2\pi / N$). However, there is only *N* distinct exponential complexes *N*. This is due to

$$e_k(n)=e^{j\frac{2\pi}{N}nk} \tag{4.1}$$

Which is periodic in *k* with a period *N*. $e_0(n)=e_N(n)$, $e_1(n)=e_{N+1}(n)$, …, $e_k(n)=e_{N+k}(n)$.
So the representation with Fourier series requires *N* terms.

$$\tilde{x}(n)=\frac{1}{N}\sum_{k=0}^{N-1}\tilde{X}(k)e^{j\frac{2\pi}{N}nk} \tag{4.2}$$

For convenience, 1/*N* is included in (4.2). In order to obtain the coefficients $\tilde{X}(k)$ from the periodic sequence, $\tilde{x}(n)$, we use the fact that

$$\frac{1}{N}\sum_{n=0}^{N-1}e^{j\frac{2\pi}{N}nr}=\begin{cases}1 & if \quad r=mN\\ 0 & otherwise\end{cases} \tag{4.3}$$

Thus, we multiply (4.2) by $e^{-j\frac{2\pi}{N}nr}$ and summing from 0 to *N*-1, we obtain

$$\sum_{n=0}^{N-1}\tilde{x}(n)e^{j\frac{2\pi}{N}nr}=\frac{1}{N}\sum_{k=0}^{N-1}\sum_{k=0}^{N-1}\tilde{X}(k)e^{j\frac{2\pi}{N}(k-r)n} \tag{4.4}$$

By changing the summation order, we can write

$$\sum_{n=0}^{N-1}\tilde{x}(n)e^{-j\frac{2\pi}{N}nr}=\sum_{k=0}^{N-1}\tilde{X}(k)\left[\frac{1}{N}\sum_{n=0}^{N-1}e^{j\frac{2\pi}{N}(k-r)n}\right] \tag{4.5}$$

Using (4.3) and the result of the geometric series i.e., $\sum_{n=0}^{N-1} q^n = \begin{cases} N & q = 1 \\ \dfrac{1-q^N}{1-q} & q \neq 1 \end{cases}$ for $k=r$, $\sum_{n=0}^{N-1} \tilde{x}(n) e^{-j\frac{2\pi}{N}nr} = \tilde{X}(r)$, the coefficients $\tilde{X}(k)$ in (4.2) are obtained by

$$\tilde{X}(k) = \sum_{n=0}^{N-1} \tilde{x}(n) e^{-j\frac{2\pi}{N}nk} \tag{4.6}$$

We observe that $\tilde{X}(k)$ of (4.6) is periodic with a period *N*, i.e., $\tilde{X}(0) = \tilde{X}(N)$, $\tilde{X}(1) = \tilde{X}(N+1)$, …. (4.2) and (4.6) form a transform pair and are called discrete Fourier series (DFS). It is more convenient to use the relation, $W_N = e^{-j\frac{2\pi}{N}}$ in (4.6). Hence

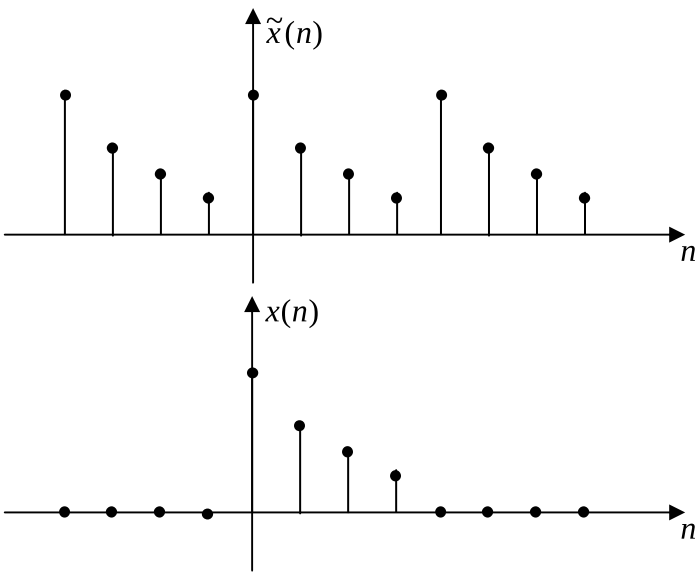
$$\tilde{X}(k) = \sum_{n=0}^{N-1} \tilde{x}(n) W_N^{kn} \tag{4.7}$$

$$\tilde{x}(n) = \frac{1}{N} \sum_{k=0}^{N-1} \tilde{X}(k) W_N^{-kn} \tag{4.8}$$

Where $\tilde{x}(n)$ and $\tilde{X}(k)$ are periodic sequences. The periodic sequence $\tilde{X}(k)$ has an adequate interpretation as the samples of the unit circle spaced equally in angle of a ZT of a period of $\tilde{x}(n)$. To get this relationship, we take

$$x(n) = \begin{cases} \tilde{x}(n) & 0 \le n \le N-1 \\ 0 & Otherwise \end{cases}$$

Is the ZT



$$X(z) = \sum_{n=-\infty}^{+\infty} x(n) z^{-n}$$

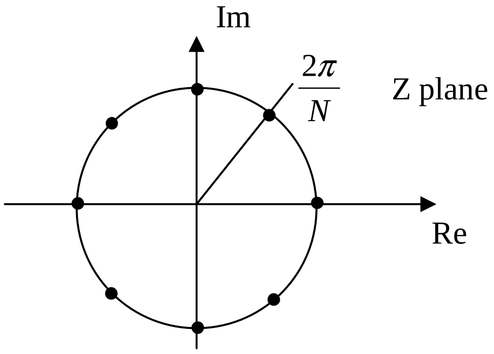
Since $x(n) = 0$ if $n>N-1$ and $n<0$ thus

$$X(z) = \sum_{n=0}^{N-1} x(n) z^{-n} \qquad (4.7)$$

Comparing (4.5) and (4.7), we have

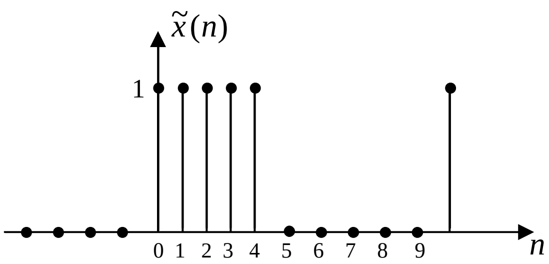
$$\tilde{X}(k) = X(z)\Big|_{z=e^{j\frac{2\pi}{N}k}=W_N^{-k}} \qquad (4.8)$$

This corresponds to a sampling of the ZT in N dots spaced at an angle around the unit circle with the first sample $z = 1$.



Example:

Given a periodic sequence, $\tilde{x}(n)$ as represented by the following figure:



The ZT evaluated on the unit circle of a period of $\tilde{x}(n)$ is

$$X(e^{j\omega}) = \sum_{n=0}^{4} z^{-n} = \frac{1-z^{-5}}{1-z^{-1}}$$

$$= e^{-j2\omega} \frac{\sin(5\omega/2)}{\sin(\omega/2)}$$

Replacing, $z = e^{j\frac{2\pi}{N}k}$, the coefficients $\tilde{X}(k)$ are

$$\tilde{X}(k)=\left.\frac{1-z^{-5}}{1-z^{-1}}\right|_{z=e^{j\frac{2\pi}{N}k}}=\frac{1-e^{-j\frac{2\pi}{10}5k}}{1-e^{-j\frac{2\pi}{10}k}}=\frac{e^{-j\frac{\pi}{2}k}\left(e^{j\frac{\pi}{2}k}-e^{-j\frac{\pi}{2}k}\right)}{e^{-j\frac{\pi}{10}k}\left(e^{j\frac{\pi}{10}k}-e^{-j\frac{\pi}{10}k}\right)}=e^{-j\frac{4\pi k}{10}}\frac{\sin(\pi k/2)}{\sin(\pi k/10)}$$

Or, we use the formula (4.5) in order to confirm the previous result

$$\tilde{X}(k)=\sum_{n=0}^{4}W_{10}^{-nk}=\sum_{n=0}^{4}e^{-j\frac{2\pi}{10}nk}=e^{-j\frac{4\pi k}{10}}\frac{\sin(\pi k/2)}{\sin(\pi k/10)}$$

4. 3 Properties of Fourier series

The following properties should be used to facilitate the calculation of DFT.

- Linearity

If we have two periodic sequences $\tilde{x}_1(n)$ and $\tilde{x}_2(n)$ of period N, we can write

$$\tilde{x}_3(n)=a\tilde{x}_1(n)+b\tilde{x}_2(n)$$

Thus, the coefficients of the DFS of $\tilde{x}_3(n)$ are given by

$$\tilde{X}_3(k)=a\tilde{X}_1(k)+b\tilde{X}_2(k)$$

- Delayed sequence

If a periodic sequence $\tilde{x}(n)$ has Fourier coefficients $\tilde{X}(k)$, then the sequence $\tilde{x}(n+m)$ has the coefficients $W_N^{-km}\tilde{X}(k)$

- Periodic convolution

$\tilde{x}_1(n)$ and $\tilde{x}_2(n)$ are two periodic squences with a period N their Fourier coefficients are respectively $\tilde{X}_1(k)$ and $\tilde{X}_2(k)$. We want to determine the sequence $\tilde{x}_3(n)$ where the DFS is $\tilde{X}_1(k)\tilde{X}_2(k)$

$$\tilde{X}_1(k)=\sum_{m=0}^{N-1}\tilde{x}_1(m)W_N^{-mk}$$

$$\tilde{X}_2(k)=\sum_{r=0}^{N-1}\tilde{x}_2(r)W_N^{-rk}$$

Hence

$$\tilde{X}_1(k)\tilde{X}_2(k)=\sum_{m=0}^{N-1}\sum_{r=0}^{N-1}\tilde{x}_1(m)\tilde{x}_2(r)W_N^{-k(r+m)}$$

Thus

$$\tilde{x}_3(n) = \frac{1}{N}\sum_{k=0}^{N-1} W_N^{-nk}\tilde{X}_1(k)\tilde{X}_2(k)$$

$$= \sum_{m=0}^{N-1}\tilde{x}_1(m)\sum_{r=0}^{N-1}\tilde{x}_2(r)\left[\frac{1}{N}\sum_{k=0}^{N-1} W_N^{-k(n-m-r)}\right]$$

If we consider $\tilde{x}_3(n)$ for $0 \le n \le N$-1, thus

$$\frac{1}{N}\sum_{k=0}^{N-1} W_N^{-k(n-m-r)} = \begin{cases} 1 & if\ r = n - m + lN \\ 0 & otherwise \end{cases}$$

Thus

$$\tilde{x}_3(n) = \sum_{m=0}^{N-1}\tilde{x}_1(m)\tilde{x}_2(n-m)$$

Table 4. 1 Summary of properties of DFS representation of periodic sequences.

Periodic sequence (period *N*)	**Coefficients of DFS**
$\tilde{x}(n)$	$\tilde{X}(k)$ with a period *N*
$\tilde{y}(n)$	$\tilde{Y}(k)$ with a period *N*
$a\tilde{x}(n) + b\tilde{y}(n)$	$a\tilde{X}(k) + b\tilde{Y}(k)$
$\tilde{x}(n+m)$	$W_N^{-km}\tilde{X}(k)$
$W_N^{\ln}\tilde{x}(n)$	$\tilde{X}(k+l)$
$\sum_{m=0}^{N-1}\tilde{x}(m)\tilde{y}(n-m)$ (periodic convolution)	$\tilde{X}(k)\tilde{Y}(k)$
$\tilde{x}(n)\tilde{y}(n)$	$\frac{1}{N}\sum_{l=0}^{N-1}\tilde{X}(l)\tilde{Y}(k-l)$
$\tilde{x}^*(n)$	$\tilde{X}^*(-k)$
$\tilde{x}^*(-n)$	$\tilde{X}^*(k)$

4. 4 Sampling of ZT

We have already seen that the values of $\tilde{X}(k)$ in the DFS representation of a periodic sequence are identical to the samples of the ZT of a period of $\tilde{x}(n)$ points equal spaced on the unitary circle. We will consider the relationship between the aperiodic sequence with its ZT $X(z)$ and the periodic sequence for which the DFS coefficients correspond to the samples of $X(z)$ equal spaced in angle on the unit circle

$$X(z) = \sum_{n=-\infty}^{+\infty} x(n) z^{-n} \tag{4.9}$$

Has a convergence region that includes the unit circle (a condition always checked for finite sequences). If we evaluate the ZT in N equal points spaced in angle on the unitary circle, we obtain a periodic sequence

$$\tilde{X}(k) = X(z)\big|_{z=W_N^{-k}} = \sum_{n=-\infty}^{+\infty} x(n) W_N^{kn} \tag{4.10}$$

where $W_N = e^{-j\frac{2\pi}{N}}$

We know that

$$\tilde{x}(n) = \frac{1}{N}\sum_{k=0}^{N-1} \tilde{X}(k) W_N^{-kn} \tag{4.11}$$

Replacing values of $\tilde{X}(k)$ of (4.10) into (4.11), we obtain

$$\tilde{x}(n) = \frac{1}{N}\sum_{k=0}^{N-1}\sum_{m=-\infty}^{+\infty} \tilde{x}(m) W_N^{km} W_N^{-kn}$$

Changing the summation order

$$\tilde{x}(n) = \sum_{m=-\infty}^{+\infty} \tilde{x}(m)\left[\frac{1}{N}\sum_{k=0}^{N-1} W_N^{-k(n-m)}\right]$$

Using (4.3), we get

$$\frac{1}{N}\sum_{k=0}^{N-1} W_N^{-k(n-m)} = \begin{cases} 1 & m = n + rN \\ 0 & autrement \end{cases}$$

Thus

$$x(n) = \sum_{r=-\infty}^{+\infty} x(n + rN) \tag{4.12}$$

If $x(n) = \tilde{x}(n)$ for $0 \le n \le N-1$

$$X(z) = \sum_{n=0}^{N-1} x(n) z^{-n} \tag{4.13}$$

Replacing (4.11) into (4.13), we obtain

$$X(z) = \sum_{n=0}^{N-1} \frac{1}{N} \sum_{k=0}^{N-1} \tilde{X}(k) W_N^{-kn} z^{-n}$$

By changing the summation order, we find by using the result of the geometric series that

$$X(z) = \frac{1}{N} \sum_{k=0}^{N-1} \tilde{X}(k) \left[\sum_{n=0}^{N-1} \left(W_N^{-k} z^{-1} \right)^{-1} \right]$$

$$X(z) = \frac{1}{N} \sum_{k=0}^{N-1} \tilde{X}(k) \frac{1 - z^{-N}}{1 - W_N^{-k} z^{-1}} = \frac{1 - z^{-N}}{N} \sum_{k=0}^{N-1} \frac{\tilde{X}(k)}{1 - W_N^{-k} z^{-1}} \quad (4.14)$$

This equation expresses $X(z)$, the ZT of a finite sequence of time N as a function of N "Frequency samples" of $X(z)$ on the unit circle.

By replacing $z = e^{j\omega}$, we can show that (4.14) will be

$$X(e^{j\omega}) = \sum_{k=0}^{N-1} \tilde{X}(k) \Phi\left(\omega - \frac{2\pi}{N} k \right) \quad (4.15)$$

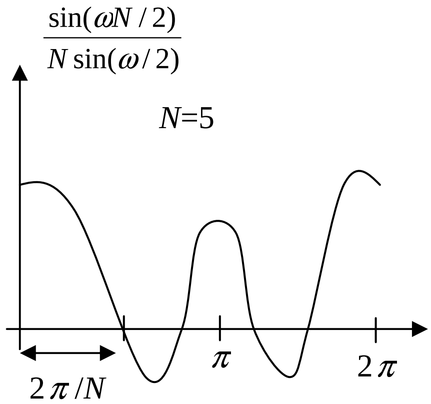
Where

$$\Phi(\omega) = \frac{\sin(\omega N/2)}{N \sin(\omega/2)} e^{-j\omega \frac{N-1}{2}} e^{j\pi k\left(1 - \frac{1}{N}\right)} \quad (4.16)$$

The function $\sin(\omega N/2)/N\sin(\omega/2)$ is plotted on the following Figure for N=5.

$$\Phi\left(\frac{2\pi}{N} k \right) = \begin{cases} 0 & k = 1, 2, ..., N-1 \\ 1 & k = 0 \end{cases}$$

$$X(e^{j\omega})\Big|_{\omega = \frac{2\pi}{N} k} = \tilde{X}(k), \quad k = 0, 1, \ldots$$



4. 5 Fourier representation of finite sequences

Finite sequences of N duration can be represented by a periodic sequence of N period, one of which is identical to the finite sequence. In the sense that the periodic sequence has a single representation in DFS, then the original finite sequence also since we can calculate a single period of the DFS periodic sequence.

We can also represent a sequence finished by the samples of its Z transform.

$$\tilde{x}(n) = \sum_{r=-\infty}^{+\infty} x(n + rN) \qquad (4.17.a)$$

$$\tilde{x}(n) = x(n \bmod ulo\ N) \qquad (4.17.b)$$

The following notation is used

$$\tilde{x}(n) = \left(x(n)\right)_N \qquad (4.18.a)$$

$$x(n) = \begin{cases} \tilde{x}(n) & if\ 0 \leq n \leq N - 1 \\ 0 & otherxise \end{cases}$$

For convenience, the following function is considered:

$$R_N(n) = \begin{cases} 1 & si\ 0 \leq n \leq N - 1 \\ 0 & ailleurs \end{cases}$$

Thus

$$\tilde{x}(n) = \tilde{x}(n) R_N(n) \qquad (4.18.b)$$

We have already seen that the DFS coefficients $\tilde{X}(k)$ of the periodic sequence $\tilde{x}(n)$ are periodical of period N. to maintain a duality between time and frequency we will choose the Fourier coefficients that we associate to the finite sequence to be a finite sequence corresponding to a single period.

.

$$\tilde{X}(k) = \left(X(k)\right)_N \qquad (4.19.a)$$

$$X(k) = \tilde{X}(k) R_N(k) \qquad (4.19.b)$$

We have

$$\tilde{X}(k) = \sum_{n=0}^{N-1} \tilde{x}(n) W_N^{kn} \qquad (4.20.a)$$

$$\tilde{x}(n) = \frac{1}{N} \sum_{k=0}^{N-1} \tilde{X}(k) W_N^{-kn} \qquad (4.20.b)$$

Since the sums in (4.20.a) and (4.20.b) take into account the interval [0, N-1], it follows that The DFT

$$\tilde{X}(k) = \begin{cases} \sum_{n=0}^{N-1} x(n) W_N^{kn} & 0 \le k \le N-1 \\ 0 & ailleurs \end{cases} \quad (4.21.a)$$

The IDFT (Inverse DFT)

$$\tilde{x}(n) = \begin{cases} \frac{1}{N} \sum_{k=0}^{N-1} X(k) W_N^{-kn} & 0 \le n \le N-1 \\ 0 & ailleurs \end{cases} \quad (4.21.b)$$

where $W_N = e^{-j\frac{2\pi}{N}}$. (4.21) represents the pair of the DFT

In a matrix form, (4.21.a) is given by

$$\begin{pmatrix} \tilde{X}(0) \\ \tilde{X}(1) \\ \tilde{X}(2) \\ . \\ . \\ . \\ \tilde{X}(N-1) \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 & \dots & 1 \\ 1 & W_N & W_N^2 & \dots & W_N^{N-1} \\ 1 & W_N^2 & W_N^4 & \dots & W_N^{2(N-1)} \\ . & . & . & & . \\ . & . & . & & . \\ 1 & W_N^{N-1} & W_N^{2(N-1)} & \dots & W_N^{(N-1)^2} \end{pmatrix} \begin{pmatrix} \tilde{x}(0) \\ \tilde{x}(1) \\ \tilde{x}(2) \\ . \\ . \\ . \\ \tilde{x}(N-1) \end{pmatrix}, \ W_N = e^{-j\frac{2\pi}{N}}$$

4. 6 Fast Fourier transform (FFT)

The invention of the FFT algorithm by Professors Cooley and Tukey in 1965 was a major advance in the field of digital signal processing. Prior to this date, the practical use of DFT was limited to problems where the number of data to be processed is relatively small. The DFT of equation (4.21) requires N multiplications to calculate a Fourier coefficient, the DFT calculation would be N^2 complex multiplications. So it's a big calculation. Cooley and Tukey have noticed that if N is a power of 2, it is possible to do a calculation much faster. Indeed, by using even and odd symmetries, it is possible to reduce the number of transactions to $N \log_2 N$ instead of N^2. For N = 1024, the FFT calculation time becomes 100 times shorter than the calculation time elapsed by the TFD. The algorithm is simple and very elegant if $N = 2^n$, there are other equivalent methods but clearly a little complex. Since the TFDI is equivalent to the TFD, with a sign and a factor of $1/N$, it is possible to generate the reverse transformation in the same way for the fast version. Attention, the name FFT does not deceive you, the FFT algorithms are only equivalent procedures of calculation of the DFT, and they are not new transformations. Currently, FFT is confidently applicable in most digital signal processing.

4. 6. 1. Cooley-Tukey's decomposition algorithm

The Cooley-Tukey algorithm for calculating DFT is based on the factorization of *N*, the size of DFT as the product of numbers below *N*. Putting *N* = PQ where both factors are greater than 1.

$$\tilde{X}(k) = \begin{cases} \sum_{n=0}^{N-1} \tilde{x}(n) W_N^{-nk} & 0 \le k \le N - 1 \\ 0 & otherwise \end{cases} \tag{4.22}$$

with $W_N = e^{j\frac{2\pi}{N}}$

Starting by dividing the domain of integer values from 0 to *N*-1 in two different ways. For the time index *n*, the division is for Q dimension intervals P. For the frequency index, *k*, the division is for P dimension intervals Q. We can then formulate the variables n and *k* as follows:

$$\begin{cases} n = Pq + p, & 0 \le \mathrm{q} \le \mathrm{Q} - 1, 0 \le \mathrm{p} \le \mathrm{P} - 1 \\ k = Qs + r, & 0 \le \mathrm{s} \le \mathrm{P} - 1, 0 \le \mathrm{r} \le \mathrm{Q} - 1 \end{cases} \tag{4.23}$$

The product n^k appears in the WN exhibitor whose TFD formula can be written according to p, q, r and s as follows

$$nk = (Qs + r)(Pq + p) = Nsq + Qsp + \Pr q + rp \tag{4.24}$$

For this

$$W_N^{-nk} = W_N^{-Nsq} W_N^{-Qsp} W_N^{-\Pr q} W_N^{-rp} \tag{4.25}$$

We have, $W_N^N = 1$, $W_N^Q = W_P$ et $W_N^P = W_Q$, thus

$$W_N^{-nk} = W_P^{-sp} W_Q^{-rq} W_N^{-rp} \tag{4.26}$$

Replacing (4.26) into (4.22), (4.22) will be

$$\begin{aligned} \tilde{X}(Qs + r) &= \sum_{p=0}^{P-1} \sum_{q=0}^{Q-1} \tilde{x}(Pq + p) W_P^{-sp} W_Q^{-rq} W_N^{-rp} \\ &= \sum_{p=0}^{P-1} W_N^{-rp} \left[\sum_{q=0}^{Q-1} \tilde{x}(Pq + p) W_Q^{-rq} \right] W_P^{-sp} \end{aligned} \tag{4.27}$$

We define the following *P* sequences of *Q* dimension

$$x_p(q) = x(Pq + p), \quad 0 \le q \le Q - 1, \ 0 \le p \le P - 1 \tag{4.28}$$

Each $x_p(q)$ is called a decimated sequence because it is obtained by choosing 'one' from P elements of x(n). For this, the selected elements are evenly spaced. The DFT of $\tilde{x}(q)$ is

$$\tilde{X}_p(r) = \sum_{q=0}^{Q-1} \tilde{x}_p(q) W_Q^{-rq} \tag{4.29}$$

Substitution of (4.29) into (4.27) gives

$$\tilde{X}_p(Qs+r) = \sum_{p=0}^{P-1} W_N^{-rp} \tilde{X}_p(r) W_P^{-sp} \tag{4.30}$$

We define for each $0 \le r \le Q-1$, the sequence of dimension P

$$\tilde{y}_r(p) = W_N^{-rp} \tilde{X}_p(r),\ 0 \le p \le P-1 \tag{4.31}$$

En remplaçant (4.31) dans (30), on obtient

$$\tilde{X}_p(Qs+r) = \sum_{p=0}^{P-1} \tilde{y}_r(p) W_P^{-sp} \tag{4.32}$$

Equation (4.32) with auxiliary definitions given by (4.28), (4.29) and (4.31) represents the Cooley-Tukey decomposition algorithm of the DFT. This algorithm can be implemented easily by the following procedure:

1. Form the decimated P-dimension sequences $x_p(q)$ and calculate the DFT associated with the Q-point for each
2. Multiply each output of each TFD by the corresponding complex number W_N^{-rp}, these numbers are called the rotated factors (twiddle factors).
3. For each r, determine the FDR associated with the P-point of the sequence $\tilde{y}_r(p)$.

As an example, the following figure represents the decomposition of Cooley-Tukey for N = 6, P = 3 and Q = 2. The three decimated sequences are, $\{x(0), x(3)\}$, $\{x(1), x(4)\}$ and $\{x(2), x(5)\}$. Each of these sequences is transformed using the DFT_2 operation. The outputs of the three DFTs are multiplied by the six factors (twiddle factors). After, the order of the numbers is changed to two sequences of the three numbers each, $\{y_0(0), y_0(1), y_0(2)\}$ and $\{y_1(0), y_1(1), y_1(2)\}$. Finally, each of these two sequences is transformed using the TFD_3 operation to find the DFT in question of the primary sequence of the signal to be processed $x(n)$. Figure 2 shows the decomposition operation of Cooley and Tukey DFT.

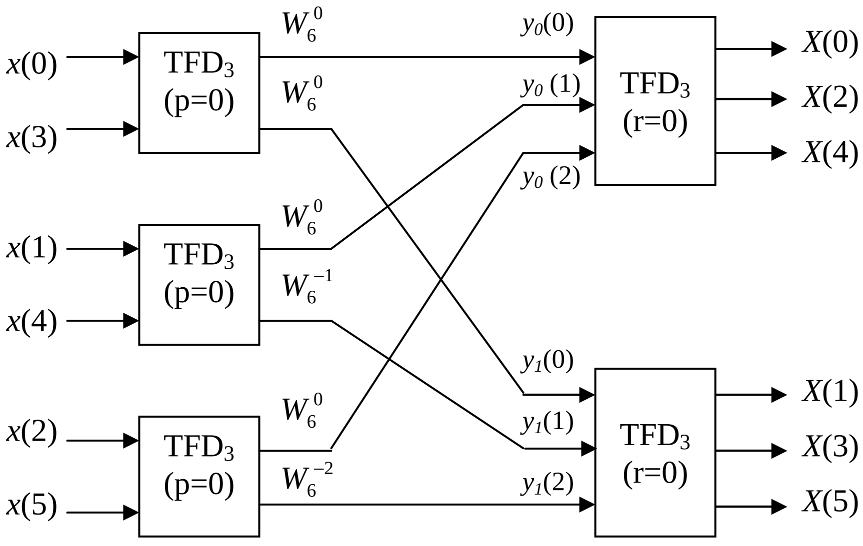


Figure. 5. 2 Cooley-Tukey's DFT algorithm illustrated for P = 3 and Q = 2.

4. 7 Recitation

Exercise #1:

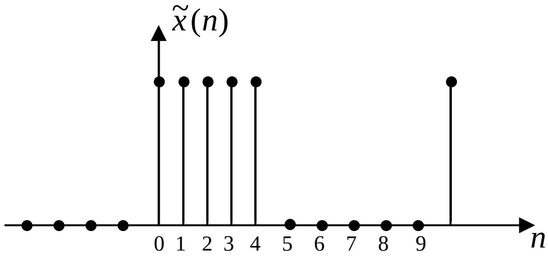
Given a periodic sequence $\tilde{x}(n)$ with a period *N*.

a- Show that the DFT of this delayed signal of *m* is given by $W_N^{-nm}\tilde{X}(k)$

b- Is there an ambiguity based on *m* values?

Exercise #2:

The periodic sequence $\tilde{x}(n)$ is represented by the following figure:



a- Determine the ZT of $\tilde{x}(n)$.

b- Calculate $\tilde{X}(k)$ using the ZT and the definition of the DFT.

Exercise #3:

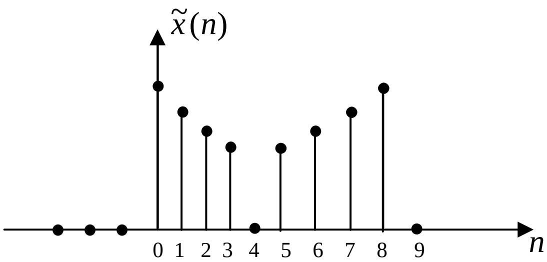
If $\tilde{x}(n) = a^n rect_N(n)$ is real signal with finite duration where $a<1$ is real.

a- Determine its DFT and discrete amplitude and phase spectrum,

where $a = 0.75$ and $N = 8$

Exercise #4:

If $\tilde{x}(n)$ is signal with finite duration $N = 8$ where its DFT is given by the following figure



We form a new signal $\tilde{y}(n)$ with period $N = 16$.

$$\tilde{y}(n) = \begin{cases} x(n/2) & for\ n\ \ even \\ 0 & for\ n\ \ odd \end{cases}$$

a- Sketch the form of $\tilde{Y}(k)$ and justify your response.