

Lab: N° 02

**Modeling and simulation of a three-level NPC**

**Lab objectives**

This lab aims to learn about:

- The modeling of a three-level NPC type inverter,
- The simulation of the three-level NPC inverter controlled by different sinusoidal modulation techniques,
- Programming in the Simulink environment using the Embedded MATLAB Function tool.

**Presentation of the three-phase NPC voltage inverter structure**

The three-level inverter presented in Figure (1) is composed of three identical arms; each consisting of four bidirectional switches and two clamped diodes. A bidirectional switch consists of a transistor and a diode in antiparallel. The different switches are assumed to be perfect, that is to say, the phenomena due to switching are neglected as well as the voltage drops across the active switches. We also assume that the voltages across the two input capacitors are constant and equal to half the DC bus voltage.

**Modeling of a three-phase three-level NPC inverter**

- Switch connection function

Each ideal switch is modeled by a connection function defined by:

$$S_{ik}^* = \begin{cases} 1 & \text{if } T_{ik}^* \text{ is closed et } T_{ik}^* \text{ is opened} \\ 0 & \text{if } T_{ik}^* \text{ is opened et } T_{ik}^* \text{ is closed} \end{cases}, \quad * = +, -; \quad i = 1, 2; \quad k = a, b, c$$

The connection functions of the switches located on the upper and lower half-arms are linked by the following complement command:

$$S_{ik}^- = \bar{S}_{ik}^+ = 1 - S_{ik}^+$$

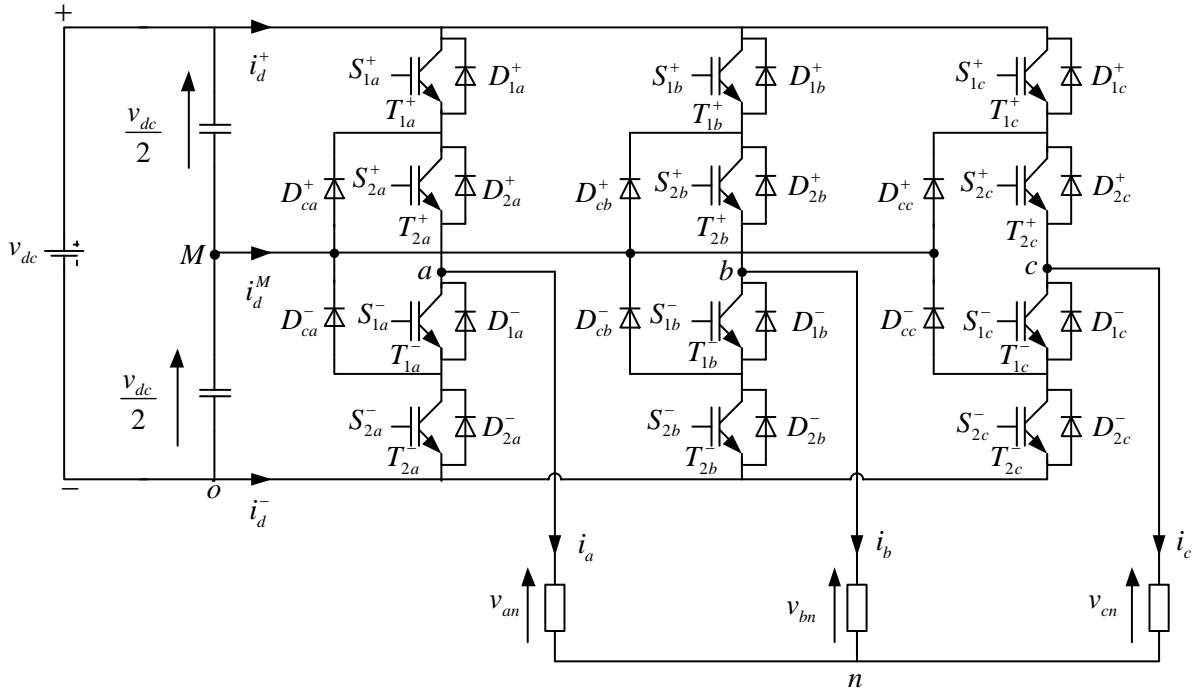


Figure (1): Diagram of the three-phase three-level NPC inverter

- Half arm connection functions

The connection function of a half-arm, denoted  $F_k^*$ , is expressed using the connection functions of the switches by:

$$F_k^* = S_{1k}^* S_{2k}^*; \quad * = +, -; \quad k = a, b, c$$

So, the connection functions of the six half-arms of a three-phase three-level NPC inverter are expressed by:

$$\begin{aligned} F_a^+ &= S_{1a}^+ S_{2a}^+, & F_a^- &= S_{1a}^- S_{2a}^- \\ F_b^+ &= S_{1b}^+ S_{2b}^+, & F_b^- &= S_{1b}^- S_{2b}^- \\ F_c^+ &= S_{1c}^+ S_{2c}^+, & F_c^- &= S_{1c}^- S_{2c}^- \end{aligned}$$

- Phase voltages relative to the midpoint of the DC bus

The potential of the node "k" of an arm of the three-level NPC inverter with respect to the midpoint "M" is given by:

$$v_{kM} = F_k^+ \frac{v_{dc}}{2} - F_k^- \frac{v_{dc}}{2} = (F_k^+ - F_k^-) \frac{v_{dc}}{2} = (S_{1k}^+ S_{2k}^+ - S_{1k}^- S_{2k}^-) \frac{v_{dc}}{2}$$

The three voltages  $v_{aM}$ ,  $v_{bM}$  and  $v_{cM}$  are therefore expressed by:

$$v_{aM} = (S_{1a}^+ S_{2a}^+ - S_{1a}^- S_{2a}^-) \frac{V_{dc}}{2}$$

$$v_{bM} = (S_{1b}^+ S_{2b}^+ - S_{1b}^- S_{2b}^-) \frac{V_{dc}}{2}$$

$$v_{cM} = (S_{1c}^+ S_{2c}^+ - S_{1c}^- S_{2c}^-) \frac{V_{dc}}{2}$$

- Line-to-line voltages

The line-to-line voltages  $u_{ab}$ ,  $u_{bc}$  and  $u_{ca}$  are obtained from the following relations:

$$\begin{cases} u_{ab} = v_{aM} - v_{bM} = (S_{1a}^+ S_{2a}^+ - S_{1a}^- S_{2a}^-) \frac{V_{dc}}{2} - (S_{1b}^+ S_{2b}^+ - S_{1b}^- S_{2b}^-) \frac{V_{dc}}{2} = ((S_{1a}^+ S_{2a}^+ - S_{1a}^- S_{2a}^-) - (S_{1b}^+ S_{2b}^+ - S_{1b}^- S_{2b}^-)) \frac{V_{dc}}{2} \\ u_{bc} = v_{bM} - v_{cM} = (S_{1b}^+ S_{2b}^+ - S_{1b}^- S_{2b}^-) \frac{V_{dc}}{2} - (S_{1c}^+ S_{2c}^+ - S_{1c}^- S_{2c}^-) \frac{V_{dc}}{2} = ((S_{1b}^+ S_{2b}^+ - S_{1b}^- S_{2b}^-) - (S_{1c}^+ S_{2c}^+ - S_{1c}^- S_{2c}^-)) \frac{V_{dc}}{2} \\ u_{ca} = v_{cM} - v_{aM} = (S_{1c}^+ S_{2c}^+ - S_{1c}^- S_{2c}^-) \frac{V_{dc}}{2} - (S_{1a}^+ S_{2a}^+ - S_{1a}^- S_{2a}^-) \frac{V_{dc}}{2} = ((S_{1c}^+ S_{2c}^+ - S_{1c}^- S_{2c}^-) - (S_{1a}^+ S_{2a}^+ - S_{1a}^- S_{2a}^-)) \frac{V_{dc}}{2} \end{cases}$$

- Phase voltages

Let "n" be the neutral point of the alternating side of the load, then we have:

$$\begin{cases} v_{aM} = v_{an} + v_{nM} \\ v_{bM} = v_{bn} + v_{nM} \\ v_{cM} = v_{cn} + v_{nM} \end{cases}$$

The load is considered to be balanced; it follows that:

$$v_{an} + v_{bn} + v_{cn} = 0$$

The sum of the voltages  $v_{aM}$ ,  $v_{bM}$  and  $v_{cM}$  leads, using the previous equation, to the expression of the voltage  $v_{nM}$  given by:

$$v_{nM} = \frac{1}{3}(v_{aM} + v_{bM} + v_{cM})$$

The three phase output voltages of the inverter as a function of the voltages  $v_{aM}$ ,  $v_{bM}$  and  $v_{cM}$  are therefore calculated by:

$$\begin{cases} v_{an} = v_{aM} - v_{nM} = v_{aM} - \frac{1}{3}(v_{aM} + v_{bM} + v_{cM}) = \frac{2}{3}v_{aM} - \frac{1}{3}v_{bM} - \frac{1}{3}v_{cM} \\ v_{bn} = v_{bM} - v_{nM} = v_{bM} - \frac{1}{3}(v_{aM} + v_{bM} + v_{cM}) = -\frac{1}{3}v_{aM} + \frac{2}{3}v_{bM} - \frac{1}{3}v_{cM} \\ v_{cn} = v_{cM} - v_{nM} = v_{cM} - \frac{1}{3}(v_{aM} + v_{bM} + v_{cM}) = -\frac{1}{3}v_{aM} - \frac{1}{3}v_{bM} + \frac{2}{3}v_{cM} \end{cases}$$

The three phase-to-neutral output voltages of the NPC inverter are expressed in terms of the connection functions of the switches by:

$$\begin{cases} v_{an} = \frac{2}{3}(S_{1a}^+S_{2a}^+ - S_{1a}^-S_{2a}^-) \frac{V_{dc}}{2} - \frac{1}{3}(S_{1b}^+S_{2b}^+ - S_{1b}^-S_{2b}^-) \frac{V_{dc}}{2} - \frac{1}{3}(S_{1c}^+S_{2c}^+ - S_{1c}^-S_{2c}^-) \frac{V_{dc}}{2} \\ v_{bn} = -\frac{1}{3}(S_{1a}^+S_{2a}^+ - S_{1a}^-S_{2a}^-) \frac{V_{dc}}{2} + \frac{2}{3}(S_{1b}^+S_{2b}^+ - S_{1b}^-S_{2b}^-) \frac{V_{dc}}{2} - \frac{1}{3}(S_{1c}^+S_{2c}^+ - S_{1c}^-S_{2c}^-) \frac{V_{dc}}{2} \\ v_{cn} = -\frac{1}{3}(S_{1a}^+S_{2a}^+ - S_{1a}^-S_{2a}^-) \frac{V_{dc}}{2} - \frac{1}{3}(S_{1b}^+S_{2b}^+ - S_{1b}^-S_{2b}^-) \frac{V_{dc}}{2} + \frac{2}{3}(S_{1c}^+S_{2c}^+ - S_{1c}^-S_{2c}^-) \frac{V_{dc}}{2} \end{cases}$$

Finally, the phase voltages across the balanced three-phase load are expressed as a function of the connection functions of the switches by the following matrix relationship:

$$\begin{pmatrix} v_{an} \\ v_{bn} \\ v_{cn} \end{pmatrix} = \frac{V_{dc}}{3} \begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix} \begin{pmatrix} S_{1a}^+S_{2a}^+ - S_{1a}^-S_{2a}^- \\ S_{1b}^+S_{2b}^+ - S_{1b}^-S_{2b}^- \\ S_{1c}^+S_{2c}^+ - S_{1c}^-S_{2c}^- \end{pmatrix}$$

- Inverter input currents

The inverter input currents  $i_d^+$ ,  $i_d^-$  and  $i_{dM}$  are expressed as a function of the phase currents  $i_k$ ,  $k = a, b, c$  and the half-arm connection functions as follows:

$$\begin{aligned} i_d^+ &= F_a^+ i_a + F_b^+ i_b + F_c^+ i_c \\ i_d^- &= F_a^- i_a + F_b^- i_b + F_c^- i_c \\ i_{dM} &= -(i_d^+ + i_d^-) \end{aligned}$$

### **Simulation of the three-level NPC inverter controlled by the LSPWM technique**

- Principle of the LSPWM technique

The sine wave SPWM modulation technique with vertically shifted carriers (Level Shifted PWM, LSPWM) of a three-level NPC inverter consists of comparing three reference voltages to two triangular carriers of the same frequency and same amplitude. The three reference voltages form a balanced three-phase system given by the following equation:

$$\begin{aligned} v_{aref} &= m_a \frac{V_{dc}}{2} \sin(2\pi f_r t) \\ v_{bref} &= m_a \frac{V_{dc}}{2} \sin(2\pi f_r t - \frac{2\pi}{3}) \\ v_{cref} &= m_a \frac{V_{dc}}{2} \sin(2\pi f_r t + \frac{2\pi}{3}) \end{aligned}$$

where  $m_a \in [0 \ 1]$  is the amplitude modulation index and  $f_r$  is the reference voltages frequency fixed in our case at 50Hz.

### **- Simulation scheme**

The simulation diagram of a three-phase three-level NPC inverter connected to a star-coupled three-phase load ( $R = 1\Omega$ ,  $L = 10mH$ ) is depicted in Figure (2). The inverter is powered by two DC voltage sources 100V each.

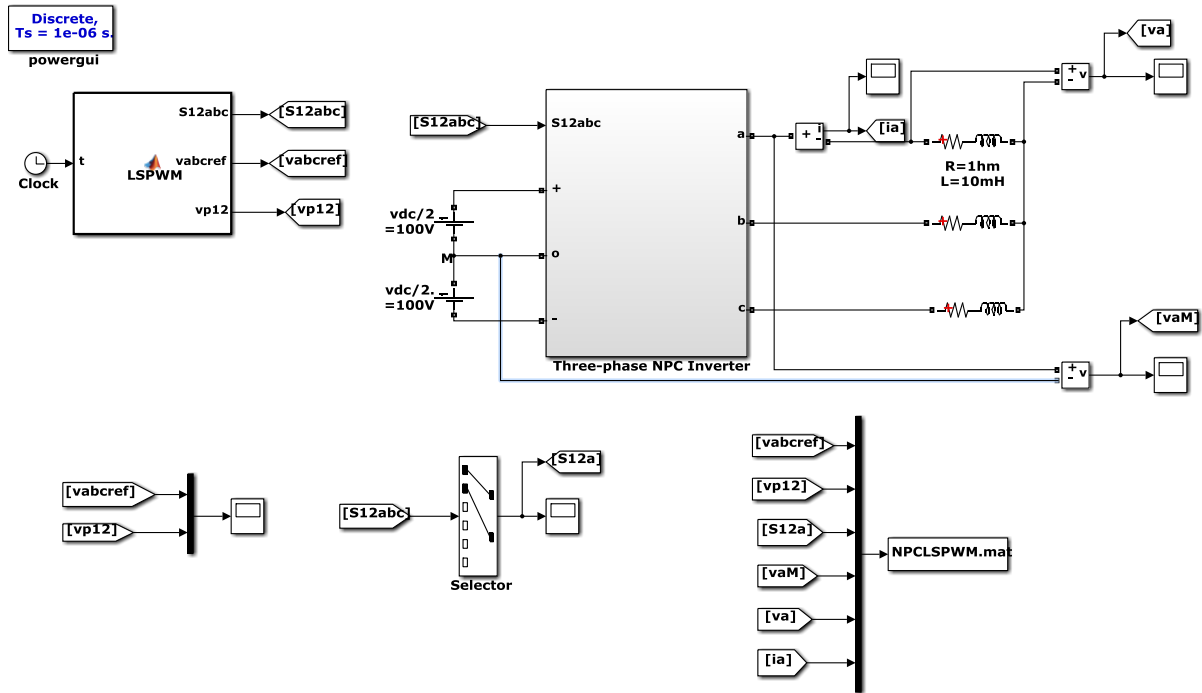


Figure (2) : Schéma block de simulation d'un onduleur triphasé à structure NPC contrôlé par la modulation LSPWM

Figure (2): Simulation block diagram of a three-phase three-level NPC inverter controlled by LSPWM modulation

The “Three-phase NPC inverter” block represents the power circuit of the inverter detailed as follows:

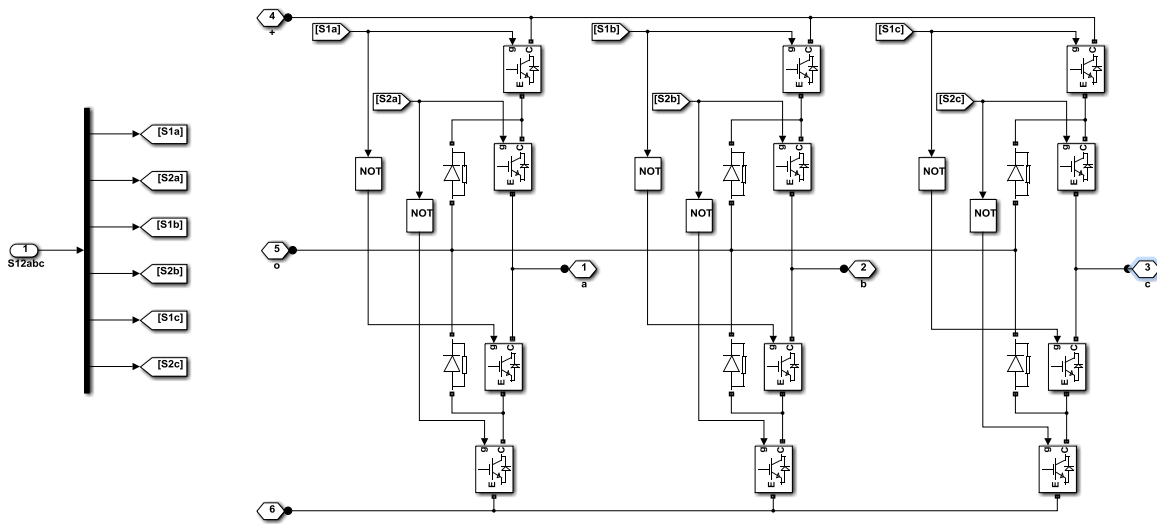


Figure (3): Block diagram of the three-phase NPC inverter

### - Implementation of the LSPWM technique

The Shifted-Carrier Sinusoidal PWM (LSPWM) technique is programmed using the Embedded MATLAB Function block (also known as Embedded MATLAB Function). This block is part of the User Defined Functions library. It can be inserted into a Simulink model in the same way as any

other block. The following code represents the implementation of the LSPWM algorithm using MATLAB Function.

```
function [S12abc,vabcref,vp12] = LSPWM(t)
%#codegen

% SPWM technique with vertically shifted carriers of a three-level NPC inverter

% Parameters

% DC voltage
vdc=200;
% Frequency modulation index
mf=9;
% Amplitude modulation index
ma=0.85;
% Reference signal frequency
fr=50;
% Carrier signal frequency
fp=m*fr;
% Carrier signal amplitude
Vpm=vdc/2;
% Reference signal frequency amplitude
Vrm=r*Vpm;
% Reference signal angular frequency
w=2*pi*fr;

% Reference voltages

varef=Vrm*sin(w*t);
vbref=Vrm*sin(w*t-2*pi/3);
vcref=Vrm*sin(w*t+2*pi/3);

% Vertically displaced triangular carriers

vp1=Vpm*(1/2-asin(cos(2*pi*fp*t))/pi);
vp2=vp1-Vpm;

% Switching signals generation

if varef > vp1, S1a=1; else S1a=0; end
if varef > vp2, S2a=1; else S2a=0; end
if vbref > vp1, S1b=1; else S1b=0; end
if vbref > vp2, S2b=1; else S2b=0; end
if vcref > vp1, S1c=1; else S1c=0; end
if vcref > vp2, S2c=1; else S2c=0; end
```

## % Outputs

```
S12abc=[S1a;S2a;S1b;S2b;S1c;S2c];
```

```
vp12=[vp1;vp2];
```

```
vabcref=[varef;vbref;vceref];
```

- Plot of results

To plot the results obtained, we can, in a first phase, save the results data in a “.mat” file. Subsequently, this file is reloaded again and used to plot the simulation results. By clicking twice on the “NPCLSPWM.mat” block, the following dialog window appears:

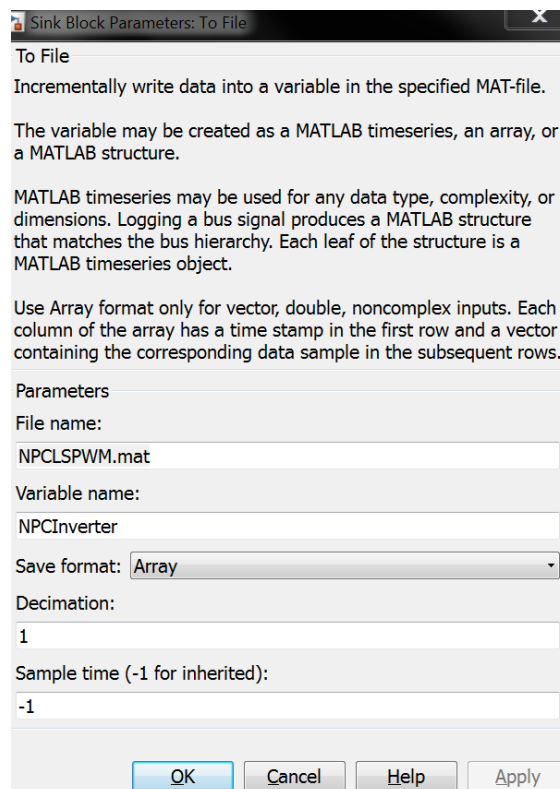


Figure (4): Saving data into a MAT file

In the dialog window presented in Figure (4), the file name and variable name must be specified. The save format should be selected as “Array”.

The following Matlab code reloads the NPCLSPWM.mat file and plots the results.

```
clc; clear all; close all;
```

```
load NPCLSPWM.mat
```

```
t=NPCInverter(1,:);
```

```
varef=NPCInverter(2,:);
```

```
vbref=NPCInverter(3,:);
```

```

vcref=NPCInverter(4,:);
vp1=NPCInverter(5,:);
vp2=NPCInverter(6,:);
S1a=NPCInverter(7,:);
S2a=NPCInverter(8,:);
vaM=NPCInverter(9,:);
va=NPCInverter(10,:);
ia=NPCInverter(11,:);

taille=9;

figure(1)
plot(t,varef,'k',t,vbref,'k',t,vcref,'k',t,vp1,'k-',t,vp2,'k-');
xlabel('Time (s)','FontSize',taille,'FontName','times new roman','FontWeight','bold');
ylabel('v_a_b_c_r_e_f      v_p_1      v_p_2','FontSize',taille,'FontName','times new roman','FontWeight','bold');
axis([0.04 0.06 -105 105])

figure(2)
plot(t,S1a,'k',t,S2a,'k-');
xlabel('Time (s)','FontSize',taille,'FontName','times new roman','FontWeight','bold');
ylabel('S_1_a, S_2_a','FontSize',taille,'FontName','times new roman','FontWeight','bold');

axis([0.04 0.06 0 1.01])

figure(3)
plot(t,vaM,'k')
xlabel('Time (s)','FontSize',taille,'FontName','times new roman','FontWeight','bold');
ylabel('v_a_M (V)','FontSize',taille,'FontName','times new roman','FontWeight','bold');
axis([0.04 0.06 -105 105])

figure(4)
plot(t,va,'k')
xlabel('Time (s)','FontSize',taille,'FontName','times new roman','FontWeight','bold');
ylabel('v_a (V)','FontSize',taille,'FontName','times new roman','FontWeight','bold');
axis([0.04 0.08 -140 140])

figure(5)
plot(t,ia,'k')
xlabel('Time (s)','FontSize',taille,'FontName','times new roman','FontWeight','bold');
ylabel('i_a (V)','FontSize',taille,'FontName','times new roman','FontWeight','bold');
axis([0.04 0.08 -30 30])

```

### **- Required work**

1- Simulation of the LSPWM technique



- a) Plot the three references signals, the two carriers as well as the two control signals  $S_{1a}^+$  and  $S_{2a}^+$
- b) Plot the waveforms of the voltage relative to the midpoint, the phase voltage and the current of the first phase.
- c) Plot the harmonic spectrum of the voltage of the first phase for the first 100 harmonics.
- d) Fill in the following table representing the variation of the total harmonic distortion (THD) and the amplitude of the fundamental of the phase voltage ( $V_{a1}$ ) as a function of the amplitude modulation index  $m_a$ .

$m_a$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5
THD															
$V_{a1}$															

Plot the variations in the harmonic distortion rate and the fundamental of the first phase voltage as a function of the amplitude modulation index. Comment on the obtained results.

- e) Compare sinusoidal modulations in terms of THD when the carriers are in phase disposition (PD) and in phase opposition disposition (POD).

## 2- Simulation of the LSPWM technique with injection of the third harmonic.

It is possible to improve the inverter control by injecting the third harmonic into the reference modulating voltages. It should be noted that this injection does not affect the quality of the phase or line-line voltage due to the fact that the output voltage of the three-phase inverter does not contain harmonics that are multiples of three. The reference signal is composed of fundamental and third harmonic frequency components as following equations:

$$v'_{kref} = v_{kref} + a \frac{V_{dc}}{2} \sin(6\pi f_r t), \quad k = a, b, c$$

- a°) What is the objective of adding a third harmonic into the fundamental reference voltages?
- b°) Show that the optimal injection rate is:  $a = 1/6$ .
- c°) Redo the work requested in the first question. Give conclusions?
- d°) By injecting the so-called "zero-sequence voltage" component  $v_{offset}$  into the sinusoidal reference modulating voltages we obtain new references defined by:

$$\begin{aligned} v''_{aref} &= v_{aref} + v_{offset} \\ v''_{bref} &= v_{bref} + v_{offset} \\ v''_{cref} &= v_{cref} + v_{offset} \end{aligned} \quad \text{with } v_{offset} = -\frac{\max(v_{aref}, v_{bref}, v_{cref}) + \min(v_{aref}, v_{bref}, v_{cref})}{2}$$

What is the purpose of this injection and how does it differ from the injection of the third harmonic.

## 3- Write a Matlab code using Embedded MATLAB Function to simulate the sinusoidal modulation with phase-shifted carriers (Phase Shifted PWM, PSPWM).