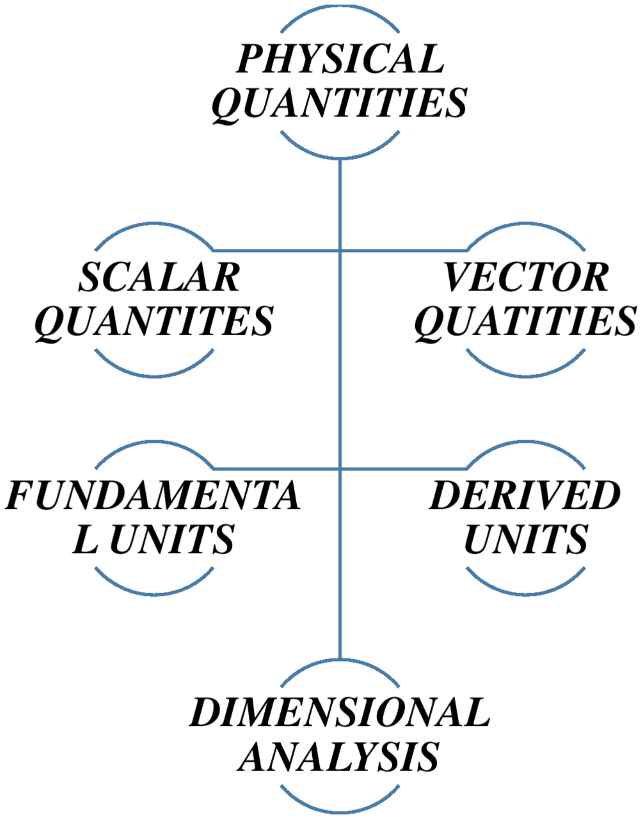
COURSE MIND MAP



Understanding physics easily, especially in classical mechanics, requires giving significant importance to building accurate concepts and providing a physical perspective on mathematical relationships. Therefore, our goal through this compilation of lessons is to reanalyse and review many of the physical principles and connect some mathematical concepts with their physical meanings.

1) PHYSICAL QUANTITY

*In physics, a **physical quantity** is any **measurable property** that describes a specific aspect of an object or a system.*

Examples of physical quantities: *mass, time, distance, area, volume, energy, density, pressure...velocity, acceleration, force and temperature …. Etc*

The physical quantities can be categorized into two types:

1-1) Scalar quantities

*Scalar quantities are properties described just by their **magnitude** (numerical value) and the appropriate **unit of measurement**. Examples of scalar quantities include:*

Mass (e.g., 15 kg), Temperature (e.g., 30°C), Energy (e.g., 100 J), Time (e.g., 10 seconds).

1.2) Vector quantities

*Vector quantities are properties described by specifying their **direction**, in addition to their **magnitude** (numerical value) and their appropriate **unit of measurement.** Examples of vector quantities include:*

Velocity, Force, Displacement, Acceleration.

2) Units of measurement

Units of measurement are symbols added to values to give them specific physical meanings. Units are divided into two main groups:

2-1) Fundamental Units: *These are the basic building blocks of measurement and are inherently independent, meaning they can't be expressed in terms of other units. They include crucial physical quantities like mass, time, energy, electric charge, length, and distance.*

2-2) Derived Units: *Derived units are units that can be expressed as a combination of fundamental units. Examples include:*

The newton (N), derived from kilogram, meter, and second, measures force.

The joule (J), a combination of newton and meter, quantifies energy.

The watt (W), obtained from joule and second, is used for power measurement.

The coulomb (C), derived from ampere and second, quantifies electric charge.

The hertz (Hz), based on the reciprocal of the second (1/s), represents frequency.

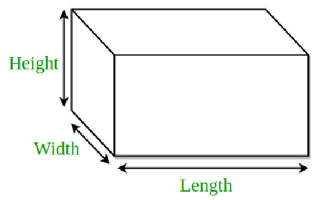
3) The International System of Units (SI)

Measurement units for physical quantities can vary from one country to another. For example, in the United States, units like inches (1 inch = 2.54 cm) and pounds (1 pound = 0.45 kg) are used to measure distance and mass. These units may not be widely recognized in many other countries, leading to challenges in scientific communication. Therefore, to establish a uniform unit system, a consensus has been reached, resulting in the International System of Units (SI), which consists of seven fundamental units as outlined in the table below.

Table01: The seven independent SI base units

Base quantity	Unit name	Unit Symbol
Length	Meter	m
Time	Second	s
Mass	Kilogram	Kg
Temperature	Kelvin	K
Electric current	Ampere	A
Amount of substance	Mole	mol
Luminous intensity	Candela	cd

Example01: *Find the derived unit for the following quantities*



- *Volume = Lenght × Width × Hight*

Volume unit ≡ Length unit × Width unit × Hight unit≡ m×m×m ≡m^3

- *Velocity = distance / time ⇒ Velocity unit ≡ distance unit / time unit = m/s*
- *Acceleration = velocity/ time ⇒ Acceleration unit ≡ velocity unit / time unit ≡ m/s^2*
- *Force = mass× acceleration ⇒ Force unit ≡ mass unit × acceleration unit ≡ $Kg.m/s^2$ ≡ **N***

4) DIMENSIONAL ANALYSIS

4-1) Dimension definition

In physics, the term "dimension" signifies the inherent property of a physical quantity. It classifies the measurement units associated with a specific physical quantity. For instance, the *measurement of distance between two points can be done using units like feet, meters, or kilometres, with each unit representing a unique aspect of the dimension known as length. Similarly, when measuring the mass of an object, it can be quantified using units like kilogram, gram, pound, or milligram. These units all share a common nature, as they represent mass, and therefore they have a mass dimension. To denote the dimension, we typically write the symbol of the quantity within square brackets* ***[quantity].***

Table02: Base Quantities and Their Dimensions

Base quantity	Symbol for dimension
Length	L
Time	T
Mass	M
Temperature	Θ
Electric current	I
Amount of substance	N
Luminous intensity	J

The dimension of the derived physical quantity (Q) can be expressed as:

$$[Q] = L^a M^b L^c T^d I^e \Theta^f N^g J^h$$

In this expression, the variables a, b, c, d, e, f, and g represent the powers associated with the fundamental physical dimensions.

4-2) The dimensional equation

The dimensional equation is an equation that expresses the relationship between different physical quantities through their dimensions side. We can obtain the dimensional equation by rewriting the original equation in a way that represents each physical quantity in terms of its dimension. In other words, the symbols used for physical quantities are replaced with symbols representing their dimensions.

Through this process, we can verify that the relationship between the physical quantities in the equation is correctly aligned in terms of dimensions and units, ensuring the accuracy and compliance of the equation with the laws of physics.

Example02: *let as assume we have a physics equation that relates force (F) to mass (m) and acceleration (a). This dimensional equation can be represented as follows:* $\vec{F} = m \cdot \vec{a}$

In this equation:

$\vec{F}$ represents force and can be expressed in units of Newton (N).

m represents mass and can be expressed in units of kilograms (kg).

$\vec{a}$ represents acceleration and can be expressed in units of meters per second squared (m/s^2).

If we analyze the dimensions in this dimensional equation, we find that they align correctly:

$[F] = [m]\ [a] = M.L.T^{2} \equiv N$

This indicates that the unit of Newton for force correctly corresponds to the units of mass and acceleration. Thus, the equation is dimensionally and physically correct.

4-3) Properties of dimensional equation

"Let F, A, B, and C are different physical quantities, and 'm' and 'n' are real numbers.

- *The equation **$F + C = B + A$** is dimensionally correct only if all the quantities F, C, B, and A, have the same dimensions(**$[F] =[C] = [B] =[A]$**).*
- *The dimension of any constant real number is dimensionless (If **$F = n \Rightarrow [F] = 1$**).*
- ***$F = n \times A$** we can analyse its dimension as **$[F] = [n] \times [A] \Rightarrow [F] = 1 \times [A]$**. This demonstrates that F possesses the same dimension as A."*
- ***$F = B \times A \times C$**, we can determine its dimension as '**$[F] = [B] \times [A] \times [C]$**.' This equation displays how the dimension of F is a product of the dimensions of A, B, and C."*
- ***$F = A^m \Rightarrow [F] = [A]^m$**. This illustrates that F's dimension is related to A raised to the power of 'm'*
- ***$F = dA/dx \Rightarrow [F] = [A]/[x]$**.*
- ***$F = \int(A\ dy) \Rightarrow [F] = [A] \times [y]$**.*
- ***$F = \sqrt{A} \Rightarrow |F| = |\sqrt{A}| = \sqrt{|A|}$***

Example03:

Find the dimension of the following quantities: velocity, acceleration, force, Charge quantity, , Kinetic Energy , and Gravitational potential energy.

- ***Velocity:** $V=x/t \Rightarrow [V]= [x]/[t]= L/T$*
- ***Acceleration:** $a=v/t \Rightarrow [a]= [v]/[t]=(L/T)/T = L.T^{-2}$*
- ***Force:** $\vec{F} = m\,\vec{a} \Rightarrow [F]= [m].[a]= M.L.T^{-2}$*
- ***Charge quantity:** $Q = i \times t \Rightarrow [Q] = [i] \times [t] \Rightarrow [Q]= I.T$*
- ***Kinetic Energy:** $E = \frac{1}{2}\,mV^2 \Rightarrow [E] = \left[\frac{1}{2}\right]\ [m][V^2] = M\ L^2T^2$*
- ***Gravitational potential energy:** $E = m.g.h \Rightarrow [E] = [m]\ [g][h] \Rightarrow [E] = M.L.T^{-2}.L = M\ L^2T^2$*

Exercise 01:

The following equation expresses the change in displacement with respect to time and the acceleration of a moving object.

$$x = \frac{1}{2}at^2$$

1 - Write the homogeneity condition of the equation.

2 - Prove its dimensional consistency using the dimensional analysis method.

Solution

1. *The homogeneity condition of the equation :*

$$[dimenion\ of\ the\ left\ side\,] = [dimension\ of\ the\ right\ side\,]$$

2- Prove its dimensional consistency using the dimensional analysis method.

The left side of the equation $[x\,] = L$

The right side of the equation $\left[\frac{1}{2}\right]\ [a][t^2] = [a][t]^2 = L\,T^{-2}T^2 = L$

The dimension of the left side equals the dimension of the right side, indicating that the equation is dimensionally correct.

Exercise 02:

What are the dimensions of both k and v_0 *constants in the following equation?*

$v = kt + v_0$ *where v represents velocity and t represents time*

Solution

The equation is dimensionally correct if both sides have the same dimension.

$$[v] = [k][t] = [v_0]$$

$$LT^{-1} = [k]T = [v_0]$$

$$LT^{-1} = [v_0]\ and\ LT^{-2} = [k]$$

Exercise 03:

Determine the physical dimension of the spring constant and then verify the homogeneity of the equation that relates the spring constant to the elastic potential energy

Solution

1) *The physical dimension of the spring constant (elasticity constant) can be found using the formula related the force exerted by a spring to its displacement*

$$F = -Kx$$

Where

F *is the force applied to the spring.*

K *is the spring constant.*

x *is the displacement from the equilibrium position.*

To find the dimension of K, *we can rearrange the equation as follows:*

$$K = -\frac{F}{x}$$

So, we can find the dimension of the spring constant K *as follows:*

$$[K] = \frac{[F]}{[x]} = \frac{M.L.T^{-2}}{L} = M.T^{-2}$$

So, the physical dimension of the spring constant (K) is $M.T^{-2}$

2) *To verify the homogeneity of the equation connecting the spring constant to elastic potential energy, you can follow these steps:*

- *Express the equation for elastic potential energy* (U) *in terms of the spring constant* (K) *and the displacement* (x).

$$U = \frac{1}{2}Kx^2$$

- *Determine the dimensions of each variable involved:*

The dimension of elastic potential energy $[U]$ *is energy, which is represented as* $[U] = M\,L^2T^2$
We already found the dimension of the spring constant $[K]$ *in a previous response:* $[K] = M.T^{-2}$
The dimension of displacement (x) is length, $[x] = L$

Now, substitute K and x dimensions into the equation for elastic potential energy:

$$[K][x]^2 = M.T^{-2}.L^2 = [U]$$

The equation is homogeneous because the dimensions on both sides match. Therefore, the equation connecting the spring constant to elastic potential energy is consistent in terms of dimensions.