UNIVERSITY OF MSILA **2023/2024**

FIRST SERIES IN ALGEBRA 1

EXERCISE01

Make a truth table for the statement $(P \vee Q) \to (P \wedge Q)$.

Make a truth table for the statement $\neg P \wedge (Q \to P)$.

What can you conclude about P and Q if you know the statement is true?

Determine whether the following two statements are logically equivalent:

Determine whether the following two statements are logically equivalent: $\neg(P \to Q)$ and $P \wedge \neg Q$.

EXERCISE02

Among the following expressions, which ones are propositions? For propositions, indicate whether they are true or false.

(a) $2+3=5$

(b) $\forall n \in \mathbb{N}, \quad n+2=4$

(c) $\exists n \in \mathbb{N} \quad n+2=3$

(d) This exercise is difficult

(e) $x \in \mathbb{N}$

EXERCISE03

Consider the following four propositions:

Are these propositions true or false? Provide their negations.

a- $\exists x \in \mathbb{R}, \forall y \in \mathbb{R} \quad x+y>0$

b- $\forall x \in \mathbb{R}, \exists y \in \mathbb{R} \quad x+y>0$

c- $\forall x \in \mathbb{R}, \forall y \in \mathbb{R} \quad x+y>0$

d- $\exists x \in \mathbb{R}, \forall y \in \mathbb{R} \quad y^2 > x.$

EXERCISE04

1) Let $n \in \mathbb{N}$. Prove by cases that $n(n^2+2)$ is a multiple of 3.

2) Prove by contradiction that $(\forall n \in \mathbb{N}^*, \exists p \in \mathbb{N}^* : n = p^2) \Rightarrow (\forall q \in \mathbb{N}^* : 2n \neq q^2)$.

SOLUTION

P	Q	$(P \vee Q) \to (P \wedge Q)$
T	T	T
T	F	F
F	T	F
F	F	T

P	Q	$\neg P \wedge (Q \to P)$
T	T	F
T	F	F
F	T	F
F	F	T

If the statement is true, then both P and Q are false.

Make a truth table for each and compare. The statements are logically equivalent.

Exercise3

(a) True
(b) False
(c) False
(d) False
Theire negation change for all by existe and existe by for all
And change $<$ by $\geq$

a. This expression is a true proposition.

b. This expression is a false proposition because for $n = 1 \in \mathbb{N}$, we have $n + 2 = 3 \neq 4$.

c. This expression is a true proposition because there exists an element $n = 1 \in \mathbb{N}$ such that $n + 2 = 3$.

d. This expression is not a proposition because we cannot assign a truth value to it.

e. This expression is not a proposition because we don't know the nature of the element x, so we cannot assign a truth value to it.

1) Let $n \in \mathbb{N}$. We have:

<u>1st case</u> : If $n = 3k$, with $k \in \mathbb{N}$, then $n(n^2 + 2) = 3k((3k)^2 + 2)$, which is a multiple of 3.

<u>2nd case</u> : If $n = 3k + 1$, with $k \in \mathbb{N}$, then $n(n^2 + 2) = (3k + 1)((3k + 1)^2 + 2) = (3k + 1)(9k^2 + 6k + 1 + 2)$

$$= 3(3k + 1)(3k^2 + 2k + 1) \text{ which is a multiple of 3}$$

<u>3rd case</u> : If $n = 3k + 2$, with $k \in \mathbb{N}$, then

$$\begin{aligned} n(n^2 + 2) &= (3k + 2)((3k + 2)^2 + 2) = (3k + 2)(9k^2 + 12k + 4 + 2) \\ &= 3(3k + 2)(3k^2 + 4k + 2) \text{ which is a multiple of 3.} \end{aligned}$$

Therefore, in all cases, $n(n^2 + 2)$ is a multiple of 3.

2) Suppose that $(\forall n \in \mathbb{N}^*, \exists p \in \mathbb{N}^* : n = p^2)$ and $(\exists q \in \mathbb{N}^* : 2n = q^2)$.

Let $n \in \mathbb{N}^*$, then $n = p^2$ and $2n = q^2$ with $p, q \in \mathbb{N}^*$, so $2p^2 = q^2$, which implies $\sqrt{2} = \frac{q}{p} \in \mathbb{Q}$, which is absurd since $\sqrt{2}$ is irrational.