## Series $\mathbf{N}^{\circ} \mathbf{0 2}$ : Kinematics of material point

## EXERCISE 01

A cannon fires cannon ball 1 of mass $m_{l}=12 \mathrm{~kg}$ horizontally at constant velocity $\boldsymbol{v}=20 \mathrm{~m} / \mathrm{s}$. At the same time, cannon ball 2 of mass $m_{2}=24 \mathrm{~kg}$ is dropped from an equal height. The fired ball lands after a time $t_{1}$, while the dropped ball lands after a time $t_{2}$.
Ignoring air resistance, which of the following is true?
$\mathbf{a}-\mathrm{t}_{1}>\mathrm{t}_{2}, \mathbf{b}-\mathrm{t}_{1}<\mathrm{t}_{2}, \mathbf{c}-\mathrm{t}_{1}=\mathrm{t}_{2}, \mathbf{d}$ - It is not possible to determine the relationship between $t_{1}$ and $t_{2}$.

## EXERCISE 02

A ball is thrown upward from the top of a building with an initial velocity $v=20 \mathrm{~m} / \mathrm{s}$. The building is 40 m high and the ball just misses the edge of the building roof on its way down; see Figure and take $g=10 \mathrm{~m} / \mathrm{s}^{2}$. Neglecting air resistance, find: (a) the time $t_{l}$ for the ball to reach its highest point, (b) how high will it rise, (c) how long will it take to return to its starting point, (d) the velocity $V_{2}$ of the ball at this instant, and (e) the velocity $V_{3}$ and the total time of flight $t_{3}$ just before the ball hits the ground.


## EXERCISE 03

A car moving along the $x$-axis starts from the position $x_{\mathrm{i}}=2 \mathrm{~m}$ when $t_{\mathrm{i}}=0$ and stops at $x_{\mathrm{f}}=-3 \mathrm{~m}$ when $t_{\mathrm{f}}=2 \mathrm{~s}$. (1) Find the displacement, the average velocity, and the average speed during this interval of time.
(2) If the car goes backward and takes 3 s to reach the starting point, Find the displacement, the average velocity, and the average speed for the whole time interval.

## EXERCISE 04 (homework)

The points A and B lie on a straight line, 240 m apart. At $\mathrm{t}=0$, a particle passes through A with velocity $4 \mathrm{~m} / \mathrm{s}$ heading towards B with constant acceleration $0.752 \mathrm{~m} / \mathrm{s}^{2}$. At $\mathrm{t}=0$, another particle passes through B heading towards A with constant velocity $5 \mathrm{~m} / \mathrm{s}$. The particles meet at point C.
1- Determine the distance $A C$.
2- On a set of suitable axes, draw a detailed displacement time graph for both particles, using $A$ as the origin.

## EXERCISE 05

Answer the following based on the velocity vs. time graph.
1- Give a written description of the motion.
2- Determine the average acceleration of the object in each part.
3- Determine the distance travelled in each part.


## EXERCISE 06

A particle moves with an acceleration given in Cartesian coordinates by:

$$
\vec{a}=e^{-t} \vec{\imath}+5 \sin (t) \vec{\jmath}-3 \cos (t) \vec{k}
$$

At $\mathrm{t}=0 \mathrm{~s}$, the particle is located at $(1,0,3)$ and its velocity is then $(1,2,-1)$.
1 - Determine the velocity and position of the particle whatever t .

## EXERCISE 07

- Plot these polar coordinate points on one graph: $(2, \pi / 3),(3, \pi / 2)$, ( $2,-\pi / 4$ ), ( $1 / 2, \pi$ ), ( $1,4 \pi / 3$ ).
- Convert the Cartesian coordinates $(2,2)$ to polar coordinates.
- Convert the polar coordinate $(4, \pi / 2)$ to a Cartesian coordinates.
- The slotted link is fixed at O and as a result of the constant angular velocity $\dot{\theta}=3 \mathrm{rad} / \mathrm{s}$ it drives the peg along the spiral path $\rho=0.4 \theta$ ( $\rho$ is in meter and $\theta$ is in radian). Determine the velocity and acceleration
 at the instant it leaves the slot in the link, i.e, when $\rho=0.5 \mathrm{~m}$


## EXERCISE 08

A boat travels around a circular path, $\rho=40 \mathrm{~m}$, at a velocity that increases with time, $\mathrm{V}=0.0625 \mathrm{t}^{2}$ 1 - Find the magnitudes of the boat's velocity and acceleration at the instant $\mathrm{t}=10 \mathrm{~s}$ by using the intrinsic coordinates.
2- Determine the curvilinear abscissa $\mathrm{S}(\mathrm{t})$. Noted that at $\mathrm{t}=0, \mathrm{~S}(\mathrm{t})=0$.

## EXERCISE 09

- Plot the points given by the cylindrical coordinates: $\mathrm{P}(3, \pi / 6,-1), \mathrm{Q}(3, \pi / 2,2)$ and $\mathrm{R}(0, \pi, 3)$.
- Convert the cylindrical point $(\mathrm{r}, \theta, \mathrm{z})=(2,-\pi / 4,1)$ to Cartesian coordinates:
- Convert the Cartesian point $(\mathrm{x}, \mathrm{y}, \mathrm{z})=(-2,2 \sqrt{3}, 1)$ to cylindrical coordinates.
- The motion of a particle moving in three-dimensions is described by the following equations:

$$
\begin{gathered}
x=R \cos \theta, \quad y=R \sin \theta, z=h \theta \\
\theta=w t, \quad w: \text { constant }, \quad h: \text { positive constant }
\end{gathered}
$$

1- Describe the motion of point M in the ( xOy ) plane?
2- Describe the motion of point M in the direction of the Oz axis?
3 - What is the resulting motion of point M ?
4- Determine the cylindrical components and modulus of the vectors: position, velocity and acceleration.
4- What are the tangential and normal components of the acceleration vector?
5- Calculate the radius of curvature of the trajectory.

## EXERCISE 10

The spherical coordinates ( $\mathrm{r}, \theta, \varphi$ ) of a moving object are given by:

$$
\mathrm{r}=\mathrm{R}, \quad \theta=\frac{\pi}{6} \quad, \varphi=\mathrm{at}^{2}
$$

1- Write the expression of the position vector in Cartesian coordinates.
2- Determine the Cartesian components and modulus of the velocity and acceleration vectors.
3- Give the equation of the trajectory. Draw the trajectory.
4 - What is the nature of the motion?

F. Mezahi, S. Hamrit

