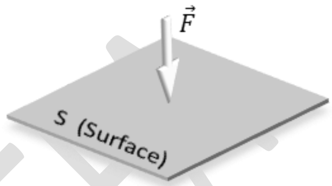
Chapter 2

Fluid Statics

Part a: Pressure in a fluid

2.1 Concepts and definitions

Fluid statics is the science that studies the equilibrium conditions of fluids at rest . More precisely, it concerns all fluid situations in statics and pseudo-statics. The quantities involved are only pressure, pressure force and Archimedes' thrust.



Static pressure (P): is defined as the quotient of a force ($\vec{F}$) by a surface (S).

- A uniform pressure applied to a surface (S), is the force $\vec{F}$acting <u>perpendicular</u> to the unit surface:

$$P = \frac{|F|}{S} \qquad [\ N/m2\ ]\ or\ [Pa]\ (Pascal) \tag{2-1}$$

- When the pressure is not uniform on S, i.e., the force acting on S is variable, and to keep the notion of pressure, we use the definitions of elementary force and elementary surface ΔS:

$$P = \lim_{\Delta S \to 0} \left(\frac{\Delta F}{\Delta S}\right) = \frac{dF}{dS}$$

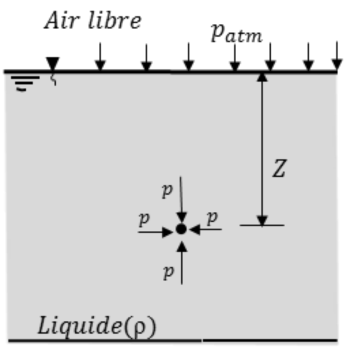
Other units

1 atm (atmospheric pressure) = 1.01325 10 ^{5} Pa ≈ 760 mm of mercury

1 bar ≈10 ^{5} Pa

Pressure characteristics

1. The pressure of a fluid always acts <u>perpendicular</u> to the surface.
2. By convention, a pressure is positive when the direction of the effect is from the interior of the fluid towards the surface studied, and vice versa.
3. The pressure acting at any point (elementary volume) in a fluid at rest <u>is the same</u> in all directions, see the figure below.



2.2 Pressure in a fluid at rest

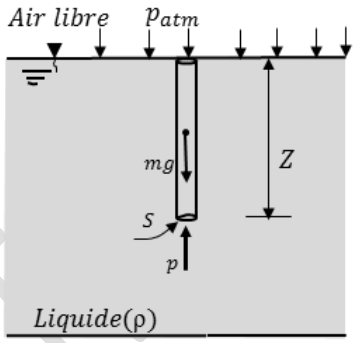
Let us take a column of fluid of depth Z *and* section S. *Let* P atm be the atmospheric pressure and P be the pressure at depth *Z.*

The force exerted on the upper section of the column is: $F_1 = P_{atm}.S$, and the force exerted by the fluid on the lower section is:$F_2 = P.S$

In static equilibrium of the fluid column along the axis$\overrightarrow{oz}$

$$F_1 + m.g = F_2$$

With: m is the mass of the fluid column. g is the gravitational acceleration (= 9.81 m/s ^{2}) .



$$P_{atm}.S + \rho.S.Z.g = P.S$$

So the pressure applied to the bottom section is:

$$P = P_{atm} + \rho.g.Z \qquad (2\text{-}1)$$

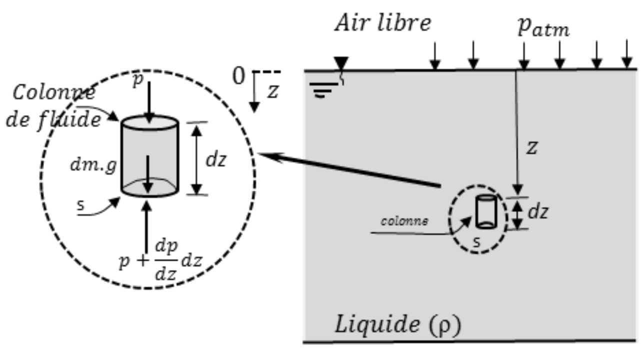
For a fluid point in the lower section, the pressure is:$P = P_{atm} + \rho.g.Z$

Noticed

The pressure does not depend on the shape or width of the column. It therefore increases linearly as a function of depth.P = f(Z).

Another demonstrative method

Consider a small column of fluid, see the figure below. Section S $_1$ is subjected to pressure P and section S $_2$ is subjected to pressure P $+\frac{\partial P}{\partial z}dz$and then gravity on the column.



The following equation is obtained from the static balance of forces according to $\overrightarrow{oz}$.

$$PS - \left(P + \frac{dP}{dz}dz\right)S + \rho.g.S.dz = 0$$

S: has no effect on pressure variation.

Fundamental equation of fluid statics is:

$$\frac{dP}{dz} = \rho.g$$

After the full: $P = \rho g \int dz = \rho g z + C^{te}$

2.2.1 Absolute pressure and relative pressure

Absolute pressure: Absolute pressure is the pressure measured relative to absolute vacuum. It is always positive, see figure 2.1 . If point 0 represents the free surface of the liquid, and z is measured downward, the above equation becomes:

$$P_a = \rho g z + P_{atm} \qquad (2\text{-}2)$$

Relative pressure (effective) : The relative pressure is defined in relation to the atmospheric pressure P_{atm} existing at the time of the measurement. This pressure can be positive if the pressure is greater than atmospheric pressure or negative if the pressure is less than atmospheric pressure. The pressure P_{atm} is used as a reference ($P_{atm} = 0$).

$$P_r = \rho g z \qquad (2\text{-}3)$$

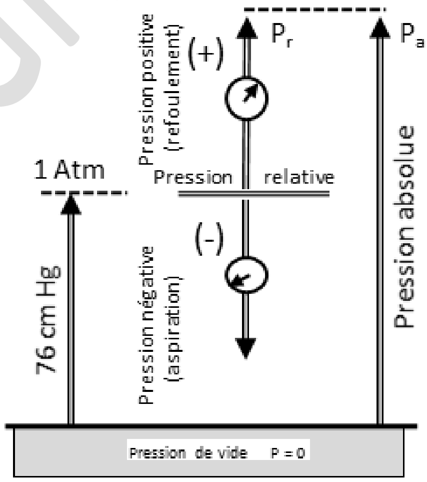


Figure 2.1: different pressure benchmarks

The two types of pressure physically correspond to the same pressure, they are simply expressed on different marks. The following relationship allows you to go from one to the other:

$$P_a = P_r + P_{atm}$$

We speak of delivery or suction when the absolute pressure is higher or lower than atmospheric pressure, respectively. Relative pressure is positive or negative in case of overpressure or underpressure, respectively.

2.2.2 Hydrostatic pressure in gases

Fundamental equation of statics for gases: $\frac{dp}{dZ} = -\rho . g$ (axis mark $\overrightarrow{oz}$ is upwards from the free surface).

For ideal gases, the density is proportional to the pressure. So the state equation is: $\rho = \frac{P}{RT}$

Between these equations: $\frac{dp}{dZ} = -\rho . g = -\frac{p}{RT} . g$

Separate the variables and integrate between points (1) and (2)

Integral equation: $\int_1^2 \frac{dP}{dZ} = ln\left(\frac{P_2}{P_1}\right) = -\frac{g}{R}\int_1^2 \frac{dZ}{T}$

An approximation in the case of the atmosphere is considered isothermal (T = T $_0$).

$$p_2 = p_1\, Exp\left[-\frac{g.(Z_2 - Z_1)}{RT_0}\right] \qquad (2\text{-}4)$$

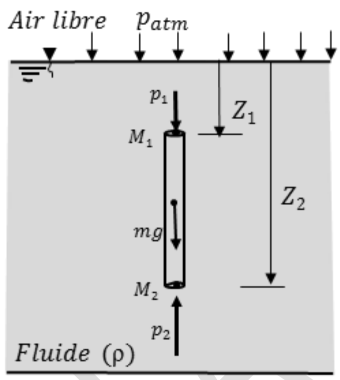
$$ln\left(\frac{p_2}{p_1}\right) = -k(Z_2 - Z_1) \qquad \text{Or} \qquad k = \frac{g}{RT_0}$$

Noticed

In the troposphere layer (about 10 km thick compared to the sea) the temperature variation is given by the formula $T = 288 - 0{,}0065 \times Z$ in [°K] which can be integrated into the previous equation to give the pressure variation? See Figure A.1.

2.2.3 Pressure difference between two points

The pressure difference between two points located at different levels of a fluid is given by:



According to formula (2-1), the pressure at point M $_1$ is:

$P_1 = \rho \, . g \, . Z_1$, the pressure at point M $_2$ is:

$P_2 = \rho \, . g \, . Z_2$

Therefore, the pressure difference between points M $_1$ and M $_2$ is:

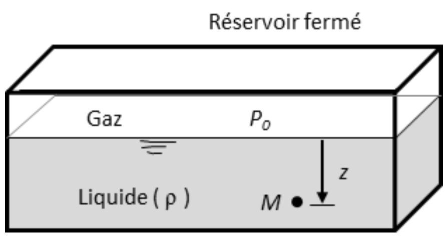
$$P_2 - P_1 = \rho \, . g \, . (Z_2 - Z_1) \qquad (2\text{-}5)$$

Noticed

To express the pressure difference between two points, we start with the lowest point, to keep the positive sign of the difference.

2.2.4 Closed system

According to Pascal's theorem (1623-1662), a pressure (P $_0$) exerted on a liquid in a closed tank, this pressure is transmitted entirely to all parts of the liquid and also to the walls of the container.



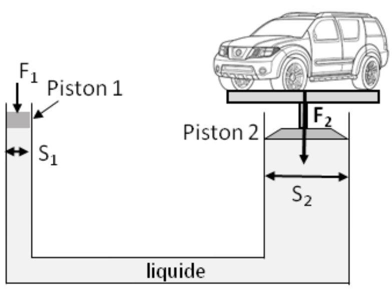
The pressure at point M in a closed system is given by the following formula:

$$P = P_0 + \rho g z \qquad (2\text{-}6)$$

P_0 is the gas pressure in the tank.

Application of Pascal's theorem:

Hydraulic press and hydraulic brake system



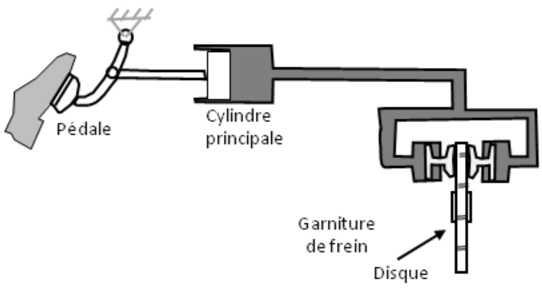


Figure 2.2: Operating mode of a braking system and a hydraulic press.

Among the applications of Pascal's theorem, the hydraulic press (lift), see the figure. As the pressure is the same everywhere, in the case of equilibrium, we can write in both sides of the hydraulic press:

Internal pressure: $P = \frac{F_1}{S_1} = \frac{F_2}{S_2}$

Hence: $$\frac{F_1}{F_2} = \frac{S_1}{S_2} = \left(\frac{D_1}{D_2}\right)^2 \quad (2\text{-}7)$$

Therefore, a small force F_1 acting on a small area S_1 of piston 1 balances a large force F_2 acting on a large area S_2 of piston 2.

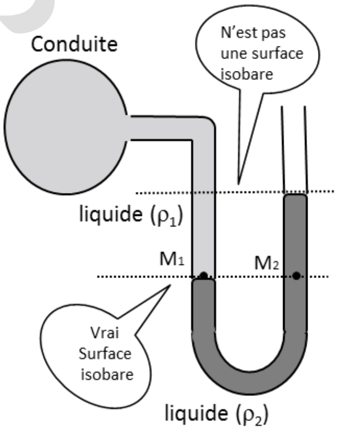
2.2.5 Isobaric surface

An isobaric surface when the pressure is the same at any point located at the same depth relative to a free (or separating) surface, see the figure.

The isobaric surface must meet the following conditions:

The points on the surface must be in contact by

THE same liquid and in continuity.



Applications

1. Perfect static case

- Connection of the vases by the same liquid, the free surface is considered as an isobaric surface. In practical life, this technique is used to maintain the horizontal level.

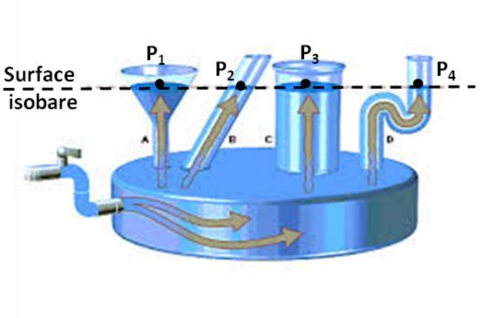


Figure 2.3: Principle of the isobaric surface rule.

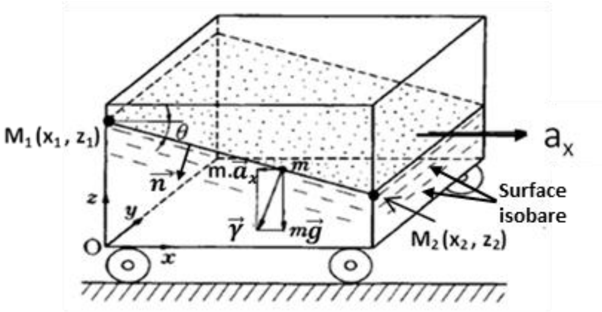
2. Pseudo-static case

- **The pressure in a container in accelerating motion** :

Considering the state of a fluid contained in a container in uniformly varied motion (a_x = Ct), see the figure.

With the new acceleration component a_x. The liquid moves like a rigid body. The slope of the free surface is given by the following expression:

$$\tan(\theta) = \frac{z_1 - z_2}{x_2 - x_1} = \frac{a_x}{g}$$

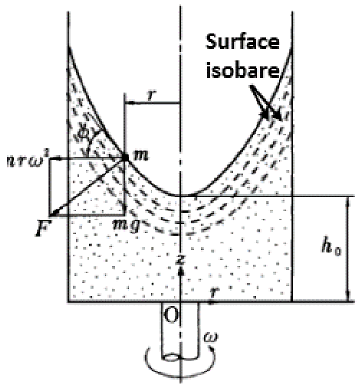


The variation of the pressure following the normal ($\vec{n}$) of the free surface is:

$$\frac{dP}{dn} = \rho.\gamma = \rho.(g^2 + {a_x}^2)^{1/2}$$

- **Parabolic surface of revolution**

A circular cylinder filled with fluid rotates uniformly with angular velocity ω. The free surface takes the form of a paraboloid of revolution under the effect of the rotational movement.



The pressure variation in the radial direction $\overrightarrow{or}$

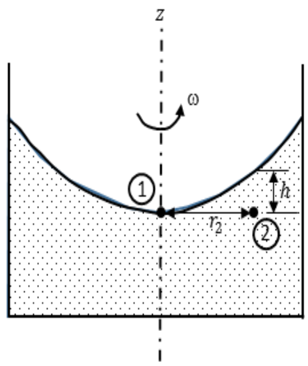
is given: $\frac{\partial P}{\partial r} = \rho\omega^2 r$

The pressure variation in the vertical direction ($\overrightarrow{oz}$)

is given: $\frac{\partial P}{\partial z} = -\rho g$

The integral of these equations gives:

$$P = P_{atm} - \rho g z + \frac{\rho\omega^2 r^2}{2}$$



The depth of point M_2 relative to the free surface is:

$$h = \frac{\omega^2 r_2^2}{2g}$$

2.3 Pressure measuring devices

The device used depends on the importance of the pressures to be measured. There are two types of pressure measuring devices:

Manometer tubes: used for measuring relatively low pressures.

Mechanical pressure gauges: used for measuring relatively higher pressures.

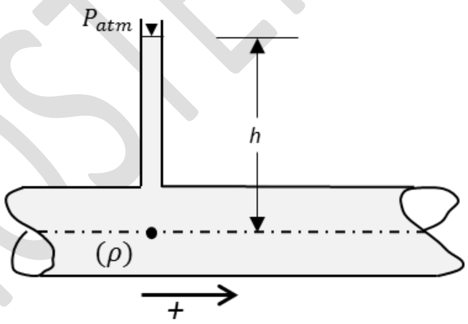
$$P_{jaugé} = P_r = P_a - P_{atm} \qquad (2\text{-}8)$$

2.3.1 Piezometers and Pressure Gauges

- **Piezometric tube**

Formula : $P_{jaugé} \ = \ \rho. g. h$ (2-9)

ρ: is the density of the fluid in the pipe, h is the manometric height, see the figure



Noticed

The correct position of the piezometric tube must be perpendicular to the axis of the flow pipe. Generally, the piezometric tube is used to measure low pressures (< 10 m).

- **U-shaped pressure gauge tube**

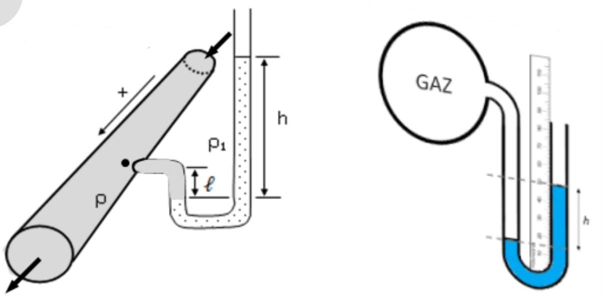
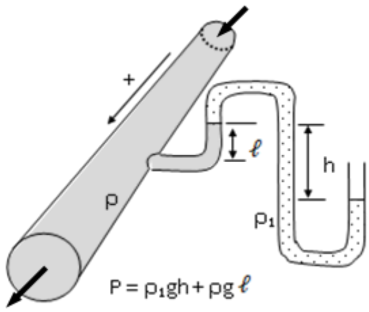


Figure 2.4: U-shaped pressure gauge measures pressure of a flow in a pipe.

- If $\rho_1 > \rho$, the pressure gauge used for high pressures:

ρ_1: is the density of the reference fluid used in the pressure tube.

Formula : $P_{jaugé} = \rho_1 . g . h - \rho . g . \ell$ (2-10)



Noticed

If the fluid with density (ρ) is a gas, its density is negligible compared to that of the gauge liquid

- If $\rho_1 < \rho$, the pressure gauge used for low pressures:

Formula : $P_{jaugé} = \rho_1 . g . h + \rho . g . \ell$ *(2-11)*

- **Differential pressure gauges**

They measure the pressure difference between two cross sections of a main pipe.

- If $\rho_1 > \rho$, the pressure gauge used for high pressures (Reducing pressure gauge), see figure.

Formula : $\frac{P_1 - P_2}{\rho g} = \left(\frac{\rho_1}{\rho} - 1\right) . \Delta h_1 = K . \Delta h_1 = \Delta h$ (2-12)

K: Manometric constant.

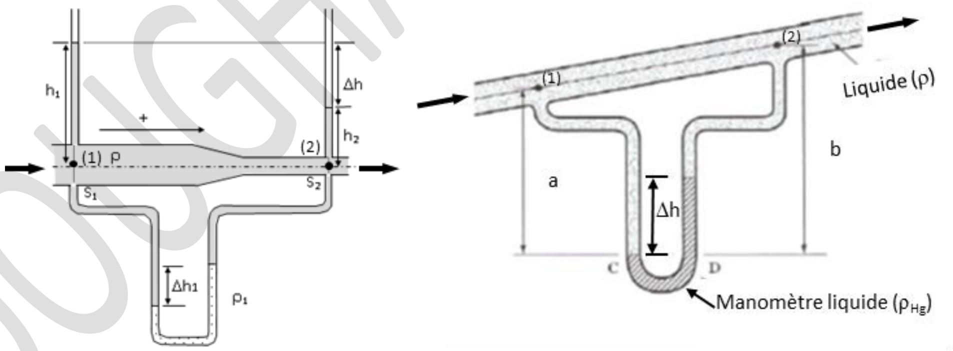
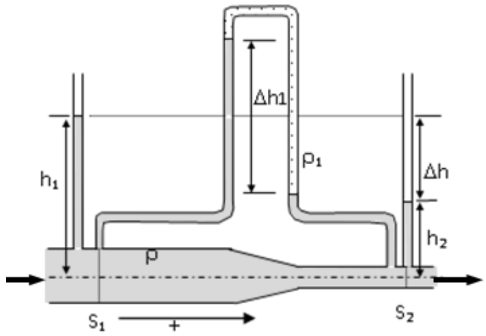


Figure 2.5: Differential pressure gauge measures the pressure difference of a flow between two cross sections.

- If $\rho_1 < \rho$, the pressure gauge is used for low pressures (amplifying pressure gauge)

Formula : $\frac{P_1 - P_2}{\rho g} = \left(1 - \frac{\rho_1}{\rho}\right) . \Delta h_1 = K_1 . \Delta h_1 = \Delta h$ *(2-13)*

K_1 : Manometric constant.



2.3.2 Mechanical pressure gauges (Bourdon gauge)

Bourdon's instrument is based on the principle that a pinched cylindrical tube tends to rectify itself and take on a circular section when subjected to internal pressure.

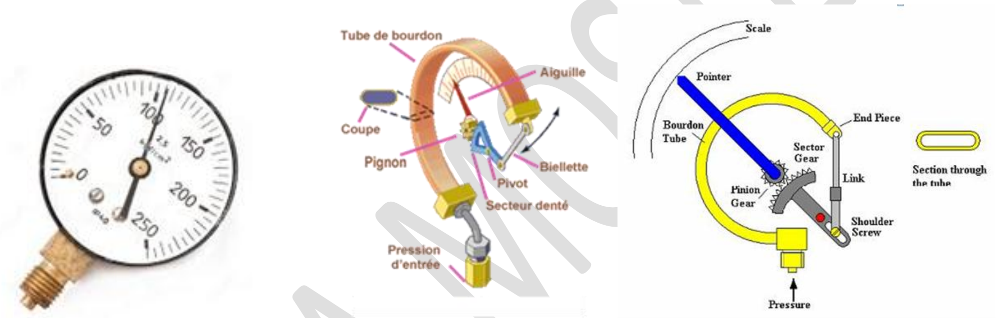


Figure 2.6: Operating principle of the Bourdon pressure gauge

2.4 Archimedes' principle

Any body immersed in a fluid receives from this fluid a vertical, upward force (thrust) whose intensity is equal to the weight of the volume of fluid displaced (this volume is therefore equal to the immersed volume of the body). *The center of thrust is the center of gravity of the volume of fluid displaced* .

$$F_A = \rho_f . g . V_{dép} \qquad [N] \qquad (2\text{-}14)$$

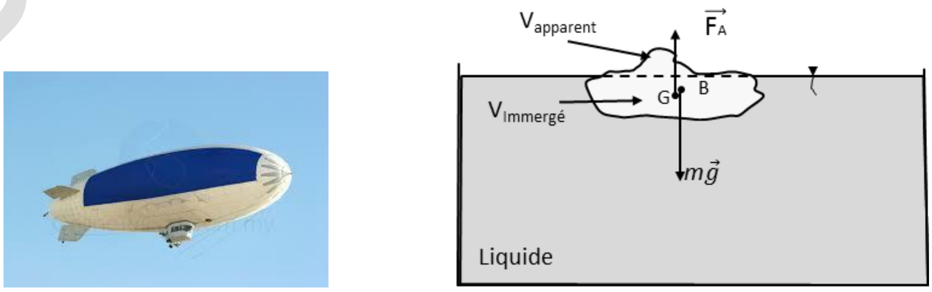


Figure 2.7: Archimedes' principle of thrust a) Airship b) Floating body .

$V_{dép}$: is the volume of fluid displaced (or immersed volume of the body), ρ is the density of the fluid displaced.

F_A : upward thrust force, applied to the center of gravity of the submerged volume B,

ρ_f: density of the fluid [kg/m ^{3}],

$\vec{g}$fuel acceleration g (9.81 m/s ^{2}) .

Applications :

- Flush mechanism

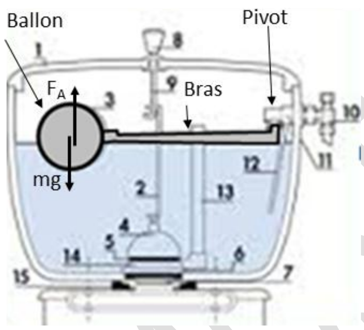


Figure 2.8: Principle of operation of the toilet flush, based on Archimedes' principle.

- Operating principle of a floating dock

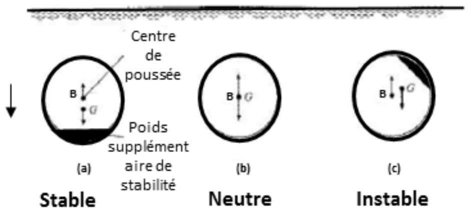
Figure 2.9: Floating dock based on Archimedes' principle

2.3.1 Stability of floating and submerged bodies

- **Submerged bodies**

To check stability, static equilibrium must be ensured:

$$\sum \overrightarrow{F_{orces}} = 0 \qquad \text{And} \qquad \sum M_{oments} = 0$$



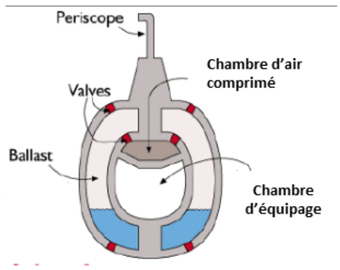
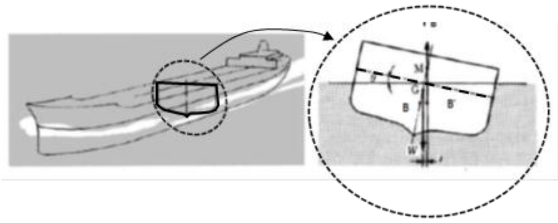


Figure 2.10: Immersed bodies a) balance ball
b) Principle of immersion/emersion of submarines

- **Floaters**



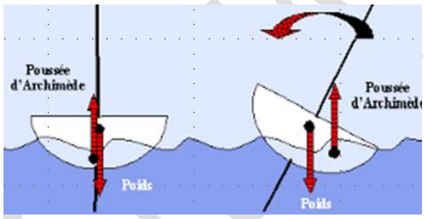


Figure 2.11: Principle of balance of a ship

Figure 2.12: a) Balance of a kayak b) Unbalance of a ship.

The equilibrium conditions are summarized by the metacentric size rule ($\overline{GM}$):

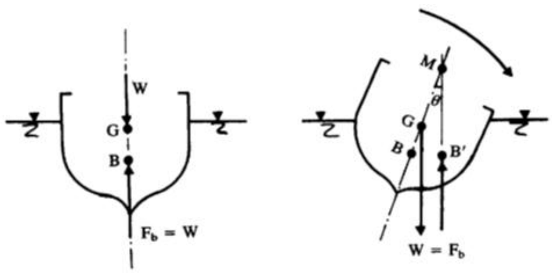
$$\overline{GM} = \overline{BM} - \overline{BG} = \frac{I}{V} - \overline{BG} \qquad (2\text{-}15)$$

I: Moment of inertia of the section studied.

V: Submerged volume or displaced volume.

$\overline{GM} < 0$ Float state is unstable.

$\overline{GM} > 0$ Float state is stable.



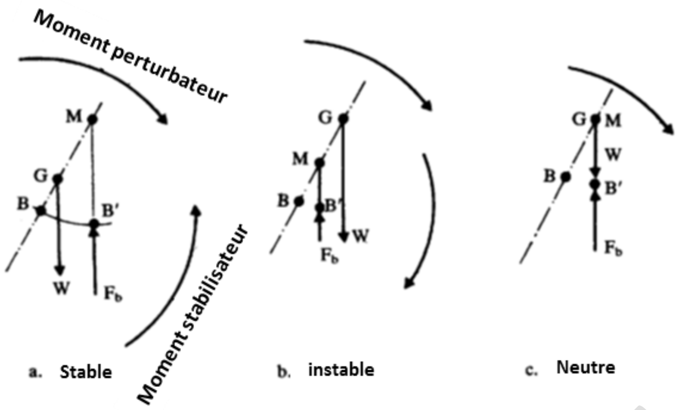


Figure 2.12: Stability conditions of a floating body.

2.5 Summary of important formulas in the chapter

Fundamental equation of statics of liquids $\frac{dP}{dz} = \rho . g$

Absolute pressure $P_a = \rho g z + P_{atm}$ (2-2)

Relative pressure (effective) $P_r = \rho g z$ (2-3)

Fundamental gas statics equation $\frac{dp}{dZ} = -\rho . g$

Pressure difference between two points $P_2 - P_1 = \rho . g . (Z_2 - Z_1)$ (2-5)

Pressure in a closed system $P = P_0 + \rho g z$ (2-6)

Isobaric surface rule (Pascal's law) The surface points must be in contact by the same liquid and in continuity.

Archimedes' force $F_A = \rho_f . g . V_{dép}$ (2-14)
