I. Numbers and Arithmetic Operations
/ˈnʌmbə(r)/ /əˈriθmətik/ /opəˈræisyen/

Numbers

Two kinds of activity made our ancestors develop numbers (cardinal and ordinal numbers). The first for comparing their things (which one has more elements), and the second for creating order.

A. Cardinal Numbers (Counting Numbers)

Example:

1	one
2	two
3	three
4	four
5	five
6	six
7	seven
8	eight
9	nine
10	ten
11	eleven
12	twelve
13	thirteen
14	fourteen
15	fifteen
16	sixteen
17	seventeen
18	eighteen
19	nineteen
20	twenty
21	twenty-one
22	twenty-two
23	twenty-three
24	twenty-four
25	twenty-five
26	twenty-six

30	thirty
40	forty
50	fifty
60	sixty
70	seventy
80	eighty
90	ninety
100	a hundred; one hundred
101	a hundred and one
110	a hundred and ten
120	a hundred and twenty
200	two hundred
300	three hundred
900	nine hundred
1 000	a thousand, one thousand
1 001	a thousand and one
1 010	a thousand and ten
1 020	a thousand and twenty
1 100	one thousand, one hundred
1 101	one thousand, one hundred and one
9 999	nine thousand, nine hundred and ninety-nine
10 000	ten thousand
15 356	fifteen thousand, three hundred and fifty-six
100 000	a hundred thousand
1 000 000	a million
1 000 000 000	a billion
1 000 000 000 000	a trillion

B. Ordinal Numbers

Example:

1st	first
2nd	second
3rd	third
4th	fourth
5th	fifth
6th	sixth
7th	seventh
8th	eighth
9th	ninth
10th	tenth

11^{th}	eleventh
12^{th}	twelfth
13^{th}	thirteenth
14^{th}	fourtheenth
15^{th}	fidteenth
16^{th}	sixteenth
17^{th}	seventeenth
18^{th}	eighteenth
19^{th}	nineteenth
20^{th}	twentieth
21^{st}	twenty-first
22^{nd}	twenty-second
23^{rd}	twenty-third
24^{th}	twenty-fourth
25^{th}	twenty-fifth
26^{th}	twenty-sixth
27^{th}	twenty-seventh
28^{th}	twenty-eighth
29^{th}	twenty-ninth
30^{th}	thirtieth
31^{st}	thirty-first
40^{th}	fortieth
50^{th}	fiftieth
100^{th}	hundredth
$1\ 000^{th}$	thousandth
$1\ 000\ 000^{th}$	miilionth

- **Natural Numbers**

  1,2,3,… one, two, three, and so forth (without end).

  1,2,3,…, 10 one, two, three, and so forth up to ten.

  Natural numbers can be divided into two sets: **Odd Numbers** and **Even Numbers**

- **Whole Numbers** Natural Numbers + 0 zero/o/nought.
- **Integers** ...,-2,-1,0,1,.. .., negative two, negative one, zero, one, ..
- **Rational numbers** are numbers that can be expressed as fraction.
- **Irrational Numbers** are numbers that cannot be expressed as fraction, such as: e,√2, π.
- **Real Numbers** are made up of rational and irrational numbers.

❖ **Complex Numbers** Complex numbers are numbers that contain **real** and **imaginary** part.

2 + 3i **2** is called the **real part**, **3** is called the **imaginary part**, and **i** is called **imaginary unit** of the complex number.

❖ **A Digit** is any one of the ten numerals 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.

Example:

3 is a single-digit number, but 234 is a three-digit number.
In 234, 4 is the units digit, 3 is the tens digit, and 2 is hundreds digit.

❖ **Consecutive numbers** are counting numbers that differ by 1.

Examples:

23, 24, 25, 26, and 27 are 5 consecutive numbers.
25, 26, 27, … are successor of 24.
23 is the immediate successor of 24.
1, 2, …, and 22 are predecessor of 23.
22 is the immediate predecessor of 23.
36, 38, 40, and 42 are 4 consecutives even numbers.

Operations on Numbers

Addition (+) , Subtraction (-) , Multiplication (x) , Division(:)

Symbols in Numbers Operation

+	added by/plus/and
-	subtracted by/minus/take away
±	plus or minus
x	multiplied by/times
÷	divided by/over

Symbols for Comparing Numbers

=	is equal to/equals/is
≠	is not equal to/does not equal
<	is less than/is smaller than
>	is greater than/is more than
≤	is less than or equal to
≥	is more/greater than or equal to
≅	is approximately equal to

The mathematical sentences that use symbols "=" are called **equation**, and the mathematical sentences that use symbols "<", ">", "≤ '", or "≥" are called **inequalities.**

Examples

$ax + b = 0$ is a linear equation.

$ax^2 + bx + c = 0$ is a quadratic equation.

$3x^3 - 2x^2 + 3 = 0$ is a cubic equation.

$\frac{a+b}{2} = \sqrt{ab}$ is called AM-GM inequality.

Examples

- $2 + 3 = 5$

two	*is added by* *plus* *and*	*three*	*is equal to* *equals* *is*	*five.*

The **sum** of two and three is five.

- $10 - 4 = 6$

Ten	*is substracted by* *minus* *take away*	*four*	*is equal to* *equals* *is*	*six.*

The **difference** between ten and four is six

- $7 \times 8 = 56$

Seven	*is multiplied by times*	*eight*	*is equal to* *equals* *is*	*fifty-six*

The **product** of seven and eight is fifty-six

- $45 \div 5 = 9$

forty-five	*is divided by* *over*	*five*	*is equal to* *is*	*nine.*

The **quotient** of *forty-five and five is nine*

Practice

1. Read out the following operations, and for every operations name each number's function.

a. 1,209 + 118 = 1,327
b. 135 + (-132) = 3
c. 2 – (-25) = 27
d. 52 – 65 = -13
e. 9 x 26 = 234
f. -111 x 99 = -10,989
g. 36 ÷ 9 = 4
h. 1375 ÷ (-25) = -55

2. Fill the blank spaces with the right words.

a. The _____ of three and seven is twenty-one.
b. The operation that uses symbol "÷" is called _____.
c. 14 is the _____of 13, and the predecessor of 13 are _____.
d. The result of division is called _____.
e. Three multiplied _____ five equals _____.
f. In 123,456,789, the hundred thousands digit is _____, and 9 is the_____.
g. We select a _____ number htu, as 100h + 10t + u, where h represents the _____ digit, t represents the _____digit, and u represents the units digit.
h. When we _____ two numbers, for example seven plus thirteen, the answer (twenty) is called _____.

Fractions

A **common (or simple) fraction** is a fraction of the form **a/b** where a is an integer and b is a counting number

Example: p/q

p is called the numerator of the fraction

q is called the denominator of the fraction

The fraction **a/b** is **simplified** ("in lowest terms") if a and b have no common factor other than 1

Saying Fraction

$\frac{1}{2}$	A/one half
$\frac{1}{3}$	A/one third
$\frac{1}{4}$	A/one quarter
$\frac{5}{6}$	Five sixths/Five over six
$\frac{22+x}{7}$	Twenty-two plus x all over seven

$13\frac{3}{4}$	Thirteen and three quarters
0.3	Nought/zero/o point three
3.056	Three point o five six
273.856	Two hundred and seventy-three point eight five six

Practice

1. Read out the following fractions

a. $\frac{2}{5}$
b. $\frac{3}{4}$
c. $\frac{5}{8} \times \frac{1}{4} = \frac{5}{32}$
d. $\frac{1}{9} - \frac{1}{8} \neq \frac{1}{24}$
e. 13,945.614
f. 43.554
g. 6.9 x 2.2 = 15.18
h. 72.4 x 61.5 = 4452.6

2. Fill the blank spaces with the right words.

a. In the fraction seven ninths, _____ is the numerator, and_____ is the _____.
b. The _____ of two thirds and a half is four over three.
c. The difference between a quarter and two is _____.

<u>Divisibility</u>

4|12

12 is **divisible** by 4.
12 is a **multiple** of 4.
4 **divides** 12
4 **is a factor** of 12
15 is not divisible by 4.
If 15 divided by 4 then the quotient is 3 and the remainder is 3.

0 is divisible by all integers

<u>Prime numbers</u> Every number is divisible by 1 and itself. These factors (1 and itself) are called **improper divisors**.

Prime numbers are numbers that have only improper divisors.

Example:

5 is a prime number, but 9 is not a prime number or a **composite number**.

Common Divisors

Example:

1, 2, 3, 4, 6, and 12 are divisors (factors) of 12.
1, 3, 5, and 15 are divisors of 15.
1 and 3 are common divisors of 12 and 15.
3 is the greatest common divisor of 12 and 15.
The **g.c.d of** 12 and 15 is 3. (gcd(12,15) = 3.)

Common Multiples

Example:

5, 10, 15, 20, 25, …are multiples of 5.
4, 8, 12, 16,20, 24, … are multiples of 4.
5, 10, 15, 20 are four first multiples of 5.
4, 8, 12, 16, 20 are five first multiples of 4.
20, 40, 60, … are **common multiples** of 4 and 5.
20 is the **least common multiple** of 4 and 5.
The **l.c.m of** 4 and 5 is 20 (lcm(4,5) = 20.)

Practice

1. Read the following conversation
A : I have two numbers, 36 and 42. Can you say their factors?
B : The factors of 36 are 1, 2, 3, 4, 6, 9, 12, 24, and 36. 1, 2, 3, 6, 7, 14, 21, and 42 are factors of 42.
A : So, what are their common factors?
B : They are 1, 2, 3, and 6.
A : And what is the greatest common divisor of 36 and 42?
B : It's 6.

2. Make a small conversation about gcd or lcm of other numbers.

Exercise 01

Write down the spelling of these mathematical sentences

- $12 + 1/3 \leq x - 7$
- $3x \times 26 > 20 \div y$
- $x (2y + 3) \neq 111.909$
- $(2 + x)/35 < 23/45$

Exercise 02

Use the right words to complete these sentences.

- 2367 is _____ by nine.
- 3 is _____ of 34.

- The _____ of three and four is twelve.
- Eighteen subtracted _____ twenty equals _____.
- 3 is the _____ and 5 is the _____ of three fifths.

Exercise 03

- Write down five first multiples of 8.
- Write down all divisors of 18.
- Find all common divisors of eighteen and thirty-three.
- Write down the simplest form of 91/234
- Find the sum of the reciprocals of two numbers, given that these numbers have a sum of 50 and a product of 25.
- What is the product of the greatest common divisor of 9633 and 4693 and the least common multiple of the same numbers?
- Let x be the smallest of three positive integers whose products is 720. Find the largest possible value of x.
- If P represents the product of all prime numbers less than 1000, what is the value of the units digit of P?
- Find a positive integer that is eleven times the sum of its digits?
- What is the greatest common divisor of 120 and 49?
- The product of 803 and 907 is divided by the sum of 63 and 37. What is the remainder?
- The average of four consecutive even integers is 17. Find the largest of the four integers.
- When the six-digit number 3456N7 is divided by 8, the remainder is 5. List both possible values of the digit N.