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SEISMIC ALEA COURSE
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Master 1

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Envelope function

envelope function was introduced to account for non-stationarity in amplitude and many suggested shapes were used (Shinozuka [12]).

Most filters used are simple 1-degree of freedom oscillators. The filtering process is obtained by solving the differential equation of the oscillator characterized by a natural frequency ω and a damping constant ε. These parameters are obtained from predominant spectral characteristics of accurate acceleration time series records. A superposition of a large number of sinusoidal functions at different intervals with equal frequency difference Δω, in which the phase angles having a uniform probability density function was random, is weighted in combination using the power of filtered stationary white noise expressed through the spectral density. This procedure allows a successful simulation of time series acceleration. Alternatively, one can generate a white noise sequence with an average spectral amplitude over frequency equal to one and multiply the spectrum of the white noise with the specified spectrum of the acceleration time series and transform it back to the time domain.

The modeling of acceleration time series has been suggested by academics. In the frequency domain approach, three methods of simulation have been used based upon assumptions of (1) stationarity in the amplitude and frequency content, (2) non-stationarity in the amplitude only, and (3) non-stationarity in both amplitude and frequency content.

1.1 Simulation in the Time Domain

The time-domain approach concerned with using ARMA models to describe acceleration time series data is relatively innovative. For example, Kozin's article [6] is an excellent methodological aid for evaluating the possibility of applying the ARMA approach to acceleration time series data. Acceleration time series records are digitized uniformly at equidistant time intervals. This set of observations forms a discrete time series. A model which describes the probability structure of a sequence of observations is called a stochastic process. Key classes of stochastic processes include autoregressive (AR), moving average (MA), and their combination (ARMA). The autoregressive model denoted by AR(p) is generally written as:

$$Z_t = \varphi_1 Z_{t-1} - \cdots - \varphi_p Z_{t-p} + a_t \qquad (1.1)$$

where (φ_i) are constant coefficients, (a_t) is a sequence of equally random distributed independent Gaussian quantities, and (Z_t) indicates the sequence of data investigated. This model is of order (p).

Another general linear model of time series analysis is the autoregressive moving average (ARMA) model. This model is obtained by adding a moving average (MA) component to the autoregressive (AR) component. It is defined by:

$$Z_t - \varphi_1 Z_{t-1} - \cdots - \varphi_p Z_{t-p} = a_t - \theta_1 a_{t-1} - \cdots - \theta_q a_{t-q} \qquad (1.2)$$

where (φ_i)) and (θ_i) are constant coefficients, and (p, q) is the order of the model.

The model contains p+q+1 unknown parameters φ_1, φ_2, ... , φ_p, θ_1, θ_2, ... , θ_q, σ_a which are usually estimated from data based on maximum likelihood and the order is based on the partial AR functions.

1.2 Stationarity Conditions

Stationarity conditions in statistical terms can be considered to mean that there is no trend, it has a constant variance over a certain period, an autocorrelation structure that is persistent over time, as well as null intervallic fluctuation. A stochastic process can be denoted as an output process (Z_t) represented by equation A1 below from a linear filter where (a_t) is the white noise input.

$$Z_t = a_t + \sum_{j=1}^{\infty} \varphi_j \, a_{t-j} \tag{1.3}$$

From equation (1.3) (a_t) consists of a sequence of uncorrelated random variables with a mean of zero and constant variance σ_a^2.

The autocorrelation of equation (1.3) is given by:

$$C(k) = E[Z_t Z_{t+k}]$$

Or (1.4)

$$C(k) = E[\sum_{i=0}^{\infty} \sum_{j}^{\infty} \text{Ø}_i \text{Ø}_j]$$

Since the random variables a_t are uncorrelated,

$$E(a_t a_{t+k}) = \begin{cases} \sigma_a^2 & for \quad k = 0 \\ 0 & for \quad k \text{ G } 0 \end{cases} \tag{1.5}$$

Therefore, equation A2 can be written as;

$$C(k) = \sigma_a^2 \sum_{j=0}^{\infty} \text{Ø}_j \text{Ø}_{j+k} \tag{1.6}$$

$$C(0) = \sigma_a^2 \sum_{j=0}^{\infty} \text{Ø}_j^2 \qquad \text{for k=0}$$

The autocovariance generating function is then to be defined as follows:

$$C(B) = \sum_{-\infty}^{\infty} C(k) B^k \tag{1.8}$$

$$C(B) = \sum_{-\infty}^{\infty} \sum_{j=0}^{\infty} B^k$$

Or simply

$$C(B) = \sigma_a^2 \text{Ø}(B) \text{Ø}(B_1) \tag{1.9}$$

If we substitute $B = \exp(-i2\pi f)$ and $B^{-1} = \exp(i2\pi f)$, half the power spectrum of the process is obtained as:

$p(\text{f}) = 2\sigma_a^2\text{Ø}(\exp(-\text{i}2\pi\text{f})\text{Ø}(\exp(\text{i}2\pi\text{f})) \qquad 0 \leq \mathbf{f} \leq 1/2$

Or (1.11)

$p(\text{f}) = 2\sigma_a^2|\text{Ø}(\exp(-\text{i}2\pi\text{f})| \qquad 0 \leq \mathbf{f} \leq 1/2$

Therefore, the variance of the process is

$$\sigma_z = 2\sigma_a^2 \int_0^{1/2} \text{Ø}(\exp(-\text{i}2\pi\text{f})\text{Ø}(\exp(\text{i}2\pi\text{f})\, d\text{f} \qquad (1.12)$$

This integral converges when the infinity series $\text{Ø}(B)$ converges for B on or within the unit circle (Box & Jenkins, 1973). Intuitively, stationarity will be definite to express that the statistical properties of the time series generating process stay constant over a certain period. Therefore, it is a common case to articulate that if a time series is considered for the stationarity condition, it is imperative to state that the conditions of constant mean and variance across the timeframe.

1.3 Invertibility Conditions

The time series of a statistical phenomenon can be considered invertible if the errors have the capability of inverting into a past observation depiction. The autoregressive moving average (ARMA) process can be written as:

$$\text{Ø}(B)Z_t = \theta(B)a_t \qquad (1.13)$$

The requirement of invertibility is required to associate current events with past events. This requirement is illustrated by assuming the following model;

$$Z_t = a_{t-}\theta_1 a_{t-1}$$

Or (1.14)

$$Z_t = a_t(1 - \theta_1 B)$$

When expressed in terms of $Z's$, the following is obtained

$$a_t = (1 - \theta_1 B)^{-1} Z_t \qquad (1.15)$$

The weights $(-\theta_1 B)^{-1} = \sum \theta_1^{\text{i}} B^{\text{i}}$

This series converges when $|B| \leq 1$. In general, the process in equation A11 is invertible if the series converges on or within the unit circle.

The excellent and indispensable statuses for the stationarity and invertibility conditions of the above simple equations and representations are much easier to attain. Still, it is conceivable that the more intricate models pose a severe problem obtaining. However, according to Quinn (1982), it is an evident occurrence that invertibility is a zero-one phenomenon instead of a stochastic phenomenon.

1.4 Previous Studies

The first work in modeling acceleration time series data as time series using linear models was completed by Robinson [15], Liu [45], and Kozin [8]. Robinson used a moving average process to generate artificial acceleration time series records for experimental purposes. Liu studied and compared several models. ARMA models were specified as a potential approach to characterize acceleration time series records. However, the concern of that paper was a general study and comparison of stochastic models. Kozin [38] proposed a continuous nonstationary time model:

$$\ddot{X}(t) + a(t)\dot{X} + b(t)X(t) = \emptyset(t)W(t) \tag{1.16}$$

where a(t) and b(t) are polynomials,

$$a(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 \tag{1.17}$$

$$b(t) = b_0 + b_1 t + b_2 t^2 + b_3 t^3$$

Ø(t) is a function obtained by a cubic spline fit to the envelope of the actual record, and w(t) is Gaussian white noise. The parameters are to be estimated from data using non-linear filtering techniques. Such techniques are extensions of the methods of recursive estimation of Kalman filtering. It was found that if the initial parameters were not close to the actual values, the recursive computation would be unstable. Since there were no convergence theorems about the non-linear method applied to estimate the desired parameters, Kozin has used maximum likelihood estimators instead of ARMA models. Two theoretical papers by Nakajima and Kozin [11] and Kozin and Nakajima [6] discussed the general problem of convergence, and a fundamental theorem that guarantees convergence of parameters was obtained. The Akaike Information Criteria was extended to include nonstationary models such as the model proposed by Kozin [4] and [5] on the modeling of nonstationary time series, which has the following form:

$$y(k) + a_1(k-1)y(k-1) + \cdots + a_L(k-L)y(k-L) = g(k)u(k) \tag{1.18}$$

where y(k) is the acceleration time series data, u(k) is white noise, and a(k), g(k) are time-varying functions estimated from the data. The coefficients a(k) was parameterized as a linear combination of discrete orthogonal functions.

Chapter 2

Chapter 2

Acceleration Time Series Process Models

2.1 Acceleration Time Series Process Models (ARMA)

The acceleration time series event is a time-dependent phenomenon initiated by regular slippage along faults for which it is not possible to write a deterministic model that allows exact calculation. Thus, an acceleration time series event is is considered as a sample of the whole set of time series that could be generated by the stochastic process. Since acceleration time series records show a highly irregular motion and have finite durations, they are modeled as a non-stationary stochastic process. As a general concept in this analysis, the damage potential is characterized by the properties of the population generated by the stochastic process model. Thus, ARMA (p, q) process model could be represented as follows:

$$Z_t - \varphi_1 Z_{t-1} - \cdots - \varphi_p Z_{t-p} = a_t - \theta_1 a_{t-1} - \cdots - \theta_q a_{t-q} \tag{2.1}$$

where (φ_i) and (θ_i) are constant coefficients, and (p, q) is the order of the model. The model contains p+q+1 unknown parameters φ_1, φ_2, ... , φ_p, θ_1, θ_2, ... , θ_q, σ_a which are usually estimated from data based on maximum likelihood and the model order is based on the Akaike information criteria (AIC) [Ref].

2.2 The Autocorrelation Function

An Autoregressive AR process is written as:

$$Z_t = \varphi_1 Z_{t-1} + \varphi_2 Z_{t-2} + \cdots + \varphi_p Z_{t-p} + a_t \qquad (2.2)$$

The theoretical autocorrelation function is found by multiplying Eq. (2.2) by $Z_{t-k}$and taking expectations, the following is found:

$$C(k) = \varphi_1 C(k-1) + \cdots + \varphi_p C(k-p) + C_{za}(k) \qquad (2.3)$$

Since Z_{t-k} involves shocks a_L up to time L=t-k, $E[Z_t a_t] = 0$.

Dividing by C(0), the following is equation is obtained

$$R(k) = \varphi_1 R(k-1) + \cdots + \varphi_p R(k-p) + R_{za}(k) \qquad (2.4)$$

In general, the solution of this equation (Box & Jenkins, 2) is given by

$$R(k) = A_1 G_1^k + A_2 G_2^k + \cdots + A_p G_p^k \qquad (2.5)$$

Where $1/G_1$, $1/G_2, \ldots, 1/G_p$ are the roots of the characteristic equation given by:

$$1 - \varphi(B) = 1 - \varphi_1 B - \varphi_2 B^2 - \cdots - \varphi_p B^p = 0 \qquad (2.6)$$

Equation C4 is stationary if $|G| < 1$

If the roots G_i are real, it can be shown that $A_i G_i^k$ goes to infinity, when k goes to infinity. If the roots G_i are complex, the following term contributes to equation (2.3).

$$G_i = x_i + iy_j$$

Equation (2.3) will follow a damped sine wave. For a stationary process, the autocorrelation function generally consists of a mixture of exponential and damped sine waves.

To provide information about the choice of the order (p, q) of the ARMA process, which may be used to represent an acceleration time series record, it is essential to refer to the behavior of the theoretical autocorrelation function R(k). The theoretical autocovariance function, C(k), may be derived in multiplying Eq.(2.1) by Z_t and taking expectations.

$$C(k) = \varphi_1 C(k-1) + \cdots + \varphi_p C(k-p) + C_{za}(k) - \theta_1 C_{za}(k-1) - \cdots - \theta_q C_{za}(k-q) \qquad (2.7)$$

where $C_{za}(k) = E[Z_{t-k}.a_t]$. The autocorrelation function R(k) is obtained by dividing Eq.(2.7) by C (0) and a similar form is obtained:

$$R(k) = \varphi_1 R(k-1) + \cdots + \varphi_p R(k-p) + R_{za}(k) - \theta_1 R_{za}(k-1) - \cdots - \theta_q R_{za}(k-q) \tag{2.8}$$

The fact that the values of Z_{t-k} are correlated only to white noise values up to time t-k implies that:

$$C_{za}(k) = 0 \quad \text{for } k > 0 \tag{2.9}$$

There are some differences in the nature of the function depending on the instrument chosen. Thus, for processes using AR, the function will take the form of an exponentially decaying decay or series of waves as lag K increases. Alternatively, for MA, when lag K reaches a specific order g, the function becomes equal to zero. Finally, when the ARMA process is used, when the first g-p lag is overcome, the function takes the form of a combination of exponentially decaying waves.

2.3 The Function of Partial Autocorrelation

Partial autocorrelation function should be understood as such a way of describing the time dependence of the series, when combined with the use of AR, it becomes possible to accurately identify the order and type of the model being estimated. For stationary autoregressive (AR) processes, the autocorrelation function is infinite in extent. Therefore, it is very convenient to describe the autoregressive process by the number of non-zero functions of the autocorrelations. The theoretical partial autocorrelation function P(k) could be found by the Yule-Walker equations (Box and Jenkins, [2]).

For the ARMA process, which can be characterized as a stationary and invertible one, the partial autocorrelation function P(k) s is dominated by a damped exponential and sine waves depending on the moving average parameters after the first p-q lags.

2.4 AIC RITERIA

A fundamental problem is estimating the number of parameters and their numerical values to fit a model to a time series. The parameter estimation is based on the non-linear least squares, but the order of the model is based on the Akaike Information Criterion (Akaike [15]), which has received wide attention. Akaike has approached this problem using the maximum entropy measure. The Akaike Information Criterion (AIC) as a methodical mathematical model aids in the evaluation of the manner in which a model data can fit the data from which it was generated. The AIC mathematical model will warrant a sure comparison of numerous probable models as well as make a decision on which model can be best suited for the data.

If we consider N independent observations of a random variable characterized by the probability density function g(x) and consider the parametric density function f(x/θ), the expected log-likelihood will be:

$$A = E[\log \mathrm{f}(x/\theta)] = \frac{1}{n} \sum_{\mathrm{i}=1}^{N} \log\ (\mathrm{f}(x/\theta)) \tag{2.10}$$

When N goes to infinity, the following is obtained:

$\lim_{N\to\infty} A = \int g(x). \log\, f(x/\theta)dx$ with prob. 1. (2.11)

Letting $s(g, f(x/\theta)) = \int g(x). \log f(x/\theta)dx$, the difference is as illustrated in equation (2.12) below.

$$I(g, f(x/\theta) = s(g, g) - s(g, f(x/\theta) \quad (2.12)$$

Equation B3 is sensitive to the average log-likelihood $s(g, f(x/\theta)$. Thus, a criterion for defining the best fit is by minimizing the entropy H as illustrated in equation B4 below:

$$H = -\int g(x). \log f(x/\theta)dx \quad (2.13)$$

One of the essential difficulties encountered during the run was the qualitative and quantitative estimation of the parameters. Specifically, it was necessary to determine the number of parameters needed as well as their numerical values in order to reduce the gap between the simulation model and the real time series. The problem is that the order of the model was previously studied with (Akaike Information Criteria, Akaike [16]), while the estimation of its parameters is based on the least squares method of a nonlinear function. Thus, Akaike has approached this problem using the maximum information criteria. If one considers independent observations entropy measure and Kullback-Liebler information criteria. Based on the AIC criteria for a stationary time series, the model to be chosen is the one that minimizes:

$$AIC(p, q) = N. Ln(\sigma_a^2) + 2(p + q) \quad (2.14)$$

where N is the sample size and σ_a^2 is the maximum likelihood estimate of the residual variance.

2.5 Modulating Function

Another difficulty in performing the simulation was estimating the variance or, in other words, the envelope function. The main problem was to quantify this function since, for given conditions, its use is critical. Thus, the variance controls the non-stationarity of the processes and also takes into account some statistical parameters, among which the response of the structure or extreme values of acceleration. The variance of (Z_t), the acceleration time series data, considered as random variables, is given as follows:

$$\sigma_a^2 = E(Z_t - \mu_t)^2 \tag{2.15}$$

The assumption that $E(Z_t) = 0$ has been commonly used in acceleration time series simulations. Ellis [17] used equally weighted two-second time windows with time intervals of 0.02 sec and estimated the variance as follows:

$$f_z^2(t) = \frac{1}{100} \sum_{i=t-50}^{i=t+50} (Z_i)^2 \tag{2.16}$$

f(t) provides an approximate estimate of the modulating function. However, this approach does not have a criterion to distinguish between stationarity and non-stationarity data. For practical purposes, it is essential to characterize the variance function with a minimum number of parameters. In this study, a moving window of time interval equal to 0.5 seconds is utilized to determine the acceleration time series variance under question. This method to determine the variance is used in all three acceleration time series used. MATLAB is used to do all necessary calculations.

2.6 Parametric Envelope Function

For practical purposes, it is essential to characterize the variance function with a minimum number of parameters. One approach used by Kozin [5] to estimate the envelope function is to use a cubic spline interpolation that follows the irregularities in acceleration time series. The spline is applied by fitting functions of the following form to a number of segments of the acceleration time series records.

$$f(t) = a.t^3 + b.t^2 + c.t + d \quad (2.17)$$

Continuity at the intersections of these functions is assured by imposing a condition of an equal slope. The need to account for a large number of parameters limits the use of the cubic spline, even though it can be an excellent tool for fitting a record. In general, a simple function with a limited number of parameters may be a satisfactory answer for the current problem, to use a model approximation. A smoothed function is used in this studies of the form:

$$S(t) = \alpha e^{-(\frac{t-\beta}{\gamma})^2} \quad (2.18)$$

where α, β, γ, are constants found by fitting the function to the estimated variance using the least squares method. This function is effective in fitting modulating functions with narrow peaks.

2.7 Modeling Procedure

Determining the subset of models and the corresponding orders was realized through the necessary to compare the estimated AR and partial AR functions with the behavior of the corresponding theoretical AR and partial AR functions, respectively. Thus, a satisfactory estimate of the AR function R(k) for a time series with zero mean at lag k is given as follows:

$$R(k) = C(k)/C(0) \tag{2.19}$$

where $C(k) = (\frac{1}{N}) \sum_{t=1}^{t=N-k} (Z_t Z_{t+k}) \qquad k = 1, 2, \ldots, k$

It is essential to compute the variance of the estimated autocorrelation coefficients as a criterion to decide that the autocorrelation function is zero after certain lag K. Standard errors of the autocorrelation estimates is given by Bartlett [1]

$$R(k) = (\frac{1}{n^{\frac{1}{2}}})(1 + (1 + 2(R^2(1) + \cdots + R^2(q))^2 \qquad k > q \tag{2.20}$$

The partial autocorrelation function may be estimated either by the Yule-Walker equations or by successively fitting autoregressive (AR) processes of order 1, 2,…,k. The Yule-Walker equations are obtained by substituting estimated autocorrelation coefficients R(k) obtained using Eq. 2.20 and solving for successive values of k = 1,2, ..., K. Using the Yule-Walker equations may lead to problems when the parameters are close to the boundary values of the non-stationarity condition. Therefore, the use of fitting autoregressive processes of k orders is adopted to estimate the partial autocorrelation function. Again, the standard errors of partial autocorrelation estimates are needed to decide if specific values may be considered to be zero beyond some lag K. Quenouille [18] has shown that for an AR process of order P, the estimates of the partial autocorrelation function of order p+1 or higher are nearly independent. The variance is given by:

$$Var(ØØ_{kk}) = 1/\text{var [kk] - 1/n} \qquad k > p+1 \tag{2.21}$$

Based on the estimated and theoretical autocorrelation and partial autocorrelation functions, subclasses from the general ARMA (p, q) process could be selected for further investigation.

The selected subclasses of ARMA models are used to model the stationary time series obtained. The idea of maximum likelihood is usually used for the estimation of the parameters in the stochastic model. The maximum likelihood function is defined as the function associated with fixed observations Z, the variable set of parameters. In this study, the set of parameters refer to the p+q+1 parameters 〖$(\varphi_i, \beta_i, \sigma_a)$ of the ARMA model. If the original observation time series (Z_i), which is in our case the stabilized acceleration record of being modeled by the ARMA (p, q) model. For a stationary invertible ARMA (p, q) model, it could be written the following:

$$a_t = Z_t - \varphi_1 Z_{t-1} - \cdots - \varphi_p Z_{t-p} + \theta_1 a_{t-1} - \cdots - \theta_q a_{t-q} \tag{2.22}$$

with the assumption that $Z_t = 0$, for $t \le 0$, and $a_t = 0$, for $t \le 0$.

For any set of parameters (φ, θ), the values a_t could be calculated successively.

$$L(£/Z) = a_t(\varphi, \theta/Z) \tag{2.23}$$

The a's are assumed to be independently normally distributed.

$$p(a_1, a_2, \ldots, a_n) = \left[\frac{1}{2\pi^{\frac{n}{2}}\sigma_a^n}\right] \exp\left(-\sum a_t^2/2\right) \qquad (2.24)$$

For any given parameters £, the probability distribution $p(a_1, a_2, \ldots, a_n)$ is associated for a given data (Z_t). In this study, the likelihood function is Eq. ($2.$ 8). It is convenient to work with log likelihood function $L(£/Z)$.

2.8 Generation of a Stationary White Noise

Stationary white noise can be generated, but it needs to meet the general condition of demonstrating the same intensity at different times, providing it with a constant power spectral density. For analytical purposes, white noise is approximated by sample functions. First, a series of pseudo-random numbers u_i are generated such that they will be repeated only after a vast cycle. Usually, the multiplicative congruence method is used, where the present pseudo-random number is u_i is related to the next one by the following relationship:

$$r_{i+1} = k.r_i(modulo\ m) \qquad (2.26)$$

Where $k\ and\ m$ are integers chosen to obtain the largest possible cycle. In this study, $k = 16807$ and $m = 2147483647$ were used. Repetition occurs after a sequence length of the order 2^{14}.

The pseudo-random number r_i generated by this method can be assumed to be independent realizations u_i of random variables u having a rectangular distribution $0 < u < 1$. These pseudo-random numbers will be used to generate a sample function that approaches white noise with a Gaussian distribution with mean μ and variance s^2. Transformation of variables used in conjunction with sample rejection method of Ahrens and Dieter [19]

Let $x_1 = 2r_1 - 1$

$x_2 = 2r_2 - 1$

$\mathrm{W} = x_1 + x_2$

If $\mathrm{W} > 1$, the procedure is repeated. Otherwise;

$$y_1 = (x_1(-\frac{2Alog(x_1)}{\mathrm{W}})^{1/2}).s + u \tag{2.27}$$

$$y_2 = (x_2(-\frac{2Alog(x_2)}{\mathrm{W}})^{1/2}).\ s\ + u$$

A sample function f_k can now be established by assigning y_1, y_2, ... , y_n to n successive ordinates spaced at equal intervals ($\delta t = 0.001\ sec$). By repeating this procedure m times, a stationary process is obtained. The process is characterized by the autocorrelation function $R_{\mathfrak{f}}(k)$, which is practically close to zero.

The generation of white noise and its correlation function are shown in figures 1 and 2.

Chapter 3

Application of ARMA Models

3.1 Models Adopted

In this study of acceleration time series, the choice of a model depends on the nature of the intended application. For design purposes, it is essential to employ the smallest possible number of acceleration time series parameters in the analysis. The model adopted is the autoregressive moving average ARMA model used in conjunction with a parametric envelope function. From previous sections, it is understood that the acceleration time series event is a non-stationary time series. As the first step in modeling procedures, the event is divided by the modulating function so as to obtain the stationary value of the series. The modulating function obtained is used to fit a smoothed parametric envelope function. A simple form for an event with a single peak, is given by:

$$S(t) = \alpha e^{-(\frac{t-\beta}{\gamma})^2} \qquad (3.1)$$

where α, β, γ, are constants found by fitting the function to the estimated variance using the least squares method. This function is effective in fitting modulating functions with narrow peaks. Subsequently, the stationary series is used to perform an estimation analysis of the model parameters.

3.2 Acceleration Time Series Modeled

It is worth noting that data from three acceleration time series were consistently used for this study. These included Afroun with 16000 data points (0.005-second digitization increment), Ain Defla with 5000 data points (0.005-second increment), and Dar Beida with 5528 data points (0.005-second increment). Thus, the critical difference between the series was the number of points to estimate. Shown in Fig. 3.1, Fig. 3.2, and Fig. 3.3 are the measured acceleration time series plots.

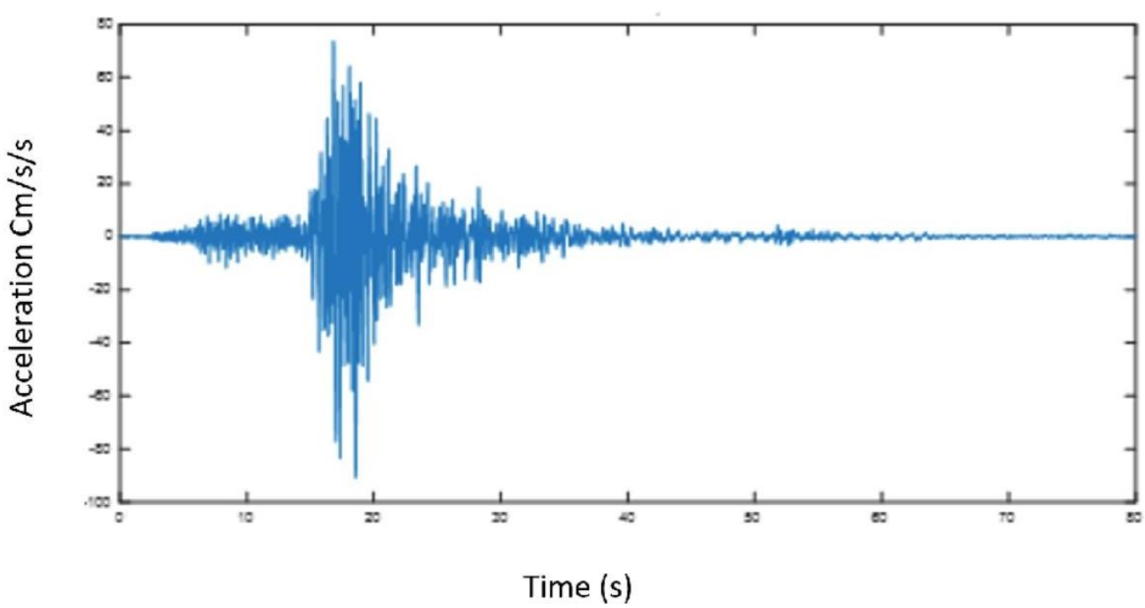

Figure 3.1 Afroun Acceleration Time Series

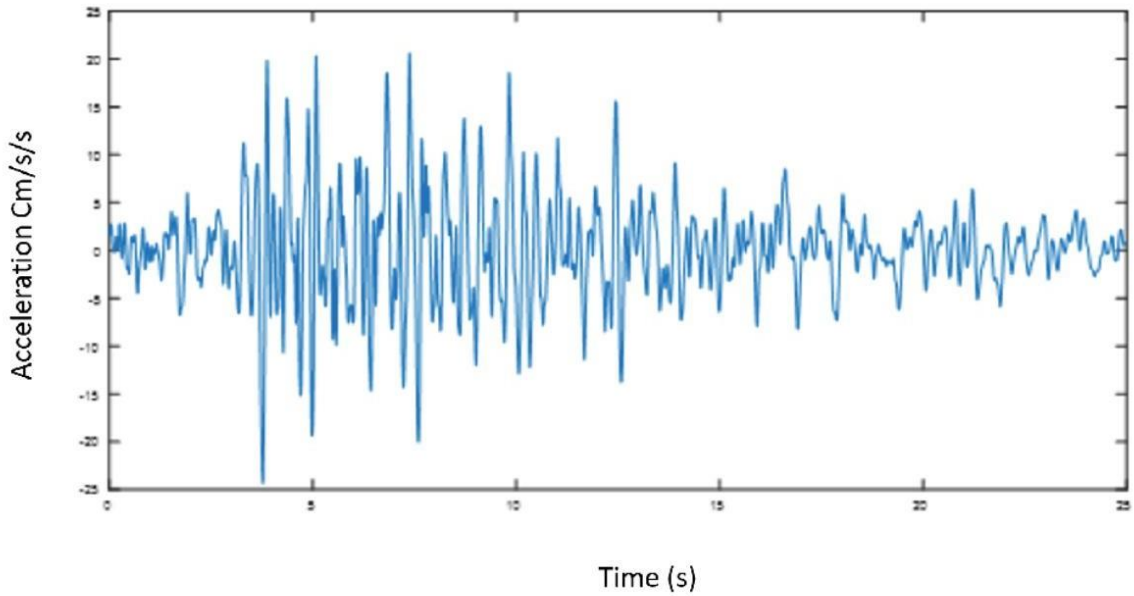

Figure 3.2 Ain Defla Acceleration Time Series

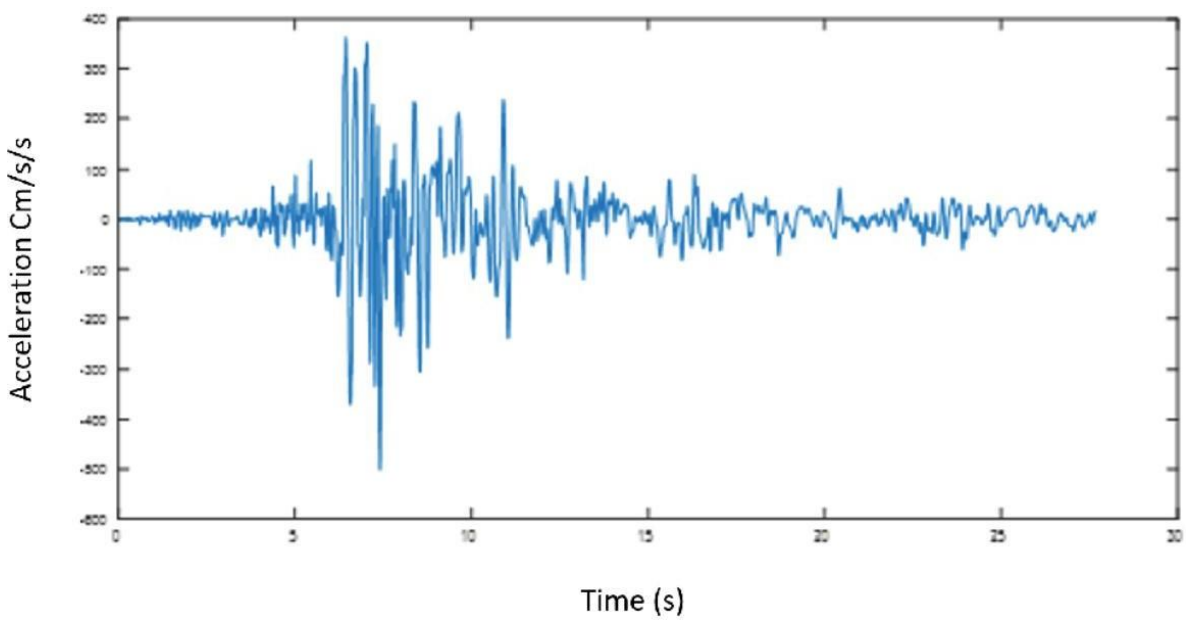

Figure 3.3 Dar El Beida Acceleration Time Series

As the first step in model identification, the modulating function f(t) was computed for each measured acceleration record using Eq. 2.7 The results are shown in Figs. 3.4, 3.5, and 3.6. It could be concluded that the non-stationarity is significant in each event.

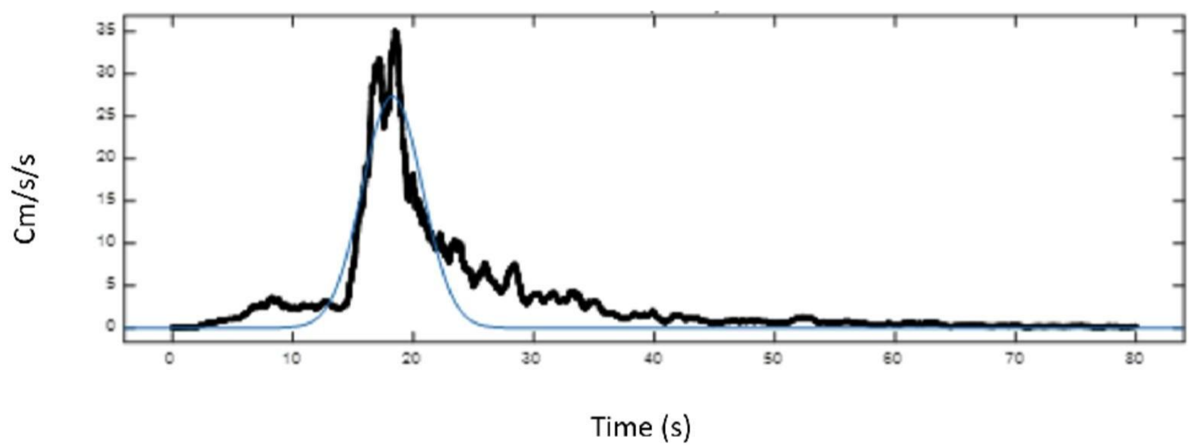

Figure 3.4 Afroun Measured and Envelope Functions.

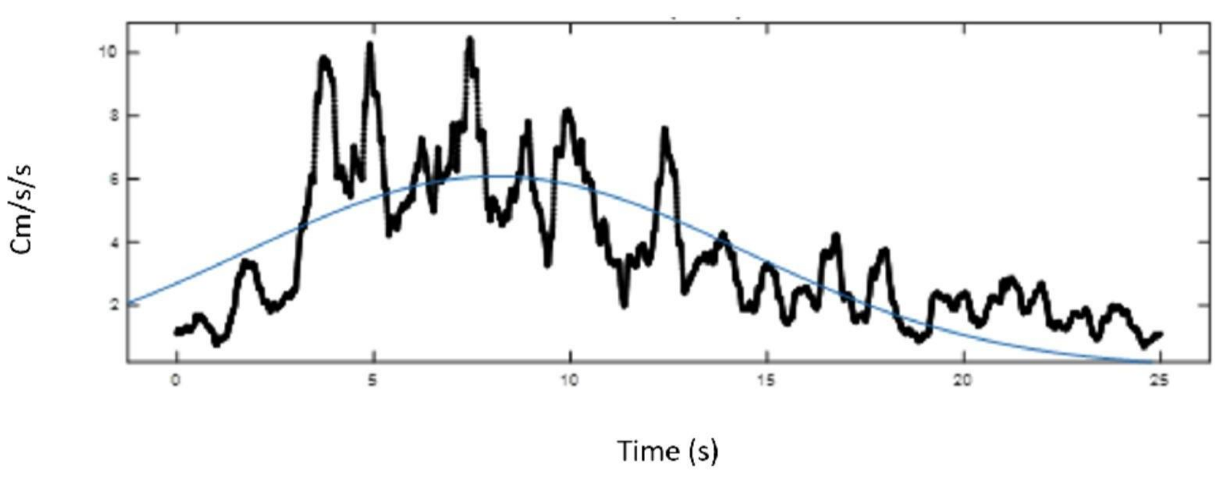

Figure 3.5 Ain Defla Measured and Envelope Functions.

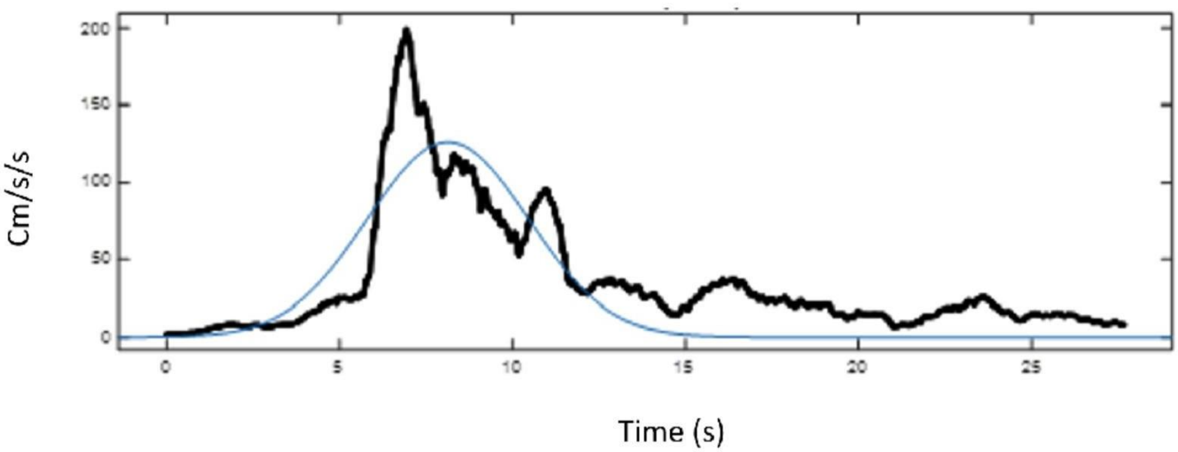

Figure 3.6 Dar Beida Measured and Envelope Functions.

The one peak envelope function proposed in Eq. 3.1 is fitted to each of the modulating functions of the measured records using the least square method. Measured and fitted functions are shown for the four acceleration time series in Figs. 3.4, 3.5, and 3.6. The original acceleration record and the modulating function obtained are then utilized to estimate the stabilized (stationary in the broad sense) acceleration time series. The stabilized time series obtained for the three events are shown in Figs. 3.7, 3.8, and 3.9 The variance of the series is approximately one with zero mean value. The frequency content of each of the time series will be included in the ARMA model parameters.

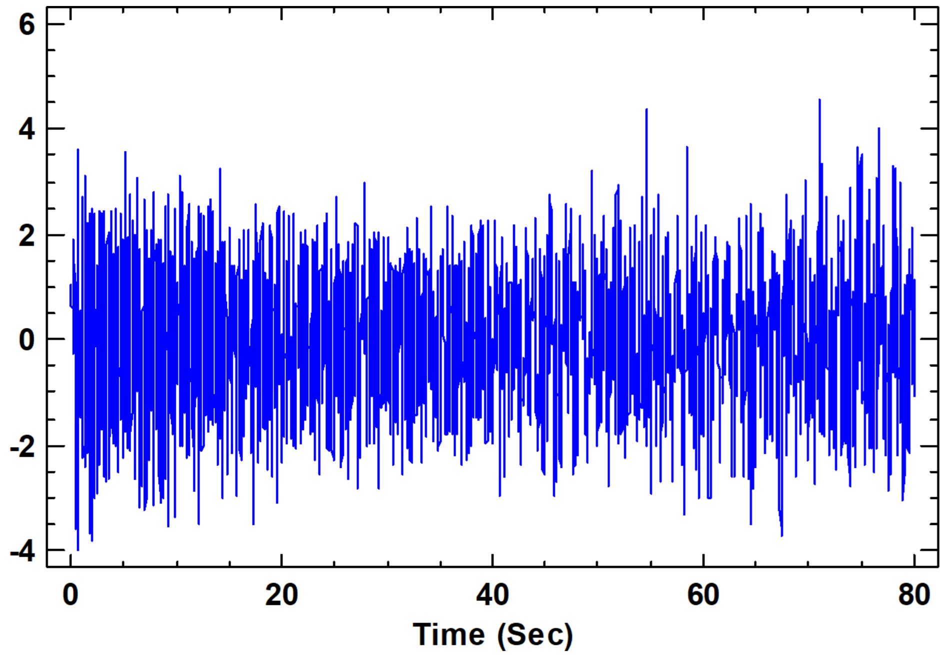

Figure 3.7 Stabilized Acceleration Series – Afroun

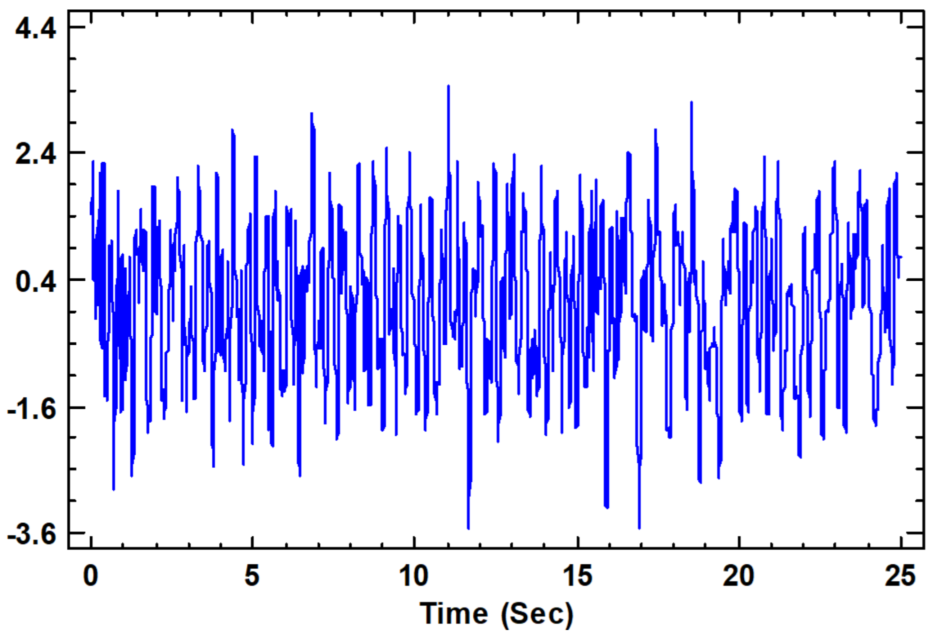

Figure 3.8 Stabilized Acceleration Series – Ain Defla

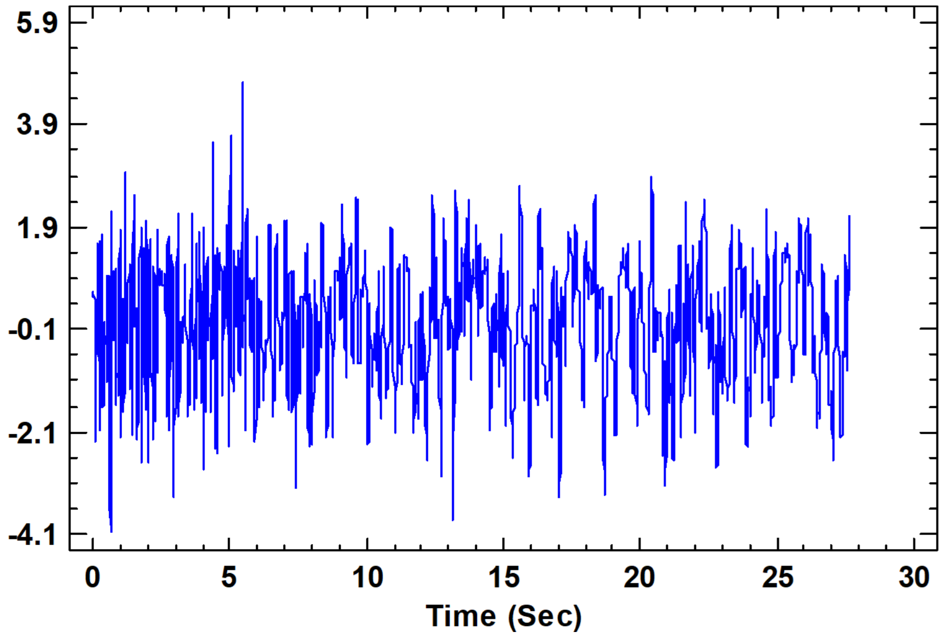

Figure 3.9 Stabilized Acceleration Series – Dar El Beida

The estimated autocorrelations for the three stabilized time series of the measured records were computed using Eq. 2.10. For illustration, the autocorrelation functions obtained are plotted in Figs. 3.10, 3.11, and 3.12. The tendency of the autocorrelation functions to die out rapidly indicates that none of the roots of the characteristic equations is close to the boundary of the unit circle. This ensures the time series stationarity obtained.

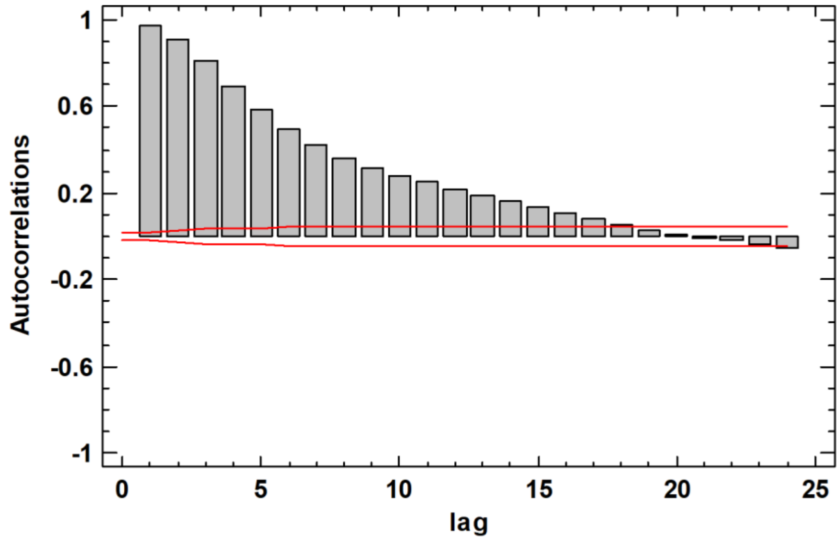

Figure 3.10 Estimated Autocorrelation for Afroun

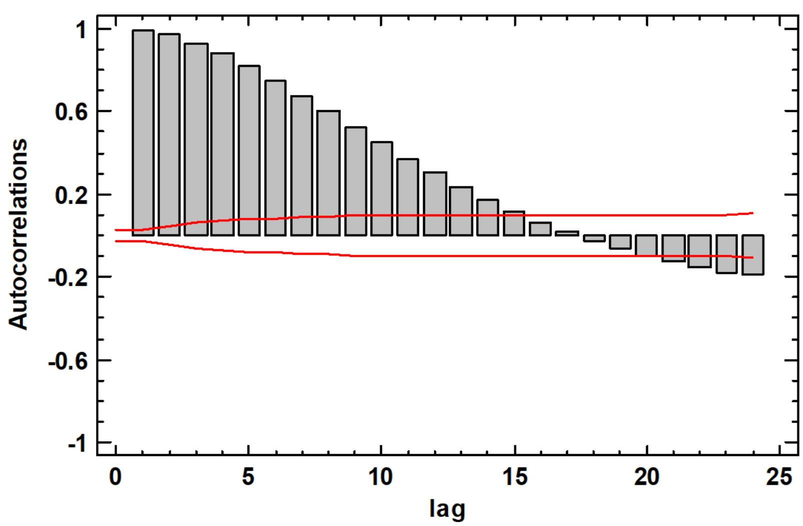

Figure 3.11 Estimated Autocorrelation for Ain Defla

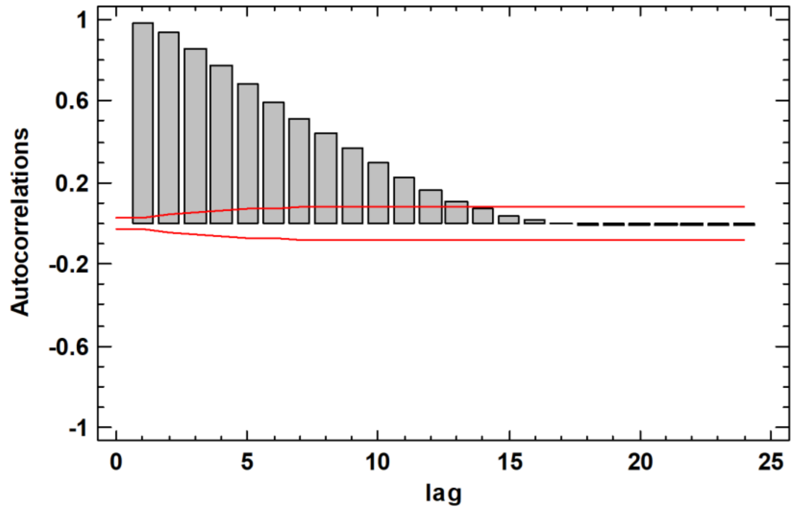

Figure 3.12 Estimated Autocorrelation for Dar El Beida

As explained previously, the partial autocorrelation functions for a record are estimated by fitting successive autoregressive processes of k orders using MATLAB. The results are plotted in Figs. 3.13, 3.14, and 3.15. From the partial autocorrelation function of Afroun, Ain Defla, and Dar El Beida time series, it is seen that after lag k = 2, or k = 3, the correlations decrease. This suggests the use of an ARMA model of order (p, q) such that p-q = 2 or 3. A model ARMA (p, q) of p-q around p-q=2 or 3 should be tried. The use of AE and partial AR functions suggested the process models which might be used. To obtain efficient estimates of the parameters, all the models suggested by the AR and partial AR functions were applied to the three events under investigation. A variety of ARMA models were fitted to the experimental records for the three acceleration time series using the maximum likelihood estimates, which could be approximated by the least squares method. The comparison needed to select the order of the model could be done through the use of the AIC criteria. Subsequently, several ARMA simulations that had one or two MAs were found during the computation phase. In other words, the AIC (p, q) comparison clearly characterizes the simulation model with the minimum AIC value as the final one. Table

3.2 displays AIC values for ARMA (p, q) models for each events. The results indicate that Afroun, Ain Defla, and Dar El Beida acceleration time series are best characterized by ARMA (2, 2) processes. Shown in Table 3.1 are the autoregressive parameters φ_1, φ_2, the moving average θ_1, θ_2, and the envelope function parameters α,β,γ, corresponding to maximum likelihood estimates for each event.

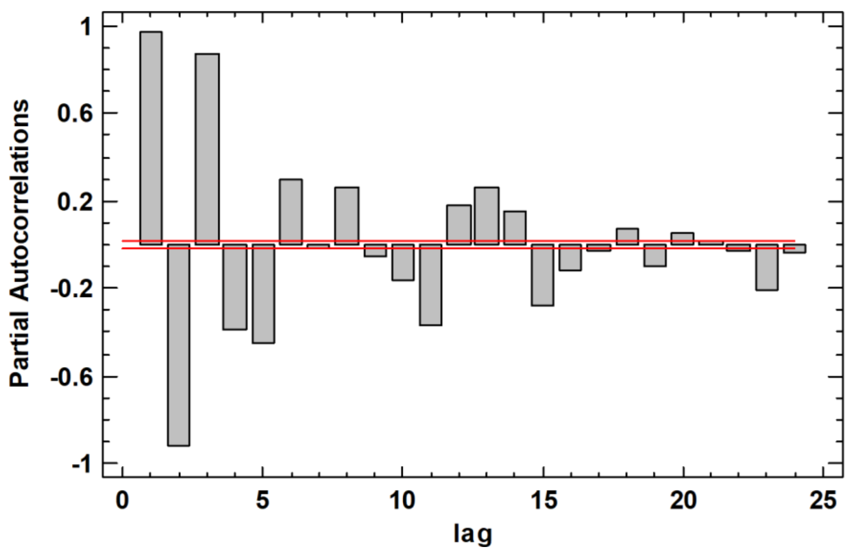

Figure 3.13 Partial Autocorrelation Function – Afroun

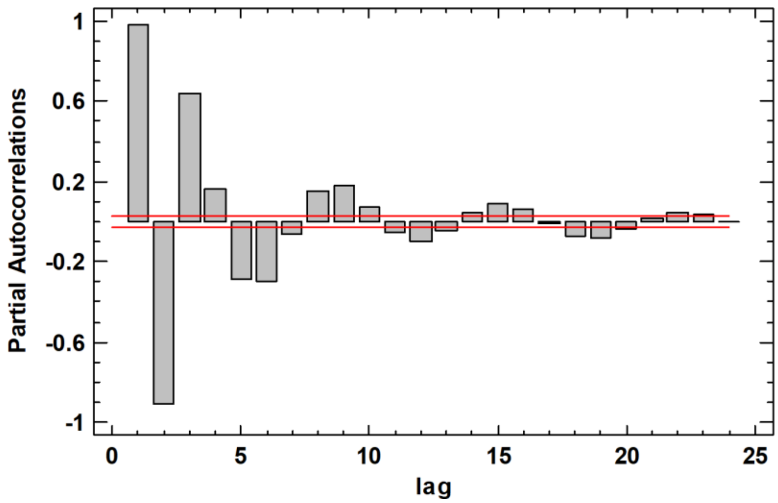

Figure 3.15 Partial Autocorrelation Function – Dar El Beida

3.4 **Acceleration Time Series Simulation [19]**

As previously stated, through the use of simulated ARMA process systems in a time-based approach, good results can be achieved with a limited number of parameters. To produce the population that describes the observed acceleration and be used for response spectra and damage, acceleration time series simulation is needed. The experimental procedure used was as follows: first of all, the station time series were generated using the ARMA model. Once this was done, the stationary series was multiplied by the function s(t), an envelope form. In addition, the assumption that ARMA is treated as a linear combination of Gaussian random variables (a_t) and already existing values (Z_t) was used: this means that the generated time series can be simulated recursively. Hence, shown in Figs. 3.16, 3.17, and 3.18 are three simulations of the acceleration time series.

Table 3.1 ARMA and Envelope Function Parameters

ARMA & Envelope Function Parameters	Afroun	Ain Defla	Dar Beida
φ_1	1.8635	0.944	1.9080
φ_2	-0.922845	-0.963	-0.9590
θ_1	-1.28836	-0.469	-1.1503
θ_2	-0.954263	-0.295	-0.8881
σ_w	0.0407843	0.0210	0.03357
α	27.36	6.081	126.2
β	18.26	8.098	8.130
$\boldsymbol{\gamma}$	3.290	8.991	3.290

Table 3.2 AIC Values for ARMA (p, q) Models

Acceleration Time Series	Model	AIC (p, q)
Afroun	ARMA (1, 1)	-3.77779
	ARMA (1, 2)	-5.24942
	ARMA (2, 1)	-5.27942
	ARMA (2, 2)	-6.39901*
	ARMA (3, 1)	-6.38901
	ARMA (3, 2)	-6.39902
Ain Defla	ARMA (1, 1)	-4.88647
	ARMA (1, 2)	-5.75858
	ARMA (2, 1)	-7.54612
	ARMA (2, 2)	-7.91847*
	ARMA (3, 1)	-7.90599
	ARMA (3, 2)	-7.90799
Dar El Beida	ARMA (1, 1)	-5.-00972
	ARMA (1, 2)	-5.30972
	ARMA (2, 1)	-6.0831
	ARMA (2, 2)	-6.79935*
	ARMA (3, 1)	-6.69935
	ARMA (3, 2)	-6.59935

**Optimal Set by AIC Criterion*

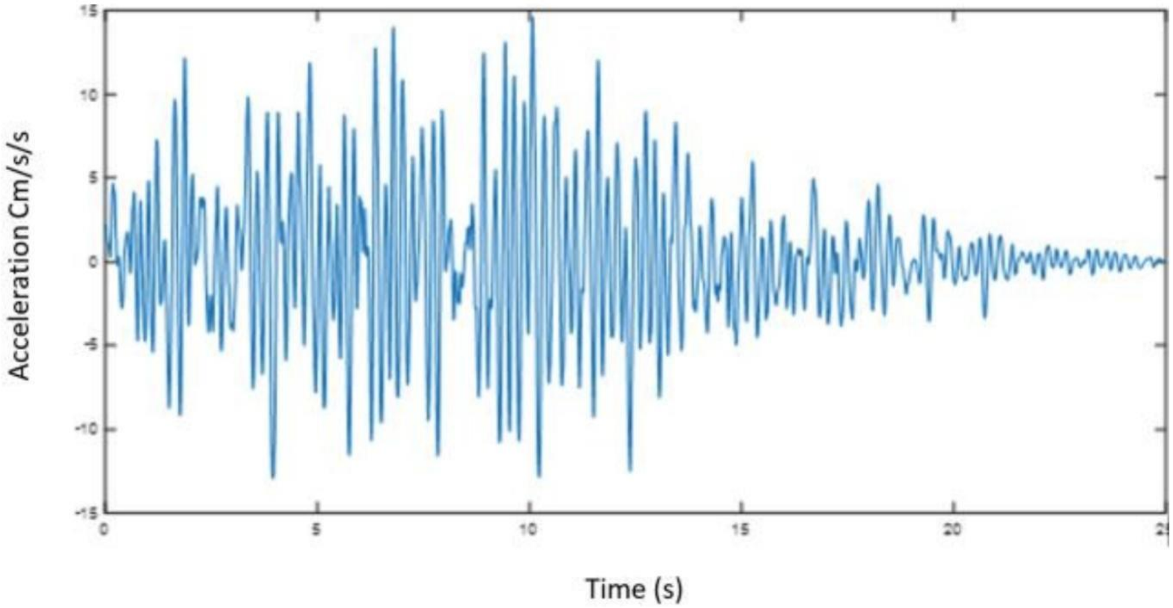

Figure 3.17 Ain Defla Simulated Acceleration