2) Motion types

The nature of particle motion is determined by the shape of its path (straight, circular, or curved) and the variation in its velocity (constant, non-uniform, or uniformly varying).

2-1) Rectilinear motion (RM)

Rectilinear motion is the type of movement that occurs along a straight path, described by a single coordinate and studied in a one-dimensional coordinate system (dimension). The expressions for the kinematics characteristics (the position, velocity and acceleration) of a moving particle are given as follows:

$$\begin{cases} \overrightarrow{OM} = x(t)\,\vec{\imath} \\ \vec{V} = \dfrac{d\overrightarrow{OM}}{dt} = \dfrac{dx(t)}{dt}\,\vec{\imath} = \dot{x}(t)\,\vec{\imath} \\ \vec{a} = \dfrac{d\vec{V}}{dt} = \dfrac{d^2x(t)}{d^2t}\,\vec{\imath} = \ddot{x}(t)\vec{\imath} \end{cases}$$

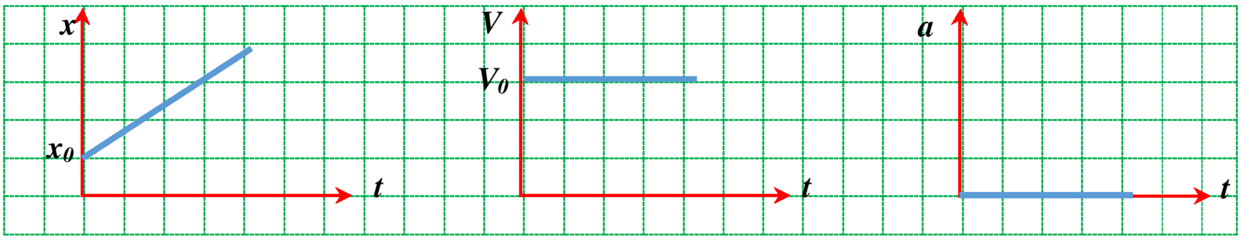
2-1-1) Uniform rectilinear motion (URM)

Uniform rectilinear motion occurs along a straight path with constant magnitude and direction of velocity (uniform velocity). This means that during this motion, the particle travels equal distances in equal time intervals. The expressions for the position, velocity, and acceleration of a moving particle are given as follows:

$$\begin{cases} \vec{a} = \dfrac{d\vec{V}}{dt} = \vec{0} \\ \vec{V} = \displaystyle\int \vec{a}\,dt = constant\,\vec{\imath} = V_0\,\vec{\imath} \\ \overrightarrow{OM} = \displaystyle\int \vec{V}\,dt = (V_0 \times t + x_0)\,\vec{\imath} \end{cases} \Rightarrow \begin{cases} \overrightarrow{OM} = x(t)\vec{\imath} = (V_0 \times t + x_0)\,\vec{\imath} \\ \vec{V} = \dfrac{d\overrightarrow{OM}}{dt} = constant\,\vec{\imath} = V_0\,\vec{\imath} \\ \vec{a} = \dfrac{d\vec{V}}{dt} = \vec{0} \end{cases}$$

The time equations of motion are given:

$$x(t) = V_0 \times t + x_0$$



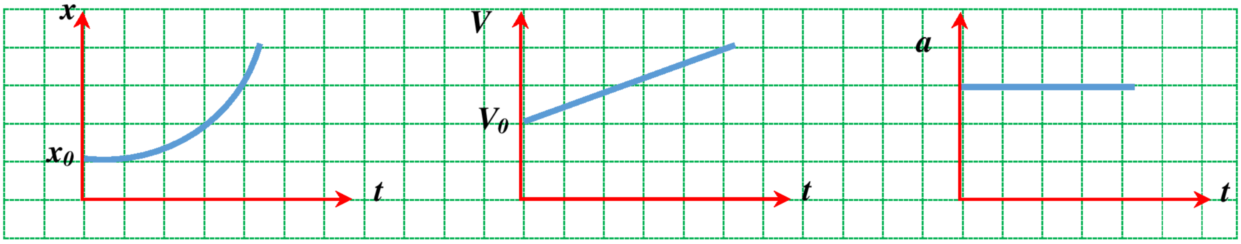
1-2-2) Uniformly varied rectilinear motion (UVRM)

Uniformly Varied Rectilinear Motion occurs when an object moves along a straight path with a uniformly changing velocity, implying a constant acceleration. In simpler terms, during this motion, the particle's velocity undergoes a consistent change at a steady rate over equal time intervals. The expressions for the position, velocity, and acceleration of a moving particle are as follows:

$$\begin{cases} \vec{a} = \dfrac{d\vec{V}}{dt} = onstant\ \vec{\imath} = a\ \vec{\imath} \\ \vec{V} = \displaystyle\int \vec{a}\ dt = (a \times t + V_0)\ \vec{\imath} \\ \overrightarrow{OM} = \displaystyle\int \vec{V}\ dt = (\frac{1}{2} a \times t^2 + V_0 \times t + x_0)\ \vec{\imath} \end{cases} \Rightarrow \begin{cases} \overrightarrow{OM} = (\dfrac{1}{2} a \times t^2 + V_0 \times t + x_0)\ \vec{\imath} \\ \vec{V} = \dfrac{d\overrightarrow{OM}}{dt} = (a \times t + V_0)\ \vec{\imath} \\ \vec{a} = \dfrac{d\vec{V}}{dt} = a\ \vec{\imath} \end{cases}$$

The time equations of motion are given:

$$x(t) = (\frac{1}{2} a \times t^2 + V_0 \times t + x_0)$$

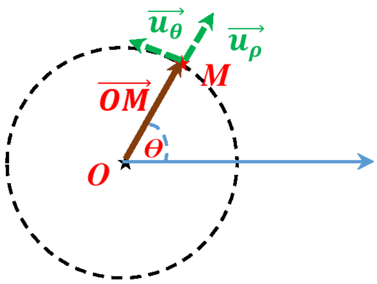


According to the direction of both the velocity and acceleration vectors of the particle's movement, we can distinguish two cases:

- ***Accelerated motion:*** *If acceleration and velocity are in the same direction* $\Rightarrow$ $\boldsymbol{\vec{v}.\vec{a} > 0}$
- ***Retarded motion:*** *If acceleration and velocity are in opposite directions* $\Rightarrow$ $\boldsymbol{\vec{v}.\vec{a} < 0}$

2-2) Circular motion

Circular motion is motion that takes place along a circular path that has a fixed radius **R**. *This movement can be studied using the polar coordinate system.*



The expressions for the position, velocity and acceleration vectors of a moving particle are given as follows:

$$\overrightarrow{OM} = \rho\overrightarrow{u_\rho} = R\,\overrightarrow{u_\rho}$$

$$\vec{V}(t) = \frac{d(\overrightarrow{OM})}{dt} = \frac{d(R\overrightarrow{u_\rho})}{dt} = R\,\frac{d\overrightarrow{u_\rho}}{dt}$$

$$\frac{d\,(\vec{u}_\rho)}{dt} = \dot{\theta}\,\vec{u}_\theta$$

$$\frac{d\,(\vec{u}_\theta)}{dt} = -\dot{\theta}\,\vec{u}_\rho$$

$$\vec{V}(t) = R\,\dot{\theta}\,\vec{u}_\theta \quad where \quad \dot{\theta} = \frac{d\theta}{dt}$$

$$\|\vec{V}(t)\| = \sqrt{(R\dot{\theta})^2} = R\dot{\theta}$$

$\dot{\theta}$ is he angular velocity

for the instantaneous ***acceleration*** *vector in* ***polar*** *coordinates is given as follows:*

$$\vec{a}(t) = \frac{d(\vec{V})}{dt} = \frac{d\,(R\dot{\theta}\,\vec{u}_\theta)}{dt}$$

$$= R\frac{d\dot{\theta}}{dt}\vec{u}_\theta + R\dot{\theta}\frac{d\vec{u}_\theta}{dt}$$

$$= R\ddot{\theta}\vec{u}_\theta - R\dot{\theta}\dot{\theta}\,\vec{u}_\rho$$

$$\vec{a}(t) = (-R\dot{\theta}^2)\,\vec{u}_\rho + (R\ddot{\theta})\vec{u}_\theta \;= (a_r)\,\vec{u}_\rho + (a_\theta)\vec{u}_\theta$$

$$\|\vec{a}(t)\| = \sqrt{= (R\dot{\theta}^2)^2 + (\,R\ddot{\theta})^2}$$

$\ddot{\theta}$ *is the angular acceleration.*

Note: *For the intrinsic coordinates, we perform the same steps, only we replace the unit vectors as follows:* $\overrightarrow{u_\rho} = -\overrightarrow{u_N}$ **and** $\overrightarrow{u_\theta} = \overrightarrow{u_T}$

$$\vec{a}(t) = a_N\,\vec{u}_N + a_T\,\vec{u}_T$$

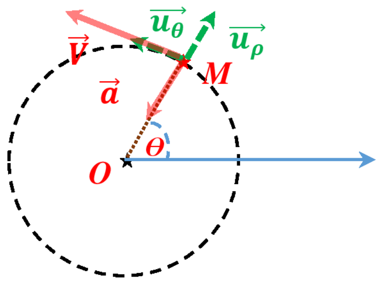
$$\begin{cases} a_{N=}R\dot{\theta}^2 \\ a_T = R\ddot{\theta} \end{cases}$$

2-2-1) Uniform circular motion UCM

The Uniform circular motion occurs along a circular path with a constant angular velocity ($\dot{\theta} = \frac{d\theta}{dt} = constant = \omega_0$)*. This means that the particle displaces at constant angles during equal time intervals.*

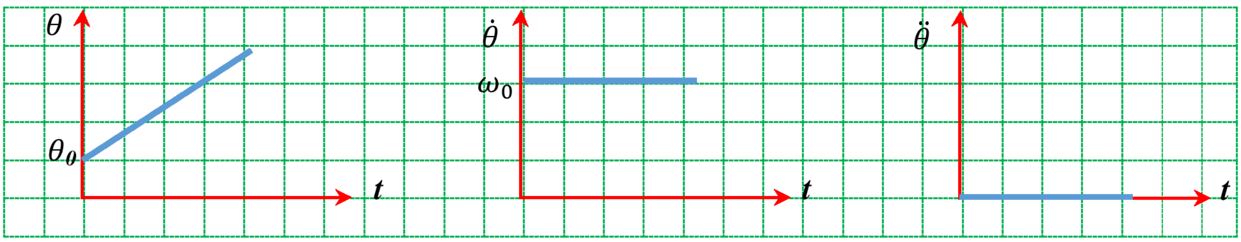
$$\begin{cases} \ddot{\theta} = \dfrac{d\dot{\theta}}{dt} = 0 \\ \dot{\theta} = \displaystyle\int \ddot{\theta} dt = constant = \omega_0 \end{cases} \Rightarrow \begin{cases} \overrightarrow{OM} = \rho \overrightarrow{u_\rho} = R\,\overrightarrow{u_\rho} \\ \vec{V}(t) = R\,\omega_0\,\vec{u}_\theta \\ \vec{a}(t) = (-R\dot{\theta}^2)\,\vec{u}_\rho \end{cases}$$

In uniform circular motion, the velocity vector stays consistently magnitude and tangential to the object's path at its position. Simultaneously, the acceleration is radial, pointing towards the center of the circle, with a zero tangential component.



The time equations of motion are given:

$$\begin{cases} \ddot{\theta} = 0 \\ \dot{\theta} = \dfrac{d\theta}{dt} = \omega_0 \\ \theta(t) = \omega_0 \times t + \theta_0 \end{cases}$$



2-2-2) Uniformly varied circular motion (UVCM)

Uniformly varied circular motion (UVCM) is movement that takes place on a circular path with angular velocities that vary regularly (constant angular acceleration). In other words, the angular velocity changes at a constant rate over equal time intervals.

$$\begin{cases} \ddot{\theta} = \dfrac{d\dot{\theta}}{dt} = constant \\ \dot{\theta} = \int \ddot{\theta} dt = \ddot{\theta} \times t + \dot{\theta}_0 \end{cases} \Rightarrow \begin{cases} \overrightarrow{OM} = \rho \overrightarrow{u_\rho} = R\, \overrightarrow{u_\rho} \\ \vec{V}(t) = R\, \dot{\theta}\, \vec{u}_\theta \\ \vec{a}(t) = (-R\dot{\theta}^2)\, \vec{u}_\rho + (R\ddot{\theta})\vec{u}_\theta \end{cases}$$

The time equations of motion are given:

$$\begin{cases} \ddot{\theta} = constant \\ \dot{\theta} = \ddot{\theta} \times t + \dot{\theta}_0 \\ \theta(t) = \dfrac{1}{2}\ddot{\theta} \times t^2 + \dot{\theta}_0 \times t + \theta_0 \end{cases}$$

During uniformly varied circular motion, the acceleration vector has both radial and tangential components