II. Powers, Roots, and Logarithm

Powers/Indices is used when we want to multiply a number by itself several times.

a^b

In this term, a is called **base/basis** and b is called **index/exponent** . The word *power* sometimes also means the exponent alone rather than the result of an exponential expression.

How to Say Powers

x^2	x squared
x^3	x cubed
x^n	x to the power of n x to the n-th power x to the n x to the n-th x upper n x raised by n
$(x+y)^2$	x plus y all squared bracket x plus y bracket closed squared x plus y in bracket squared

Practice

A. Read out the following terms and say their values.

1. 2^6
2. $\left(\frac{2}{3}\right)^3$
3. $x^5 \div x^2$
4. $(3ab)^2$
5. $\left(\frac{x}{3y}\right)^3$
6. $(9x)^0$

B. Read these expressions and simplify them.

1. $5^3 \times 5^{13}$
2. $8^{14} \times 8^{11}$
3. $(2^4)^3$
4. $\left(\frac{x^7}{x^3}\right)^2$

LAWS FOR POWERS

For equal exponents

➢ **First Law for Power:**

$$(ab)^n = a^n b^n$$

A product raised by an exponent is equal to product of factors raised by same exponent

$$(a/b)^n = a^n/b^n$$

For equal basis

➢ **Second Law for Powers:**

$$a^m a^n = a^{m+n}$$

- *The product of two powers with equal basis equals to the basis raised to the sum of the two exponents*
- *When expressions with the same base are multiplied, the indices are added*

How can we say this rule?

$$a^m \div a^n = a^{m-n}$$

➢ **Third Law for Powers :**

$$(a^m)^n = a^{mn}$$

Exponentiating of powers equals to the basis raised to the product of the two exponents

Practice

Try to express in words these other rules of powers:

1. $a^0 = 1$, $a \neq 0$.
2. $a^{-n} = 1/a^n$, $a \neq 0$.
3. $(a/b)^n = a^n/b^n$
4. $a^m \div a^n = a^{m-n}$

Roots and Radicals

Root is inversion of exponentiation

$$\sqrt[n]{a} = b \leftrightarrow b^n = a$$

$\sqrt[n]{a}$ is called **radical expression** (or **radical form**) because it contains a root.

The radical expression has several parts :

- The **radical sign** √
- The **radicand** : the entire quantity under the radical sign
- The **index**: the number that indicates the root that is being taken

Example:

$\sqrt[3]{a+b}$ $a+b$ is the radicand, 3 is the index.

The radical expression can be written in **exponential form** (**powers with fractional exponents**)

example:

$$\sqrt[n]{x} = x^{1/n}$$

So the law of powers can be used in calculating root

Examples:

$$\sqrt[n]{ab} = (ab)^{1/n} = a^{1/n}\, b^{1/n} = \sqrt[n]{a}\sqrt[n]{b}$$

A number is said **perfect square** if its roots are integers.

Example:

9, 16, 36, and 100 are perfect squares, but 12 and 20 are not.

How to Say Radicals

$\sqrt{x}$	(square) root of x
$\sqrt[3]{y}$	cube root of y
$\sqrt[n]{z}$	n-th root of z
$\sqrt[5]{x^2y^3}$	fifth root of (pause) x squared times y cubed fifth root of x squared times y cubed in bracket

Square Root

The square root is in **simplest form** if :

a. the radicand does not contain perfect squares other than 1.
b. no fraction is contained in radicand.
c. no radicals appear in the denominator of a fraction.

Example

$\sqrt{24}$ is not a simplest form because we can write it as $\sqrt{6\times 4}$ where 4 is a perfect square. We can simplify the radical into $2\sqrt{6}$

A radical and a number is called a **binomial** . The **conjugate** of binomial is another binomial with the same number and radical, but the sign of second term is changed.

Example

$2 + \sqrt{6}$ is a binomial and its conjugate is $2 - \sqrt{6}$

Practice

a. Read out the following radical expressions and say theirs exponential notation.

1. $\sqrt{4\,x^4}$

2. $\sqrt[4]{m^3y^8}$

3. $\sqrt[5]{a^3}$

4. $\sqrt[8]{x^6y^9}\,3$

5. $\sqrt{x^2 + y^2}$

b. Read out the following terms and say what their values are:

1. $243^{1/5}$
2. -4^{-2}
3. $125^{1/3}$
4. $(-5)^{-1}$
5. 3^{-3}

c. Simplify these radicals

1. $\sqrt{72}$
2. $\sqrt{234}$
3. $\frac{5}{2+\sqrt{3}}$
4. $\frac{\sqrt{3}}{\sqrt{6}-\sqrt{2}}$

d. Find the conjugate of these binomials

1. $2+\sqrt{5}$
2. $6 -\sqrt{4}$

Logarithm

How to Say Logarithm

$^{n}\log x$	log /lɒg/x to the base of n log base n of x
ln 2	natural log of two **"L N" of two**
$^{5}\log^{2} 25$	log squared of twenty-five to the base of five log base five of twenty-five all squared

Practice

Read out the following terms:

a. $a\,^{x}\log b$

b. $\log a^{2}$

c. $^{2}\log (1/6)$

d. $^{5}\log (x^{2}+y)$

e. $(^{n}\log x)^{2}$

f. $^{6}\log^{2} 22 - {}^{6}\log x^{2} -1$

Laws for Logarithm

- First Law for logarithm:

The logarithm of a product is equal to the sum of the logarithm of the factors

$$^{b}\log(xy) = {}^{b}\log x + {}^{b}\log y$$

- Second Law for logarithm:

The logarithm of a quotient is equal to the difference of the logarithms of the dividend and divisor

$$^{b}\log(x/y) = {}^{b}\log x - {}^{b}\log y$$

- Third Law for logarithm:

The logarithm of a power is equal to the exponent times the logarithm of the basis

$$^{b}\log(x^{a}) = a\,{}^{b}\log x$$

More Examples

$^{2}\log (x+y)+2\,{}^{2}\log 4x >4$	*log base two of x plus y in bracket plus two times log base two of four x's is greater than four*
$x^{2} + \frac{1}{\sqrt{x}} = 1$	*x squared plus (pause) one over root of x equals one*
$3^{x} + 9^{x-1} > 27$	*three upper x plus (pause) nine upper x minus one (pause) is more than twenty- seven*
$9^{x} - 1 < 2$	*nine to the x (pause) minus one is less than two*

Some Algebraic Processes

1. Expand (x-3)(x+2) into x^2-x-6.
2. Simplify (2x+2)/(x+1) into 2
3. Factorize x^3-2x^2+3x-2 into (x-1)(x+1)(x-2)
4. Cancel (x+1) from (2x+2)/(x+1) to get 2
5. Add/subtract/multiply/divide both side

   Examples: multiply both side of equation ½x= 4 with 2 to get x=8
6. Subtitute y=4 into equation 2x+y=12
7. Collect (x+2) from $(x+2)^3$-2(x+2)(x+1) to get (x+2)[$(x+2)^2$-2(x+1)]

Example

Find x that satisfy equation 3x-3x-1=162.

Answer

First, we multiply both side with 3 to get $3.3^x-3^x=486$.

Then, we collect 3^x and we have $3^x(3-1)=486$, which can be simplified into $2.3^x=486$.

Divide both side by 2, we get $3^x=243$.

We know that 243 is 3^5, so we can write $3^x=3^5$.

According to the rule of powers, x must be equal to 5.

Exercise

a. How do we say these mathematical terms?

1. $({}^3\log x)^2 + {}^3\log x^2 = \sqrt[4]{4 - x^3}$
2. $x^{\,{}^{n}\log (x+1)} = 0$
3. $\sqrt{2\sqrt{2}} = \log \left(\frac{x}{5}\right)$

b. Read and complete answers.

1. 13^2=...
2. 2^9=...
3. Every positive real numbers has .... real-numbered square roots.
4. The cube root of two hundred and sixteen is ....

5. If the root of eighty-one is raised by three, then we have ……….

6. 7 is the log base ten of …………..

c. Solve this problem and try to explain it.

1. Which is greater, $2^{95}+2^{95}$ or 2^{100}?

2. Which is the larger, $10^{1/10}$ or $2^{1/3}$?

3. Let A and B are real numbers greater than 1. If there is positive number $C \neq 1$ such that $2({}^{A}\log C + {}^{B}\log C) = 9\ {}^{AB}\log C$, then find the largest possible value for ${}^{A}\log B$.

4. Given ${}^{9}\log 20 = a$ and ${}^{3}\log n = 4a$. What is the value of n?

5. In March, the number of students was a perfect square. At the end of the semester, with 100 new students, the number of students became 1 more than a perfect square. At the end of the year with an additional 100 new students, the number of students is a perfect square. How many students were there in September?

d. Explain the process to find the solution x that satisfies each equation, inequality, or system of equations.

1. $1 - 4x \leq x + 11$
2. $\frac{2}{3}y - \frac{5}{3} = \frac{4-2y}{5}$
3. $x + y = 2$

   $2x - y = -5$