University of M'sila. 2023/2024
Faculty of Technology
Domaine: Engineering
Level: First year
Analysis 1

Tutorial Series 03(Real Functions)

Exercise 01

1. Find the domain of definition of the following functions

$$f(x) = \sqrt{x^2+3x-4}, \quad g(x) = \ln(x^2+3x-4), \quad h(x) = \frac{\ln(1+x)}{\sqrt{1-x^2}}, \quad k(x) = \frac{1}{[x]-2023}$$

2. Evaluate the following limits

$$(a) \lim_{x\to\infty} \frac{x+\sqrt{x^4-x^2+1}}{2x^2+1+\sqrt{x^4+1}}; \quad (b) \lim_{x\to 0}\left(\frac{\sin px}{\sin qx}\right); \quad (c) \lim_{x\to\pi}\left(\frac{\sin(x)}{x-\pi}\right);$$
$$(e) \lim_{x\to 0}\left(\frac{\ln(1+x)}{x}\right) \quad (f) \lim_{x\to 0}\left(x\left[\frac{1}{x}\right]\right)$$

3. Evaluate the limit $\left(\lim_{x\to\infty} \frac{n}{2} r^2 \sin\frac{2\pi}{n}\right)$, where r is a constant. Interpret this limit geometrically.

Exercise 02

1. Sketch the graph of the following function on $[0, 2]$

$$f(x) = \begin{cases} \sqrt{1-x^2} & \text{for} \quad 0 \le x < 1 \\ 1 & \text{for} \quad 1 \le x < 2 \\ 2 & \text{for} \quad x = 2 \end{cases}$$

2. For what values of c in the domain does $\lim_{x\to c} f(x)$ exist?
3. At what points does only the left-hand limit exist?
4. At what points does only the right-hand limit exist?

Exercise 03

Given the function $y = f(x)$ defined as follows

$$f(x) = \begin{cases} 0 & \text{if} \quad x^2 = 1 \\ 1 & \quad \text{otherwise} \end{cases}$$

Sketch the function. At what points is the function f discontinuous? Explain

Exercise04

Let f be the function defined on $\mathbb{R}^*$ by $f(x) = x^2 \sin\left(\frac{1}{x^2}\right)$

1. Show that f can be extended to be continuous on $\mathbb{R}$ and give its extention $\widetilde{f}$
2. Study the differentiability of $\widetilde{f}$ and calculate its derivative $\widetilde{f}'$
3. Is $\widetilde{f}$ of class $C^1(\mathbb{R})$

Exercise 05

Given $f(x) = \begin{cases} \frac{1}{2}\left(x^2-4\right) & \text{for} \quad 0 < x < 2 \\ 0 & \text{for} \quad x = 2 \\ \frac{2}{x^2}\left(x^2-4\right) & \text{for} \quad x > 2 \end{cases}$

Show that f is differentiable at $x = 2$.

Exercise 06

Discuss the continuity and the differentiabilty of the following functions:

$f(x) = \begin{cases} x^3 \sin\left(\frac{1}{x}\right) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$, $g(x) = \begin{cases} x+1 & \text{if } x \leq 0 \\ \cos^2\left(\frac{\pi x}{2}\right) & \text{if } x > 0 \end{cases}$,

$h(x) = \begin{cases} 1 - \left(\frac{1}{x}+1\right)e^{-\frac{1}{x}} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$

Exercise 07

Let f the function defined by $f(x) = \begin{cases} \dfrac{x+1}{2} & \text{si } 0 \leq x \leq 1 \\ \alpha x + 2 & \text{si } 1 < x \leq 2 \end{cases}$ where α is a real parameter

1. Find the value of α for which f is continuous.
2. Discuss the differentiability of f (with the found value of α)