

✿ Class: Numerical Methods Lab ✿

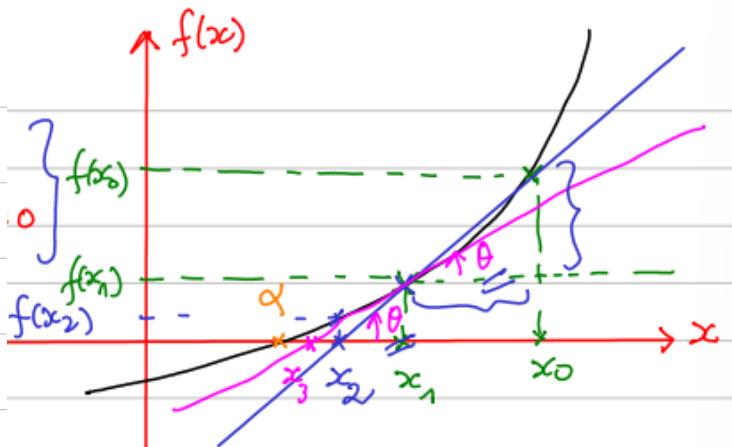
Lab 5 : Secant method

The objectives of this lesson:

- Understand the bisection method.
- Write an algorithm/flowchart for this method.
- Write a Matlab script for this method.
- Be able to apply this method to solve a non-linear equation $f(x) = 0$.
- Be able to use various stopping criteria to exit the algorithm of the bisection method.

Basic ideas and fundamental concepts:

$$\begin{aligned} ? f(x) = 0 \\ \rightarrow \alpha \text{ s.t. } f(\alpha) = 0 \end{aligned}$$



$$tg \theta = \frac{f(x_1) - 0}{x_1 - x_0} = \frac{f(x_0) - f(x_1)}{x_0 - x_1}$$

$$\hookrightarrow x_2 = x_1 - f(x_1) \cdot \frac{x_1 - x_0}{f(x_1) - f(x_0)} \longrightarrow ①$$

By following the same steps, we can get

$$x_3 = \dots$$

$$x_4 = \dots$$

:

Now, one can generalize this formula to get

$$x_{k+1} = x_k - f(x_k) \frac{(x_k - x_{k-1})}{f(x_k) - f(x_{k-1})} \longrightarrow ②$$

→ This formula concern the secant method.

An algorithm for the method

Comparison b/w Newton's method & Secant method formulas

$$\text{Newton} \rightarrow x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)} \quad \text{--- (3)}$$

$$\text{Secant} \rightarrow x_{k+1} = x_k - f(x_k) \frac{(x_k - x_{k-1})}{f(x_k) - f(x_{k-1})} \quad \text{--- (4)}$$

By comparing (3) & (4), we conclude that $f'(x_k)$ is estimated by

$$f'(x_k) = \frac{f(x_k) - f(x_{k-1})}{x_k - x_{k-1}} \quad \text{--- (5)}$$

Stopping Criteria :

Criteria #1. The length of the interval $[a, b]$ ecc $\rightarrow |a - b| < \epsilon$

\rightarrow Stop if $\text{abs}(a - b) < \text{epsilon} \rightarrow \text{while } (\text{abs}(a - b) > \text{eps}) \downarrow$

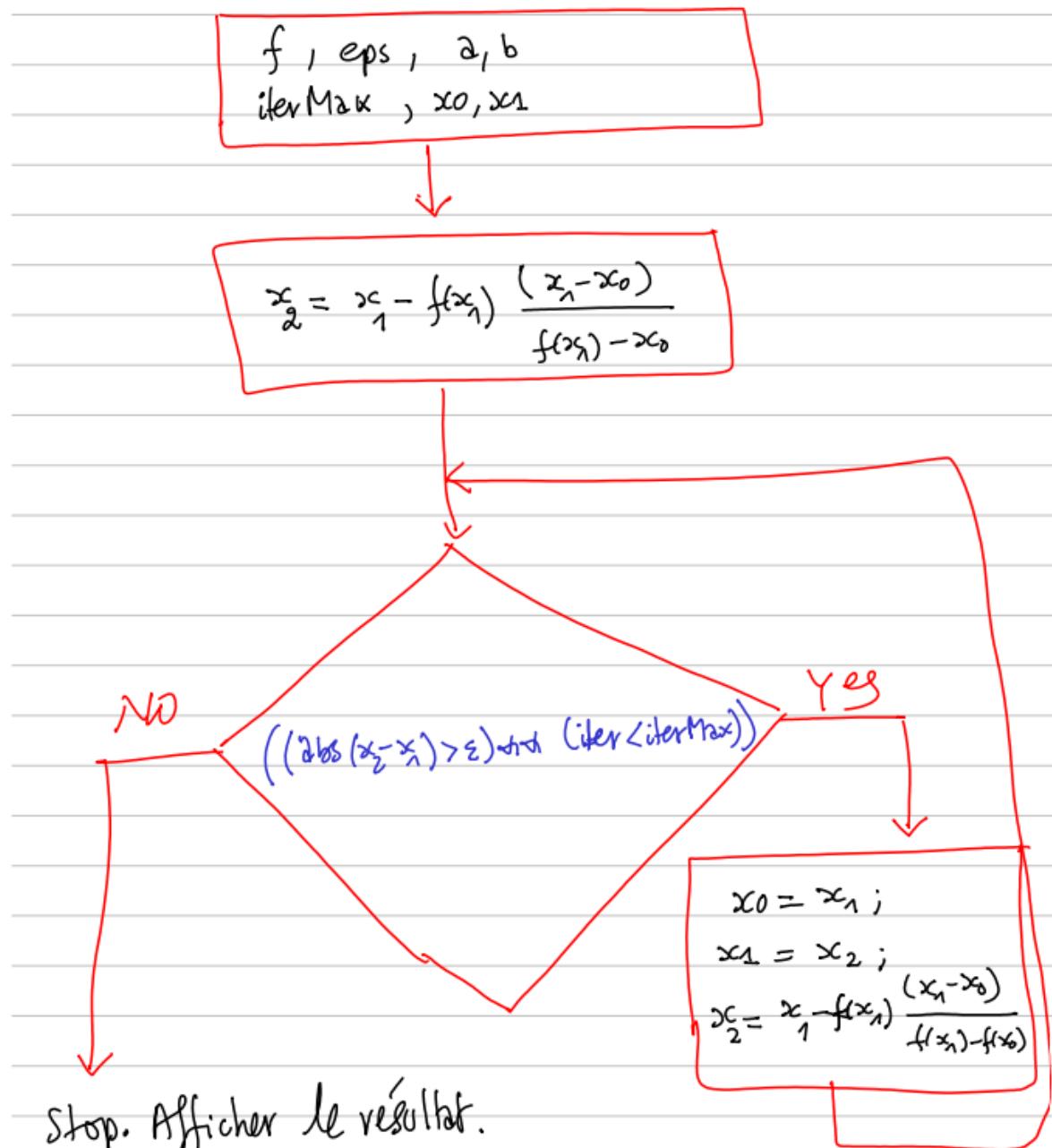
Criteria #2. The number of iterations (MaxIter)

\rightarrow for iter = 1 : MaxIter \downarrow

Criteria #3. Both criteria combined \rightarrow

$\text{while } ((\text{abs}(a - b) > \text{eps}) \text{ and } (\text{iter} < \text{MaxIter})) \downarrow$

A flowchart for the method



Matlab script(s) for the method:

```
1
2 %f = inline('cos(x)-x.^3');
3 f = inline('x.^3 + 4*x.^2 - 10');
4 %-----
5
6 a=1;
7 b=2;
8 x0 =0;
9 x1=1;
10 x2 = x1-((f(x1)*(x1-x0))/(f(x1)-f(x0)));
11
12 iterMax=5;
13 if ((f(a)*f(b))< 0)
14 for iter=1:iterMax
15
16 x0=x1;
17 x1=x2;
18 x2 = x1-((f(x1)*(x1-x0))/(f(x1)-f(x0)));
19
20 fprintf('For iteration =%d \t , the solution is x0=%f\n',iter,x0)
21 end
22 fprintf('The final solution is x0 = %f \n',x0) ;
23 else
24 disp('There is no solution in [a,b]')
25 end
```

```

1
2
3 %f = inline('cos(x)-x.^3');
4
5 f = inline('x.^3 + 4*x.^2 - 10');
6 a=1; b=2;
7 x0 =0; x1=1;
8 x2 = x1-((f(x1)*(x1-x0))/(f(x1)-f(x0)));
9 iter=0;
10 eps=10^(-6);
11 if ((f(a)*f(b))< 0)
12 while (abs(x2-x1)>eps)
13 iter=iter+1;
14 x0=x1;
15 x1=x2;
16 x2 = x1-((f(x1)*(x1-x0))/(f(x1)-f(x0)));
17 fprintf('For iteration =%d \t , the solution is x0=%f\n',iter,x0)
18 end
19 fprintf('The final solution is x0 = %f \n',x0) ;
20 else
21 disp('There is no solution in [a,b]')
22 end

```