Homework #2

Pascal's triangle is a triangular array of binomial coefficients found in probability theory, combinatorics, and algebra. It bears the name of the French mathematician Blaise Pascal. The numbers in Pascal's triangle are arranged so that each number is the sum of two numbers just above it. Pascal's triangle is commonly used in probability theory, combinatorics, and algebra.

In general, we can use Pascal's triangle to find the coefficients of binomial expansion, the probability of heads and tails in a coin toss, the probability of certain combinations of things, and so on. In the following section, we will go over the concept of Pascal's triangle in depth.

Pascal's triangle is represented by a triangular matrix P whose elements p(i,j) are defined as follows:

- P(i,j)= 1 if j=1 (the first column is always 1).
- P (i,j)= 1 if i =j (the diagonal is always 1).
- P (i,j)= P (i-1,j-1) + P (i-1,j) if j > 1 and j < i (the element in row i and column j is obtained by adding the values of the elements in row i-1 and column j-1, and in row i-1 and column j).
- P (i,j)= 0 si j > i (upper part of the matrix is zero).



Pascal's triangle determines the coefficients which arise in binomial expansions. For example, consider the expansion

$$(x+y)^2 = x^2 + 2xy + y^2 = \mathbf{1}x^2y^0 + \mathbf{2}x^1y^1 + \mathbf{1}x^0y^2.$$

The coefficients are the numbers in the second row of Pascal's triangle:

$$\begin{pmatrix} 2\\1 \end{pmatrix}=2, \begin{pmatrix} 2\\2 \end{pmatrix}=1.$$

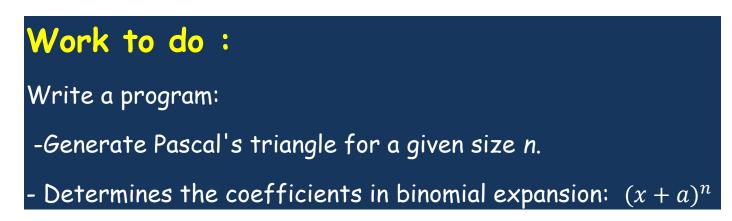
In general, when a binomial like x + y is raised to a positive integer power of n we have:

$$(x+y)^n = \sum_{k=0}^n a_k x^{n-k} y^k = a_0 x^n + a_1 x^{n-1} y + a_2 x^{n-2} y^2 + \ldots + a_{n-1} x y^{n-1} + a_n y^n$$
,

where the coefficients a_k in this expansion are precisely the numbers on row n of Pascal's triangle. In other words,

$$a_k = \binom{n}{k}$$
.

This is the binomial theorem.



Important Notes

- ✓ Answers will be received before <u>December 21, 2023</u>. No answers will be accepted after this date
- \checkmark The work is **individual**
- ✓ Works are **sent via e-mail** to professors, each according to their group :

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