Université de M'Sila
Faculté des MathÃẹmatiques et d'Informatique / Département de Mathématiques
Master Maths. **S 1/ Analyse Mathématiques et Numériques**
– ***Distributions Analyse de Fourier I*** –

Homework (Devoir maison) January (Janvier) 2023

** (Utiliser les notations du cours)
** (On tiendra compte de la prśentation des copies)

Exercise 1. (1) Give the definition of : – a regular distribution, – a singular distribution - the norm in $\mathcal{D}(\mathbb{R})$, – the derivation in $\mathcal{D}'(\mathbb{R})$, – the convolution in $\mathcal{D}'(\mathbb{R})$, – the distributions $vp\frac{1}{2x+2}$, H and δ.

Exercise 2. Prove that $\mathrm{pf}\frac{H}{x^2} \in \mathcal{D}'(\mathbb{R})$:

$$\langle \mathrm{pf}\frac{H}{x^2}, \varphi \rangle = \lim_{\epsilon \downarrow 0} \Big\{ \int_\epsilon^\infty \frac{\varphi(x)}{x^2} \mathrm{d}x - \frac{\varphi(0)}{\epsilon} + \varphi'(0) \log \epsilon \Big\}, \quad \forall \varphi \in \mathcal{D}(\mathbb{R}).$$

Exercise 3. Let $m, k \in \mathbb{N}$ and $f(x) := e^{mx}\delta^{(k)}$. Prove that $T_f \in \mathcal{D}'(\mathbb{R})$. Calculate T_f.

Exercise 4. Let

$$\langle T, \varphi \rangle = \int_{-\infty}^{\infty} \varphi(2x+3, -x) \mathrm{d}x, \quad \forall \varphi \in \mathcal{D}(\mathbb{R}^2).$$

Prove that $T_f \in \mathcal{D}'(\mathbb{R}^2)$, and calculate $(\partial_x - \partial_y)T$ in $\mathcal{D}'(\mathbb{R}^2)$.

Exercise 5. Let $f(x) := \begin{cases} 1 & if \quad -1 \leq x \leq 1 \\ 0 & otherwise \end{cases}$

Calculate $f * f$, $f * f * f$.

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