Tutorial (TD): Series of Exercises n°4

Exercise 1:

1. Prove the following equality by truth table:
$$\bar{A}B + A\bar{B} = (A + B)(\bar{A} + \bar{B})$$

2. Demonstrate the following equalities using the properties (laws) of Boole algebra :
$$AB + A(B + C) + B(B + C) = B + AC$$
$$(A\bar{B}(C + BD) + \bar{A}\,\bar{B})C = \bar{B}\,C \qquad\qquad \overline{(AB + AC)} + \bar{A}\bar{B}C = \bar{A} + \bar{B}\bar{C}$$

3. Determine the complement ($\bar{F}$) of the following functions :
$$F1 = A + \bar{B}C \qquad\qquad F3 = (\bar{A} + B + \bar{C})(\bar{A} + B + C)(A + B + C)$$
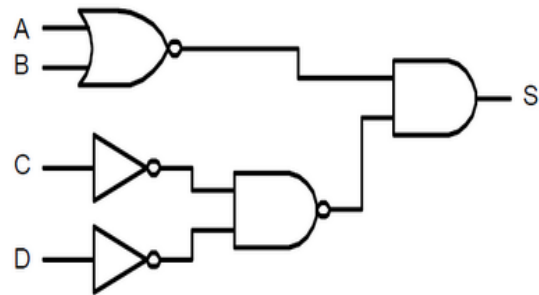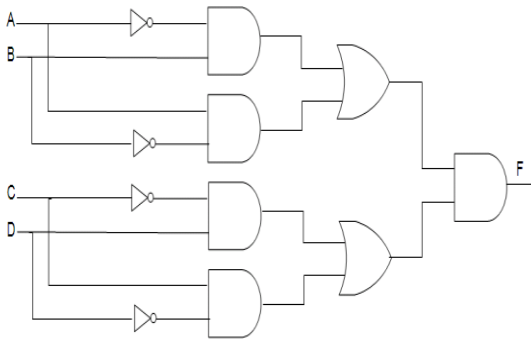$$F2 = \bar{A}\bar{B}\bar{C} + \bar{A}BC + ABC \qquad F4 = (\bar{A} + \bar{B} + \bar{C})(\bar{A} + B + C) + ABC$$

Exercise 2:

1. Draw the logic diagram (logigram) of $F1(A, B, C) = (A + B)(\bar{A} + B + C)$

2. Determine the functions of the circuits (output F and S) and write its truth tables:

Exercise 3:

1. Reduce (Simplify) the following Boolean functions using Algebraic simplification :

$$T1(X, Y, Z) = X + XY\bar{Z} + \bar{X}Y\bar{Z} + \bar{X}YZ \qquad\qquad T2(X, Y, Z) = \bar{X}Y\bar{Z} + XY\bar{Z} + XYZ$$

$$T3(X, Y, Z) = XYZ + Z(X\bar{Y} + \bar{X}Y) \qquad T4(X, Y, Z, W) = YW + ZW + \bar{Z}W + \bar{X}Y\bar{Z}\bar{W} + XY\bar{Z}$$

$$T5(X, Y, Z) = (\bar{X} + \bar{Y} + \bar{Z})(\bar{X} + Y + Z)(X + Y + Z) \quad T6(X, Y, Z, W) = (\bar{X} + Y)(X + Y + W)\bar{W}$$

$$T7(A, B, C) = \bar{A}BC + AC + A\bar{B}\,\bar{C} + \bar{A}\,\bar{B} \qquad\qquad T9\ (A, B, C) = ABC + A\bar{B}C + AB\bar{C}$$

$$T8(A, B, C) = (A + B)(A + C) + (B + A)(B + C) + (C + A)(C + B)$$

$$T10(A, B, C) = AB + C + \bar{C}\,(\bar{A} + \bar{B}) \qquad\qquad T11(A, B) = (A + \bar{B})(\bar{A} + B)(\bar{A} + \bar{B})$$

2. Write the truth tables for the two functions T7 and T8.

3. Find the two canonical forms of T7.

Exercise 4:

a) Convert F1 and F2 functions to 1st canonical form, F3 and F4 to the 2nd canonical form.
$$F1\ (A, B) = \bar{B} + A \qquad\qquad F2\ (A, B, C) = AB + \bar{B}C + \bar{C}$$
$$F3\ (A, B) = \bar{A} \qquad\qquad F4\ (A, B, C) = (\bar{B} + A)(A + \bar{C})$$

b) Find the other numerical form for the following Boolean functions :

F1 (A,B,C) = $\sum$ (0, 2, 4,7) F2 (A,B,C,D) = $\sum$ (0, 2, 6, 10, 11, 14) F3 (A,B,C,D) = $\prod$ (0, 3, 5, 6)

Exercise 5: Considering the Boolean functions given by the truth table:

X	Y	Z	F1	F2
0	0	0	1	1
0	0	1	0	1
0	1	0	1	1
0	1	1	0	0
1	0	0	1	1
1	0	1	0	0
1	1	0	1	1
1	1	1	0	0

1. Find the two canonical forms of F1 and F2 and $\overline{F1}$ and $\overline{F2}$.
2. Simplify F1 and F2 using the rules of Boole algebra.
3. Use the Karnaugh map (Karnaugh table) to simplify F1 and F2 in the form of **Sum of Products** (**SoP**) and **Product of Sums** (**PoS**).
4. Draw the logigram (logic diagram) of F1 and F2 (with the minimum of logical gates).
5. Draw the logigram of F2 only with NAND gates.

Exercise 6:

1. According to Karnaugh maps (tables), make the groupings and simplify the logical functions :

F1

C \ AB	00	01	11	10
0	1	0	1	0
1	1	0	1	0

F3

CD \ AB	00	01	11	10
00	1	1	1	1
01	1	0	0	1
11	1	0	0	1
10	1	1	1	1

F5

CD \ AB	00	01	11	10
00	1	1	1	1
01	0	0	1	1
11	0	0	1	1
10	0	0	0	1

F2

C \ AB	00	01	11	10
0	1	1	0	0
1	1	1	0	1

F4

CD \ AB	00	01	11	10
00	1	0	0	1
01	0	1	1	0
11	0	1	1	0
10	1	0	0	1

F6

CD \ AB	00	01	11	10
00	1	X		
01				X
11				X
10	1	X		

2. Using the Karnaugh method, simplify in the form of Sum of Products (**SoP**) and Product of Sums (**PoS**) the following functions:

T9, T7, T11, T5 et T10   from exercise 3.

$F1\ (A, B, C, D) = \bar{A} + AB + A\bar{B}C + A\bar{B}CD$

$F2\ (A, B, C, D) = \bar{A}\bar{B}\bar{D} + \bar{A}\bar{C}\bar{D} + \bar{A}BC\bar{D} + ABD + \bar{B}\bar{C}\bar{D} + A\bar{B}C\bar{D}$

$F3\ (A, B, C, D) = (A + \bar{B} + C)(A + \bar{B})(A + \bar{C} + D)(\bar{A} + B + C + \bar{D})(B + \bar{C} + \bar{D})$

$F4\ (A, B, C, D, E) = \bar{A}BE + BCDE + B\bar{C}\bar{D}E + \bar{A}\bar{B}D\bar{E} + \bar{B}\bar{C}D\bar{E} + \bar{B}CD\bar{E}$

G (A,B,C) = $\sum$ (2, 3, 4, 5)   H (A,B,C) = $\prod$ (4, 6, 7)

M (A,B,C,D) = $\sum$ (4, 8, 10, 11, 12, 14, 15)   N (A,B,C,D) = $\prod$ (1, 3, 5, 7, 9, 11, 13, 15)

R (A,B,C,D) = $\sum$ (0, 1, 4, 5, 7, 12, 13) + Φ (2, 15)

S (A,B,C,D) = $\sum$ (0, 1, 3, 5, 6, 10, 15) + Φ (2, 4, 7, 11, 14)

T (A,B,C,D) = $\prod$ (6, 7, 8, 9) + Φ (10, 11, 12, 13, 14, 15)