M'sila University Faculty of Mathematics and Computer Department of Mathematics Year 2023/2024

TD Number 4

Exercise 1.

Let A be a 3 × 3 matrix, such that $A = \begin{bmatrix} 1 & 5 & -2 \\ 1 & 2 & -1 \\ 3 & 6 & -3 \end{bmatrix}$

- 1. Verify that A is nilpotent.
- 2. Using exponential formula, solve the differential system $X' = \frac{dX}{dt} = AX.$

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Exercise 2.

Let A be a 4 × 4 matrix such that:
$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

- 1. Find the characteristic subspaces of A.
- 2. Find the new basis B' such that $A = PDP^{-1}$.

Exercise 3.

Find the Jordan Form of the following matrices:

1.
$$A = \begin{bmatrix} -1 & 1 \\ -1 & 1 \end{bmatrix} A = \begin{bmatrix} 11 & 4 \\ -4 & 3 \end{bmatrix}$$

2. $A = \begin{bmatrix} 5 & -9 & -4 \\ 6 & -11 & -5 \\ -7 & 13 & 6 \end{bmatrix} A = \begin{bmatrix} 3 & 0 & -1 \\ -2 & 1 & 1 \\ 3 & -1 & -1 \end{bmatrix}$