University of M'sila. 2023/2024
Faculty of Technology
Domaine: Engineering
Level: First year

Tutorial Series 01(Analysis 1)

Part 1: Real number system

Exercise 01

Answer "True" or "False" to the statements below. If the statement is False, explain why.
1.) -5 is a rational number. ________
2.) $\sqrt{8}$ is rational. ________
3.) $\sqrt{16}$ is a natural number ____________
4.) $\sqrt{2.25}$ is rational.. ________
5.) $\dfrac{22}{7}$ is a rational number________
6.) π is a rational number. ________
7.) $\sqrt[3]{9}$ is rational. ________
8.) $\sqrt{16}$ is an irrational number ____________
9.) $10\dfrac{3}{4}$ is rational.. ________
10.) $\dfrac{\pi}{2}$ is a rational number ________

Exercise 02

1. Let $n \in \mathbb{N}$. Prove that if n^2 is even then n is even

2 Prove that $\sqrt{2}$ is irrational.

Part 2: Supremum and Infimum

Exercise 03

Find the infimum and the supremum, whenever they exist, of the following sets

1. $A_1 = [-2, 0[$
2. $B_1 =]-\infty, 2[$
3. $B_2 = [-1, 0] \cup [3, 4]$
4. $A_2 = \left\{1 - \dfrac{(-1)^n}{n},\ n \in \mathbb{N}\right\}$
5. $A_3 = \left\{x \in \mathbb{R}^*,\ x < \dfrac{1}{x}\right\}$
6. $A_4 = \left\{x \in \mathbb{R},\ x + 2 \geq x^2\right\}$
7. $A_5 = \left\{\dfrac{1}{n} - \dfrac{1}{m},\ n, m \in \mathbb{N}\right\}$
8. $A_6 = \left\{x \in \mathbb{Q},\ x^2 \leq 2\right\}$
9. $A_7 = \left\{x \in \mathbb{R} \setminus \mathbb{Q},\ x^2 \leq 2\right\}$

Exercise 04

Given nonempty subsets A and B of $\mathbb{R}$ and $k \in \mathbb{R}$, we define the following subsets of $\mathbb{R}$:

$$\begin{array}{l} kA := \{k.a,\ a \in A\} \\ k + a := \{k + a,\ a \in A\} \\ A + B := \{a + b,\ a \in A, b \in B\} \end{array}$$

Assume that A and B are nonempty bounded subsets of $\mathbb{R}$. Prove (any two of) the following

1. If $k > 0$, then $\inf(kA) = k \inf(A)$, $\sup(kA) = k \sup(A)$.
2. If $k < 0$, then $\inf(kA) = k \sup(A)$, $\sup(kA) = k \inf(A)$.
3. $\sup(A + B) = \sup A + \sup B$, $\inf(A + B) = \inf A + \inf B$
4. $\sup(A \cup B) = \sup\{\sup A, \sup B\}$, $\inf(A \cup B) = \inf\{\inf A, \inf B\}$

Part 3: The Modulus (Absolute value)

Exercise 5

Let $a, b \in \mathbb{R}$

1. Prove that $|a + b| \leq |a| + |b|$ (Triangle Inequality)
2. Prove that $|a + b| \geq ||a| - |b||$ (Reverse Triangle Inequality)

Exercise 6

Solve the following equations

1. $|2x + 5| - 2 = 2$
2. $\frac{1}{4}|2x - 6| + 1 = 2$
3. $\left|6 - x^2\right| + 1 = 3$
4. $|x + 3| = x^2 - 4x - 3$
5. $\left|x^2 + 1\right| = 2x$

Exercise 7

Find which $x \in \mathbb{R}$ meet

1. $|x - 4| \leq 2$
2. $|x + 3| > 4$
3. $|x + 2| \leq -1$
4. $3|2x - 5| \geq -6$
5. $\left|x^2 - 1\right| > 2x$
6. $\left|x^2 - 3\right| < 2$

Part 4: Integer Part

Exercise 08

Find the integer part and fractional part of the following numbers

$$14, \quad 4.1, \quad \frac{35}{4}, \quad \frac{-22}{3}, \quad \sqrt{14}, \quad \pi, \quad \frac{\pi}{2}, \quad \frac{\pi}{3}$$

Exercise 09

Find the smallest positive $x \in \mathbb{R}$ such that $\left[x^2\right] - x\left[x\right] = 2019$