FIRST PART: CARTESIAN, POLAR AND CYLINDRICAL COORDINATE SYSTEMS

Exercise 01:

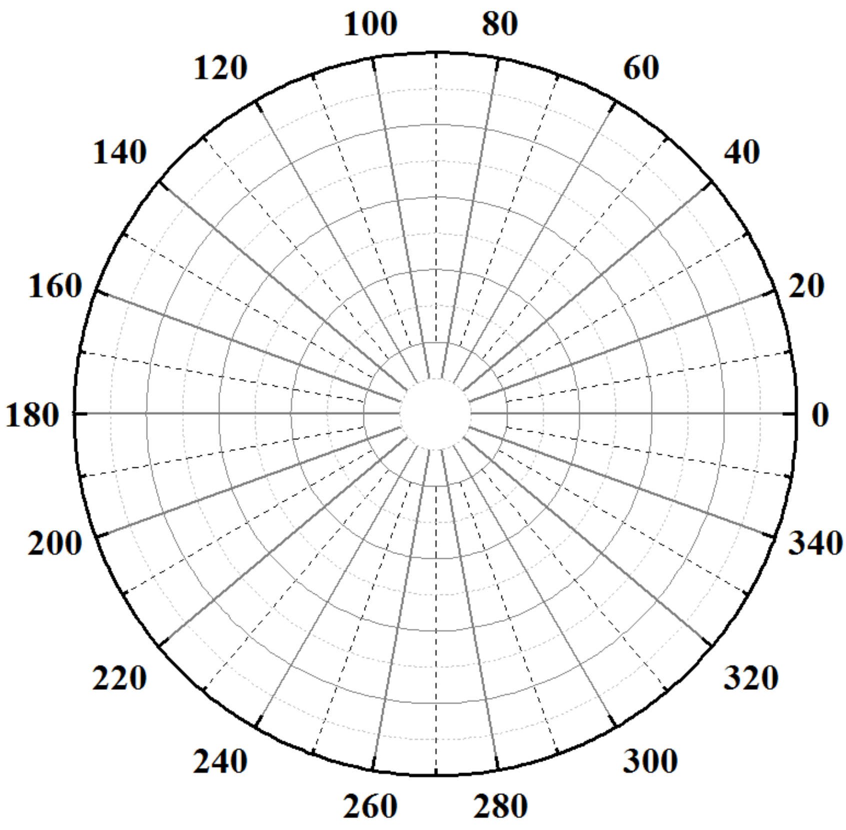
Using $\vec{\imath}$ and $\vec{\jmath}$ as the unit vectors for the Cartesian coordinate system, and $\vec{u}_r$ and $\vec{u}_\theta$ as the unit vectors for the polar coordinate system (where θ is time-dependent),

1) *Write the expressions for the unit vectors $\vec{u}_r$ and $\vec{u}_\theta$ in terms of $\vec{\imath}$ and $\vec{\jmath}$.*
2) *Calculate the derivatives of these unit vectors with respect to both time and θ.*
3) *Express the unit vectors $\vec{\imath}$ and $\vec{\jmath}$ in terms of $\vec{u}_r$ and $\vec{u}_\theta$.*
4) *Compute the derivatives of $\vec{\imath}$ and $\vec{\jmath}$ unit vectors with respect to both time and θ*

Exercise 02:

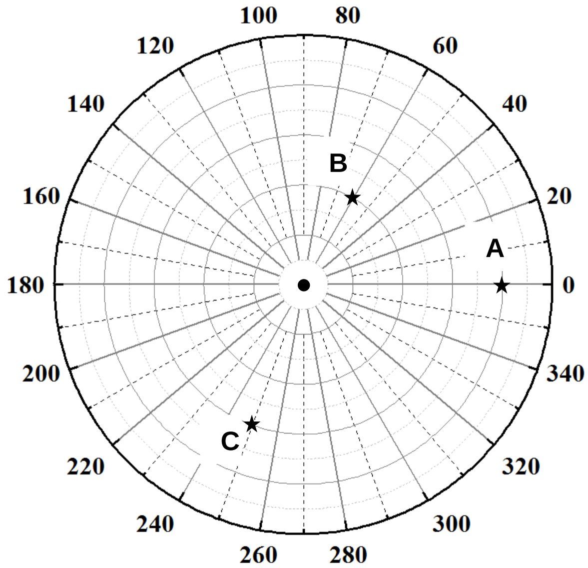
In the polar coordinate system with unit vectors $\vec{u}_r$ and $\vec{u}_\theta$, the positions of the moving object M at two different moments t_1 and t_2 are given as follows : M_1 (3, π/6) M_2 (2, 2π/3)

1) *Represent the positions of the moving object M in the polar coordinate system.*
2) *Provide the expressions for the position vector at t_1 and t_2 moments.*
3) *Determine the expression for the displacement vector from M_1 to M_2.*
4) *Convert the coordinates of the two positions from polar to Cartesian coordinates, and rewrite the previous expressions in Cartesian coordinates.*



Exercise 03:

- *Identify the coordinates of points A, B, and C presented in the following polar coordinate system.*
- *Represent D, E, and F points on the same polar coordinate system.*
  D(5 cm,150^0); E(1 cm,90^0); F(3.5 cm,320^0)
- *From the cylindrical coordinate system shown in the attached diagram, calculate the coordinates for points K, L, and M.*



Exercise 03:

Consider a point particle M described by the following Cartesian coordinates:

$x = R(1 + \cos(2\theta)),\ y = R\sin(2\theta),\ \theta = \omega t$

1- Find in Cartesian coordinates:

The equation of the trajectory and plot it.

The position, velocity, and acceleration vectors. Calculate the magnitudes of the velocity and acceleration.

2- Find in polar coordinates:

The equation of the trajectory $\rho = f(\theta)$ and plot on the trajectory curve the polar and intrinsic unit vectors.

The position, velocity, and acceleration vectors. Calculate the magnitudes of the velocity and acceleration.

Exercise 04:

The Cartesian coordinates of a material point M in terms of time (t) are given as follows:

$$\begin{cases} x(t) = 2t \\ y(t) = \sqrt{4(1-t^2)} \end{cases}$$

1) Write the path equation, the vector of its position, the vector of its instantaneous velocity and acceleration.

Exercise 05:

The coordinates of a material point (M) moving in the (xOy) plane are given in terms of time (t) as follows:

$$\begin{cases} x(t) = 2t^2 + 2t - 2 \\ y(t) = 3t + 2 \end{cases}$$

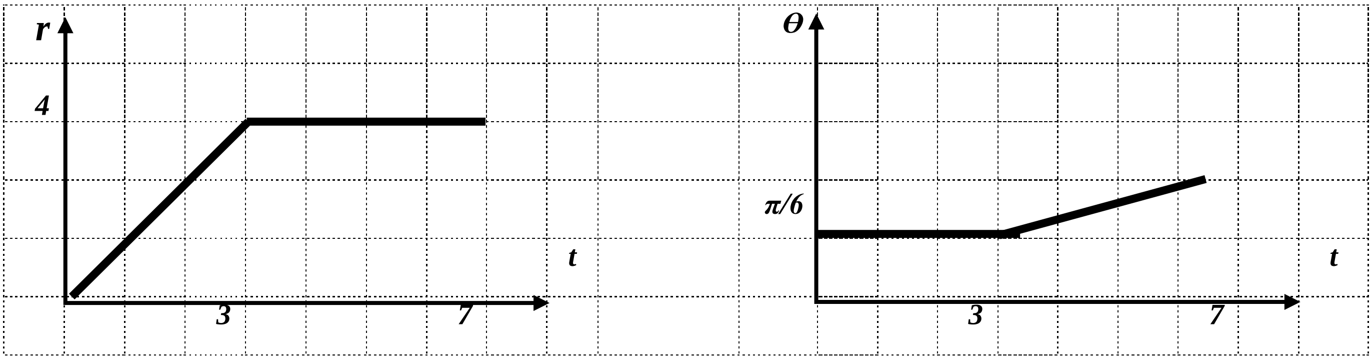
Write the expression of : a) the position vector. b) the path equation. c) the instantaneous velocity vector and its instantaneous acceleration vector

calculate each of them at the instant 2 s.

Exercise 06:

The motion of a particle (M) is defined by its polar coordinates r(t) and θ(t) given by the graphs

1- *write the equations for each phase of the motion*
2- *Express the position vector for each phase in the polar coordinate system and compute the velocity and acceleration components within this coordinate system, thereby deriving their magnitudes*



Exercise 07:

The motion of a material point M in the plane is described using the following polar coordinates:

$$\begin{cases} r = a\, e^{\left(-\frac{t}{\tau}\right)} \\ \theta(t) = \omega t \end{cases} \quad \text{where } a, \tau \text{ and } \omega \text{ are positive constants}$$

a) Calculate the velocity and acceleration of point M in the polar coordinates

b) Calculate the velocity and acceleration of point M in the Cartesian coordinates