SECOND PART: particle kinematics

Exercise 01:

Consider a point particle M described by the following Cartesian coordinates:

$x = R(1 + \cos(2\theta))$, $y = R\sin(2\theta)$, $\theta = \omega t$

1- Find in Cartesian coordinates:

The equation of the trajectory and plot it.

The position, velocity, and acceleration vectors. Calculate the magnitudes of the velocity and acceleration.

2- Find in polar coordinates:

The equation of the trajectory $\rho = f(\theta)$ and plot on the trajectory curve the polar and intrinsic unit vectors.

The position, velocity, and acceleration vectors. Calculate the magnitudes of the velocity and acceleration."

4.3.1-Corrigé

1-L'équation de la trajectoire :

On a :

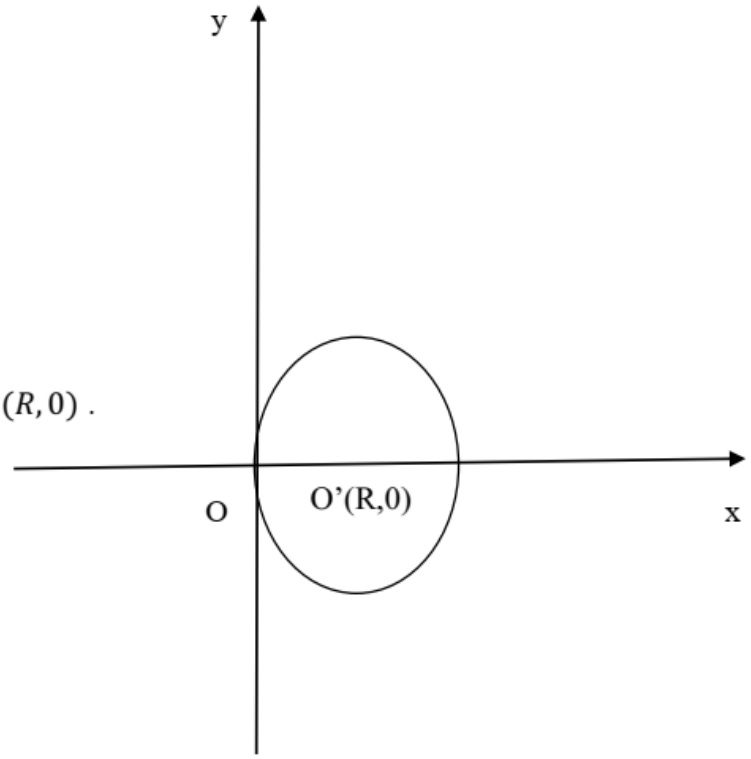
$$\begin{cases} x = R(1 + cos2\theta) \\ y = Rsin2\theta \end{cases}$$

1-l'équation de la trajectoire

$$\begin{cases} x - R = Rcos2\theta \\ y = Rsin2\theta \end{cases}$$

$$\Rightarrow (x - R)^2 + y^2 = R^2$$

C'est un cercle de rayon R et de centre $(R, 0)$.



Le vecteur position :

$\vec{r} = x\vec{\imath} + y\vec{\jmath}$

$\vec{r} = R(1 + cos2\theta)\vec{\imath} + Rsin2\theta\vec{\jmath}$

Le vecteur vitesse :

$$\vec{v}(t) = \frac{d\vec{r}(t)}{dt}$$

$$\vec{v}(t) = 2R\omega(-sin2\theta\vec{i} + cos2\theta\vec{j}+)$$

$$\|\vec{v}\| = 2R\omega$$

Le vecteur accélération :

$$\vec{\gamma}(t) = \frac{d\vec{v}(t)}{dt}$$

$$\vec{\gamma}(t) = -4R\omega^2(cos2\theta\vec{i} + sin2\theta\vec{j}+)$$

$$\|\vec{\gamma}\| = 4R\omega^2$$

2-en coordonnées polaires :

-l'équation de la trajectoire $\rho = f(\theta)$:

$$\rho = \sqrt{x^2 + y^2}$$

$$\Rightarrow \rho = R\sqrt{(1 + cos2\theta)^2 + (sin2\theta)^2}$$

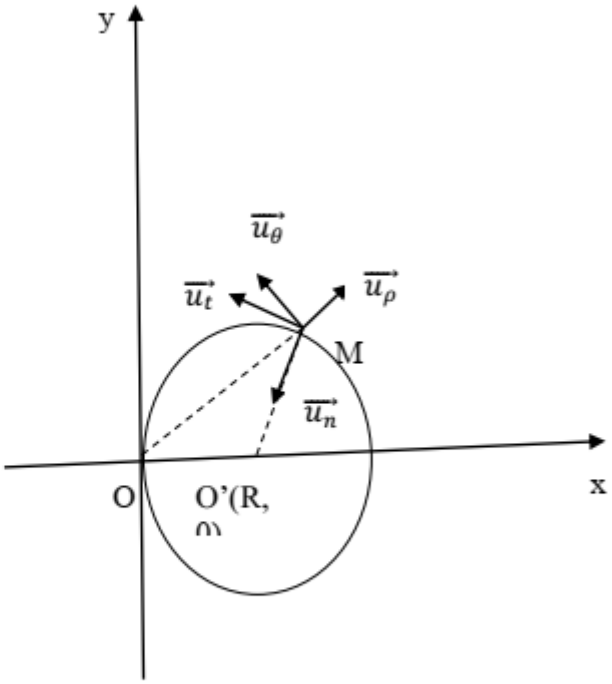
$$\Rightarrow \rho = R\sqrt{2(1 + cos2\theta)}$$

On a : $cos2\theta = cos^2\theta - sin^2\theta$

$$\Rightarrow cos2\theta = 2cos^2\theta - 1$$

$$\Rightarrow \rho = R\sqrt{4cos^2\theta}$$

$$\Rightarrow \rho = 2R|cos\theta|$$



Le vecteur position :

$$\vec{r} = \rho\overrightarrow{u_\rho}$$

$$\Rightarrow \vec{r} = 2Rcos\theta\overrightarrow{u_\rho}$$

Le vecteur vitesse :

$$\vec{v}(t) = \frac{d\vec{r}(t)}{dt}$$

$$\Rightarrow \vec{v}(t) = 2R\omega\left[-sin\theta\overrightarrow{u_\rho} + cos\theta\overrightarrow{u_\theta}\right]$$

$$\|\vec{v}\| = 2R\omega$$

Le vecteur accélération :

$$\vec{\gamma}(t) = \frac{d\vec{v}(t)}{dt}$$

$$\Rightarrow \vec{\gamma}(t) = -4R\omega^2\left[cos\theta\overrightarrow{u_\rho} + sin\theta\overrightarrow{u_\theta}\right]$$

$$\|\vec{\gamma}\| = 4R\omega^2$$