جامعة محمد بوضياف - المسيلة
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Artificial Learning Models

Lecture 5 : Support Vector Machine

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Agenda

Introduction

- Supervied learning model
  - Used for classification
  - But also for regression

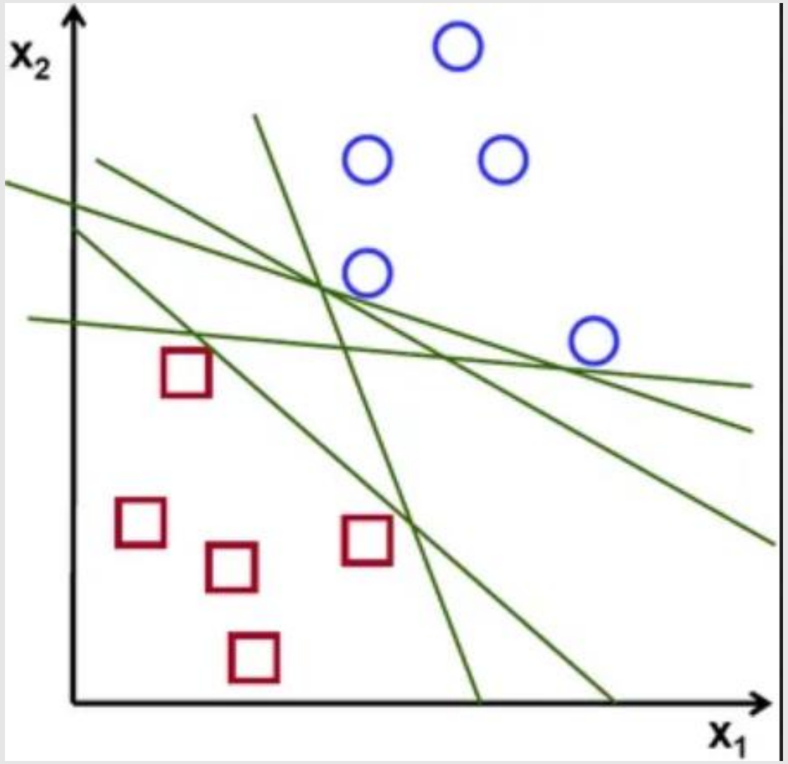
- Developed at **AT&T Bell Laboratories** by **Vladimir Vapnik** with colleagues (Boser et al., 1992, Guyon et al., 1993, Cortes and Vapnik, 1995, Vapnik et al., 1997)

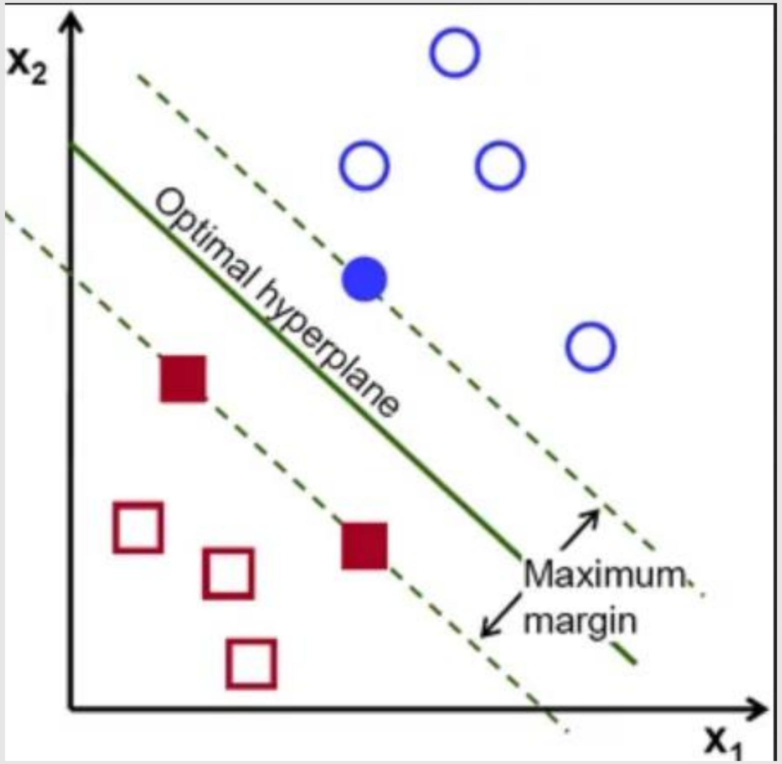
- Is not a probabilistic classifier

What is SVM ?

- During training:
  - SVM constructs a hyperplane or set of hyperplanes in a high or infinite-dimensional space
  - SVM maps training examples to points in space so as to maximize the width of the gap between the classes.
- During test or prediction:
  - New examples are then mapped into that same space and predicted to belong to a category based on which side of the gap they fall.

What is SVM ?





How does SVM work ?

- It is defined in terms of the support vectors only,
- Don't have to worry about other observations

↓

- The margin is made using the points which are closest to the hyperplane (support vectors)
- Remember
  - In logistic regression, the classifier is defined over all the points.

What is SVM ?
SVM vs Logistic regression

- Don’t get confused
  - Both the algorithms try to find the best hyperplane

- The difference :
  - logistic regression is a probabilistic approach
  - SVM is based on statistical approaches.

- SVM or Logistic regression
  - SVM works best when the dataset is small and complex.
  - Use logistic regression first and see how does it perform, if it fails to give a good accuracy you can go for SVM without any kernel
  - Logistic regression and SVM without any kernel have similar performance

Types of SVM algorithms

- Linear SVM
  - The data is perfectly linearly separable
  - Performs linear classification
- Non-linear SVM
  - Data is not linearly separable
  - Perform a non-linear classification using some advanced techniques like kernel tricks to classify them.
  - kernel tricks ??!!

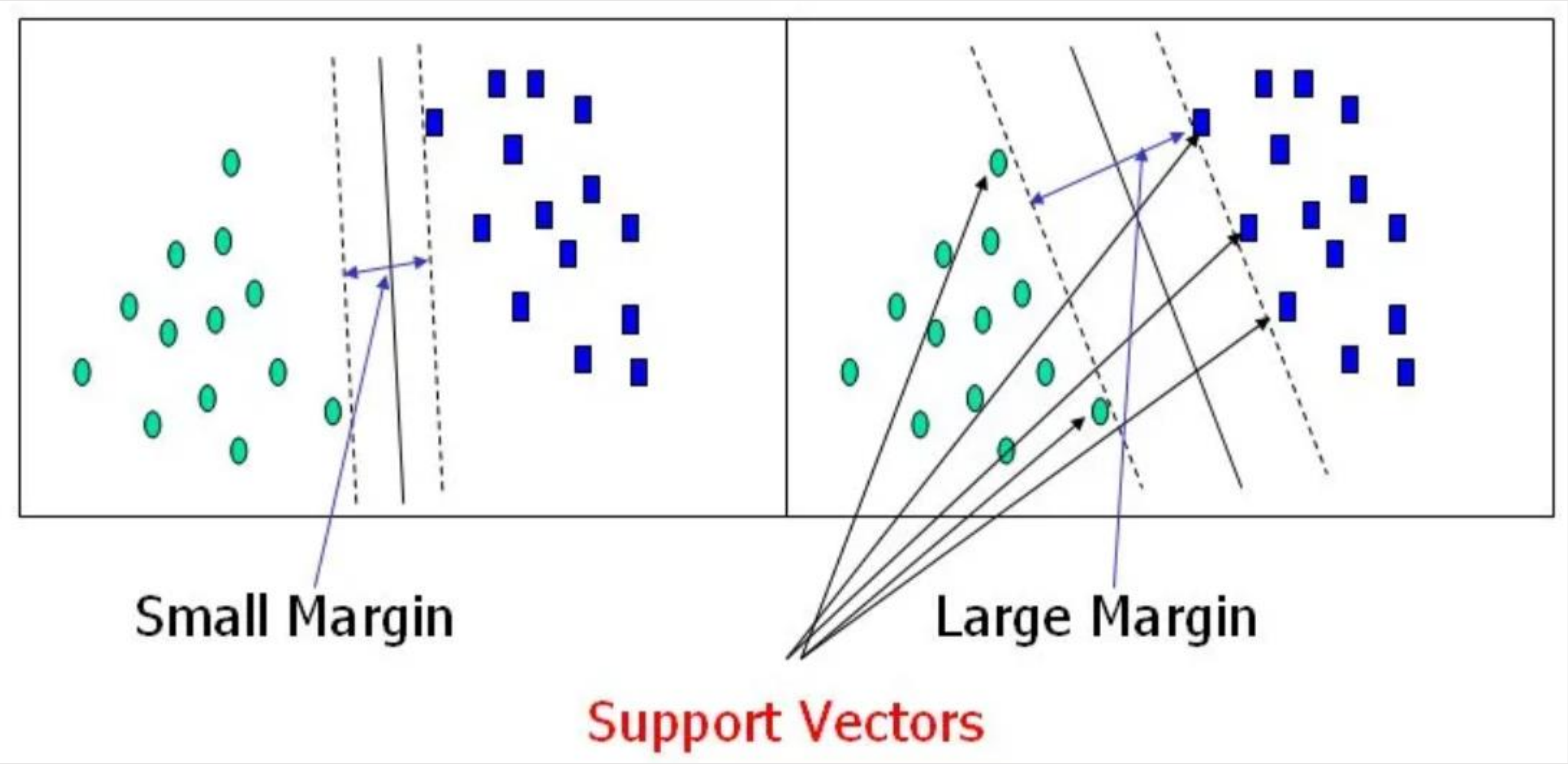
SVM Terminology

- **Hyperplane**
  - Hyperplane is the decision boundary that is used to separate the data points of different classes in a feature space.
- **Support vectors**
  - These are the points that are closest to the hyperplane.
  - A separating line will be defined with the help of these data points.
- **Margin**
  - It is the distance between the hyperplane and the support vectors.
  - Large margin is considered a good margin.
  - Two types of margins **hard margin** and **soft margin.**

SVM Terminology

- Kernel
- Hard Margin
- Soft Margin
- C
- Hinge Loss

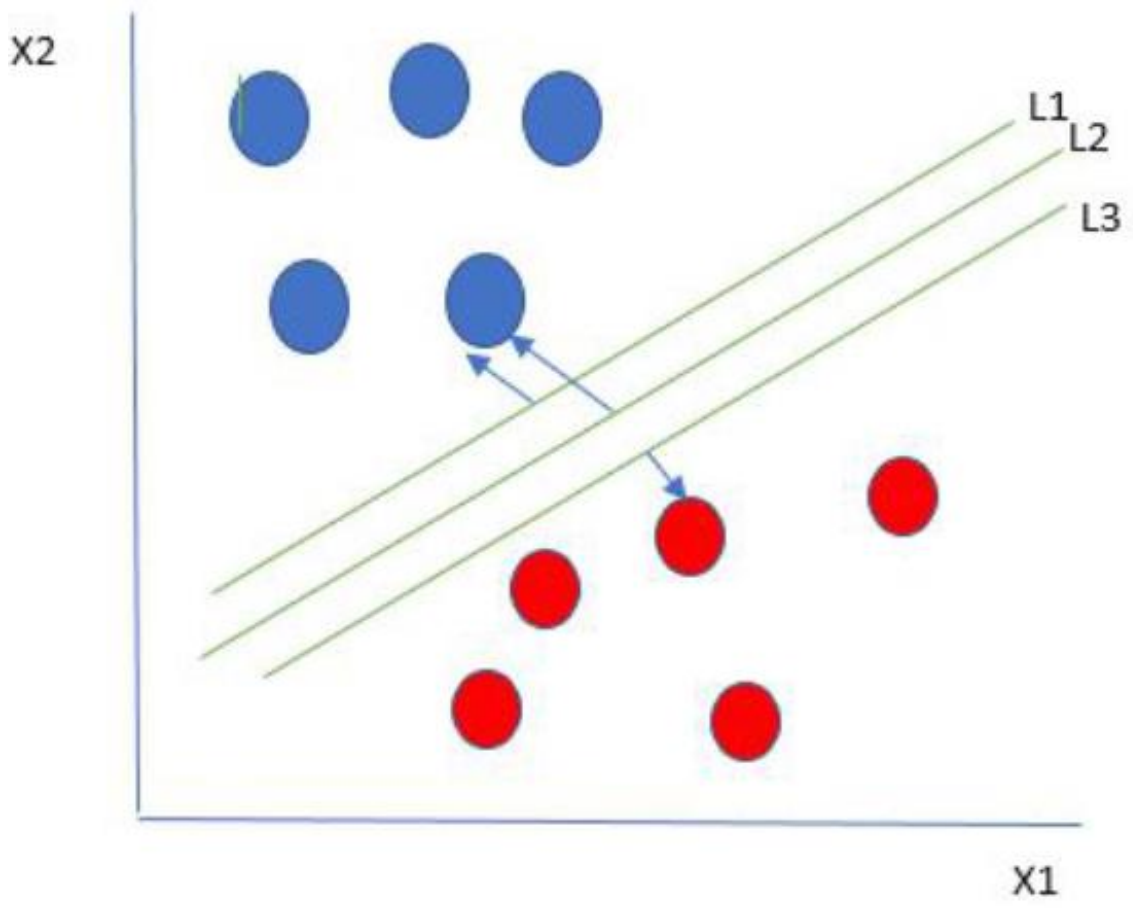
Support vectors and Margin



Hard margin

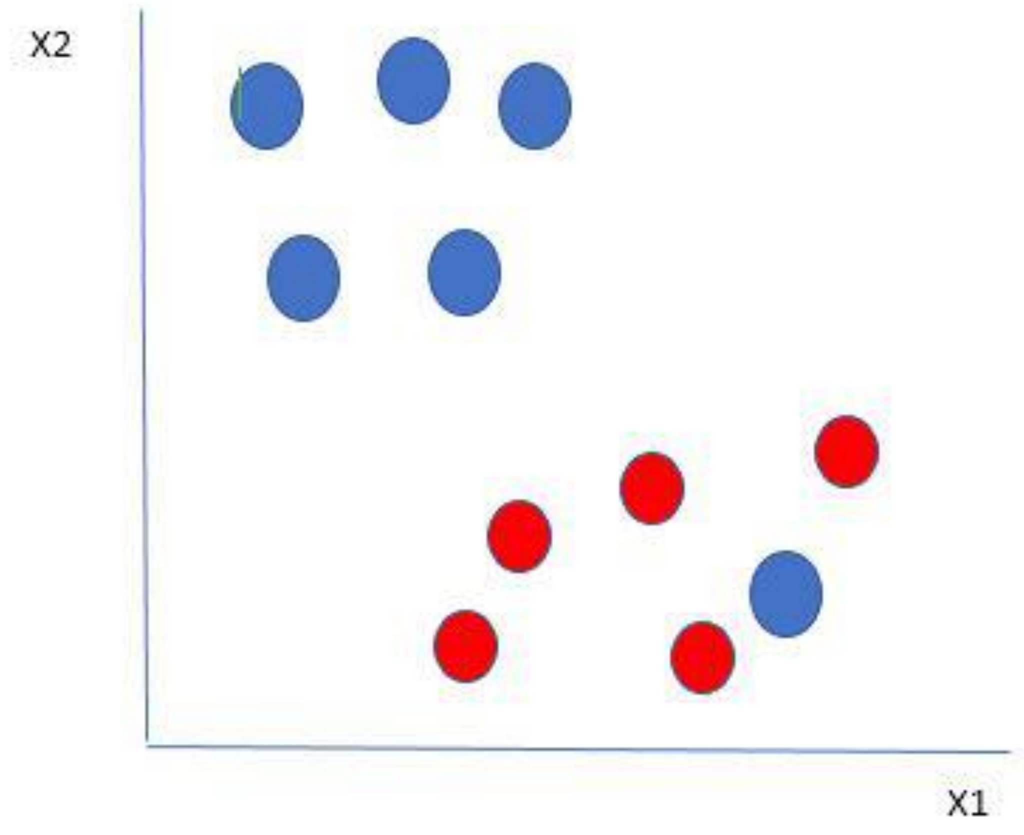
- Hyperplane with :
  - Perfect separation
  - Margin is maximized

Hard Margin



Soft margin

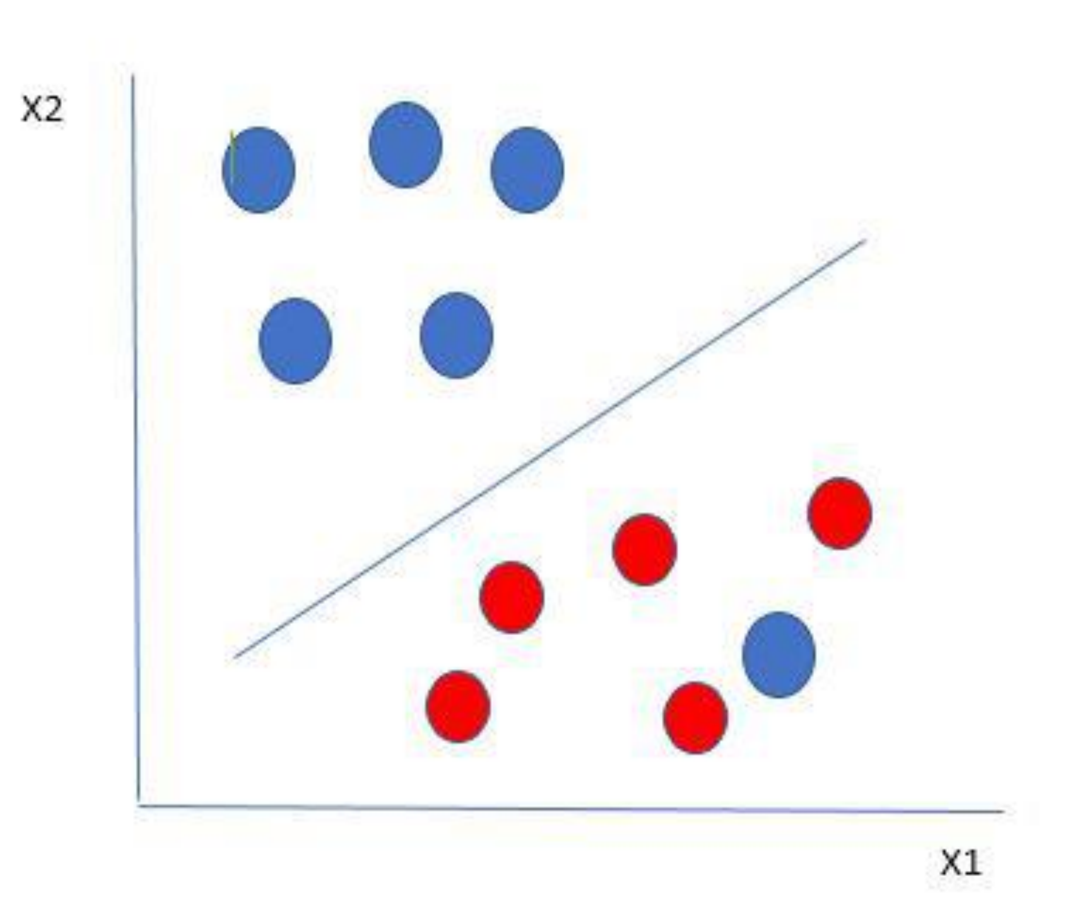
How does SVM classify these data?



Soft margin

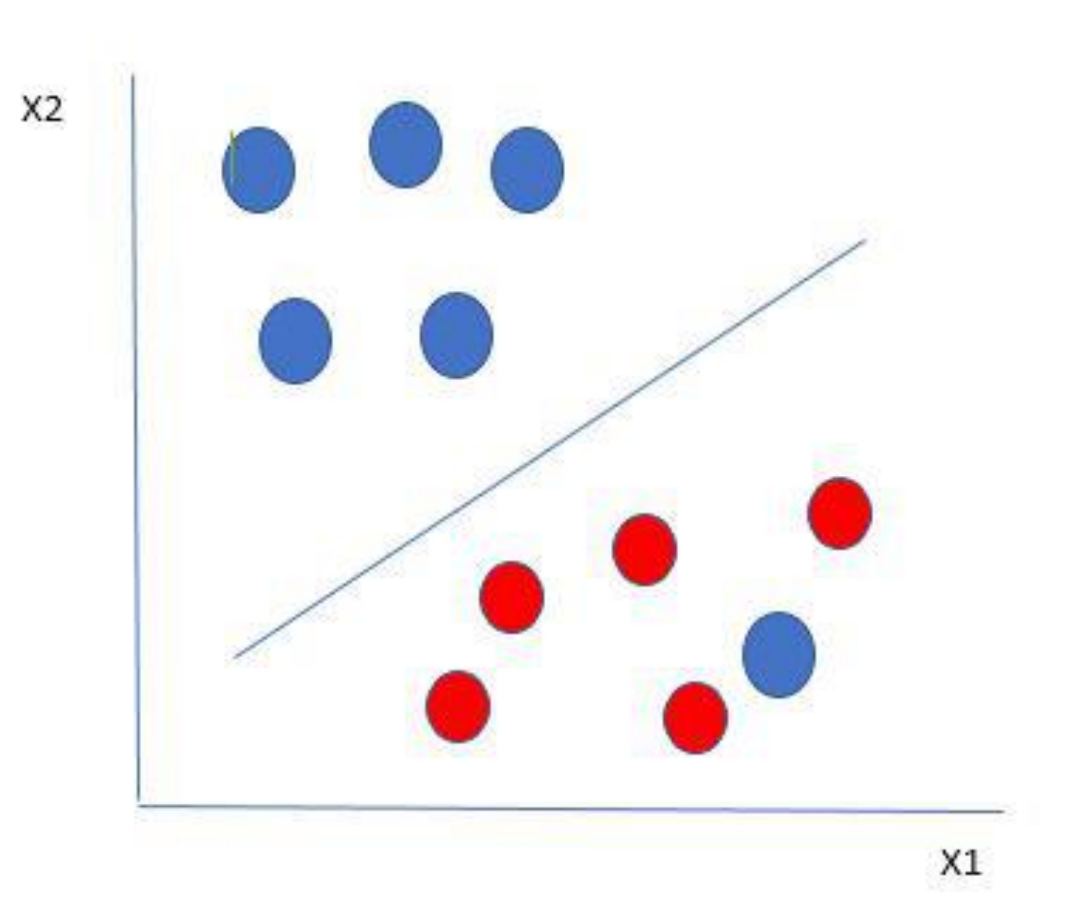
- The blue ball in the boundary of red ones is an outlier of blue balls.
- SVM ignore the outlier and finds the best hyperplane that maximizes the margin.
- SVM is robust to outliers.

How ??



Soft margin

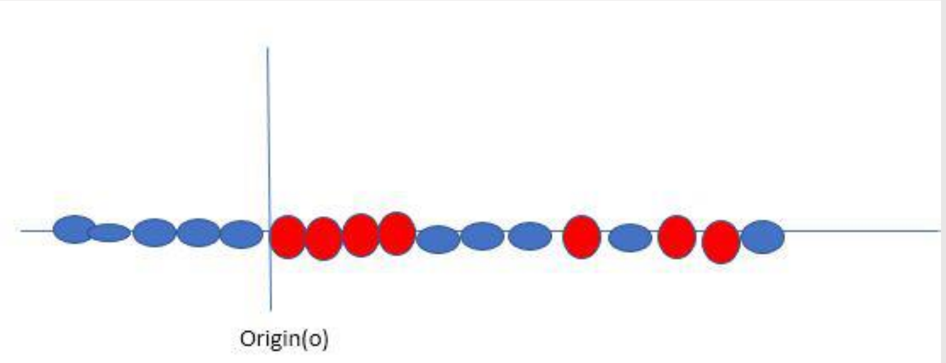
- Finds the maximum margin
- Adds a penalty each time a point crosses the margin.



The kernel trick

- Till now, we were talking about linearly separable.

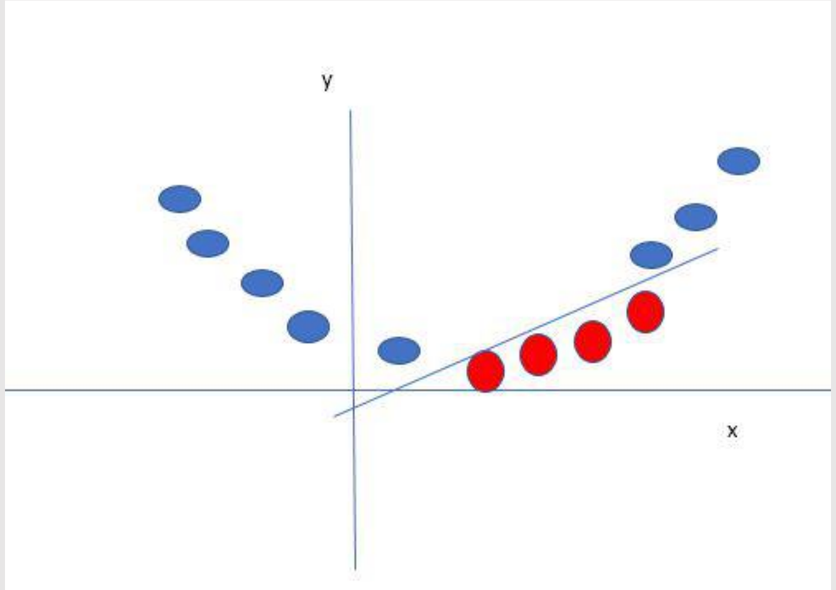
What to do if data are not linearly separable?



Create a new variable using a **kernel**.

The kernel trick

- create a new variable y_i as a function of distance from origin o.
- A non-linear function that creates a new variable is referred to as a kernel.



The kernel trick

- Representing data in higher dimensional spaces to find hyperplanes that might not be apparent in lower dimensions

Kernel functions

- **Polynomial Kernel**
- **Sigmoid Kernel**
- **RBF Kernel**
- **Bessel function kernel**
- **Anova Kernel**

$$\text{Linear}: \; K(w,b) = w^T x + b$$

$$\text{Polynomial}: \; K(w,x) = (\gamma w^T x + b)^N$$

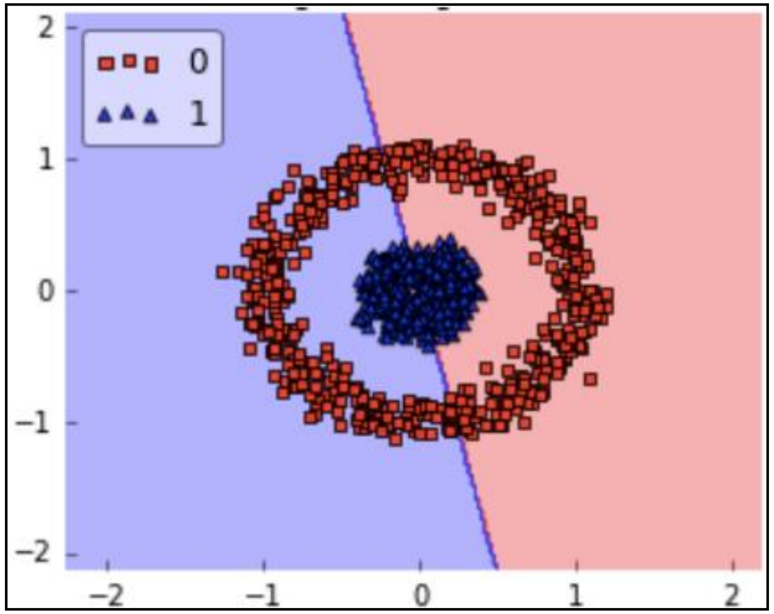
$$\text{Gaussian RBF}: \; K(w,x) = \exp(-\gamma||x_i - x_j||^n$$

$$\text{Sigmoid}: K(x_i,x_j) = \tanh(\alpha x_i^T x_j + b)$$

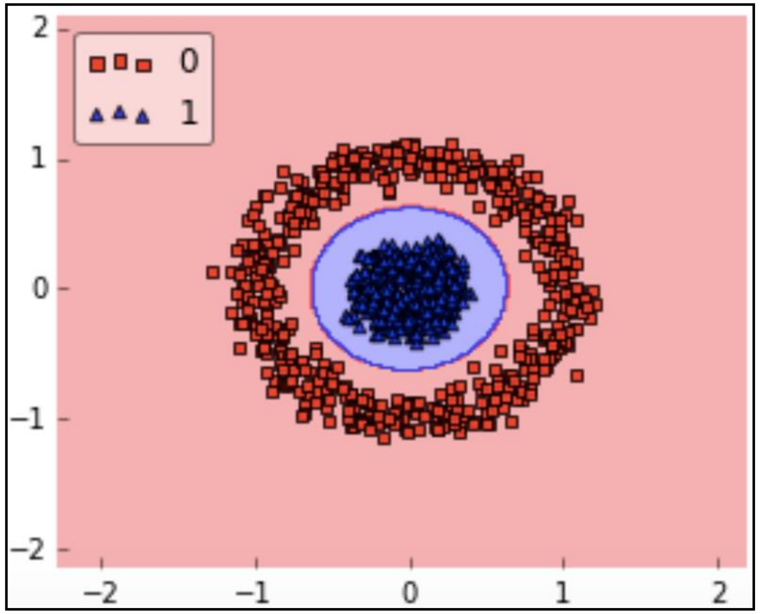
How to choose the right kernel ?

- The performance of the model depends on chosen kernel function
- Choosing a kernel totally depends on what kind of dataset are you working on
  - If it is linearly separable then you must opt for linear kernel function
  - It is recommended to start with a hypothesis that the data is linearly separable
- Usually, we use SVM with RBF and linear kernel function

How to choose the right kernel ?
Example



for this kind of dataset, we can use RBF without even a second thought because it makes decision boundary like this:



Mathematical intuition of SVM

- Consider a binary classification problem with two classes, labeled as +1 and -1.
- The equation for the linear hyperplane can be written as:

$$w^T x + b = 0$$

- The vector W represents the normal vector to the hyperplane. i.e the direction perpendicular to the hyperplane.
- The distance between a data point x_i and the decision boundary can be calculated as:

$$d_i = \frac{w^T x_i + b}{||w||}$$

- where ||w|| represents the Euclidean norm of the weight vector w.

Mathematical intuition of SVM
Optimization

- **For Hard margin linear SVM classifier:**

$$\underset{w,b}{\text{minimize}}\frac{1}{2}w^Tw = \underset{W,b}{\text{minimize}}\frac{1}{2}\left\|w\right\|^2$$
$$\text{subject to } y_i(w^Tx_i + b) \geq 1 \; for \; i = 1, 2, 3, \cdots, m$$

- **For Soft margin linear SVM classifier:**

$$\underset{w,b}{\text{minimize}} \; \frac{1}{2}w^Tw + C\sum_{i=1}^{m}\zeta_i$$
$$\text{subject to } y_i(w^Tx_i + b) \geq 1 - \zeta_i \;\; and \; \zeta_i \geq 0 \;\; for \; i = 1, 2, 3, \cdots, m$$

Advantages and Disadvantages of SVM

- **Advantages of SVM**
  - SVM works better when the data is Linear
  - It is more effective in high dimensions
  - SVMs are less prone to overfitting than other algorithms such as neural networks.
  - SVM is not sensitive to outliers
  - With the help of the kernel trick, we can solve any complex problem
  - Can help us with Image classification

- **Disadvantages of SVM**
  - Choosing a good kernel is not easy
  - It doesn’t show good results on a big dataset
  - The SVM hyperparameters are Cost -C and gamma. It is not that easy to fine-tune these hyper-parameters. It is hard to visualize their impact

SVM with sklearn

- **Library** : *from sklearn import svm*
- **define the model** : model = svm.SVC(kernel='linear', C=1.0)
- **train the model :** model.fit(training_X, training_y)
- **make non-linear algorithm :** for model : nonlinear_model = svm.SVC(kernel='rbf', C=1.0)