



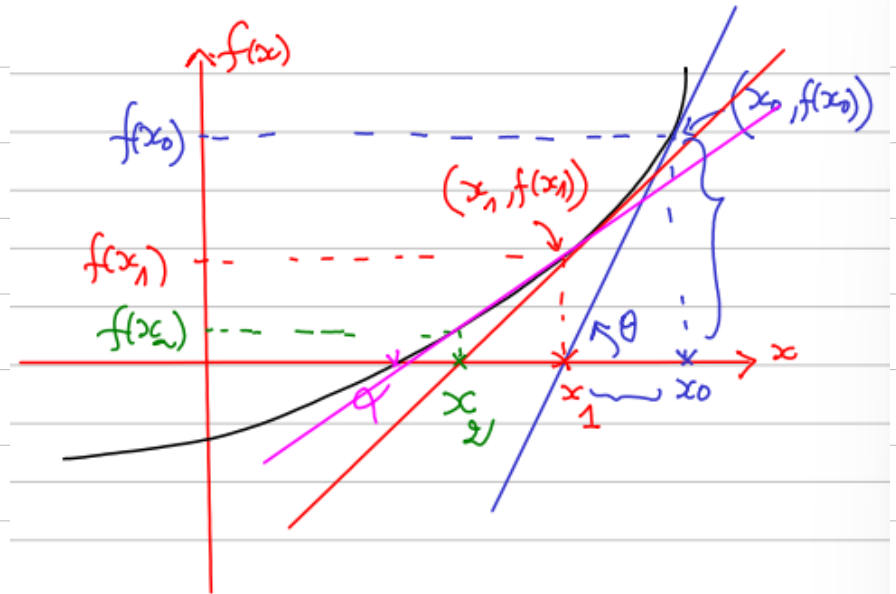
Lab 4 : Newton method

The objectives of this lesson:

- Understand the bisection method.
- Write an algorithm/flowchart for this method.
- Write a Matlab script for this method.
- Be able to apply this method to solve a non-linear equation $f(x) = 0$.
- Be able to use various stopping criteria to exit the algorithm of the bisection method.

Basic ideas and fundamental concepts:

$f(x) = 0$
 search α
 s.t. $f(\alpha) = 0$



$$\operatorname{tg} \theta = \frac{f(x_0) - 0}{x_0 - x_1} = f'(x_0)$$

$$\Rightarrow \boxed{x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}} \quad \text{--- (1)}$$

Following the same steps, one can write

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

⋮

An algorithm for the method

We can generalize Eq (1) to get

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)} \quad (1)$$

This formula concerns Newton's method.

NB. One can also derive the same formula as in Eq (1), by performing a Taylor's series expansion near to x_0 . It means

$$f(x) = f(x_0) + \frac{f'(x_0)}{1!} (x-x_0) + R \rightarrow 0$$

Stopping Criteria:

Criteria # 1. The length of the interval $[a, b]$ is $\ll \epsilon \rightarrow |a-b| < \epsilon$

\rightarrow stop if $\text{abs}(a-b) < \text{epsilon} \rightarrow \text{while}(\text{abs}(a-b) > \text{eps}) \downarrow$

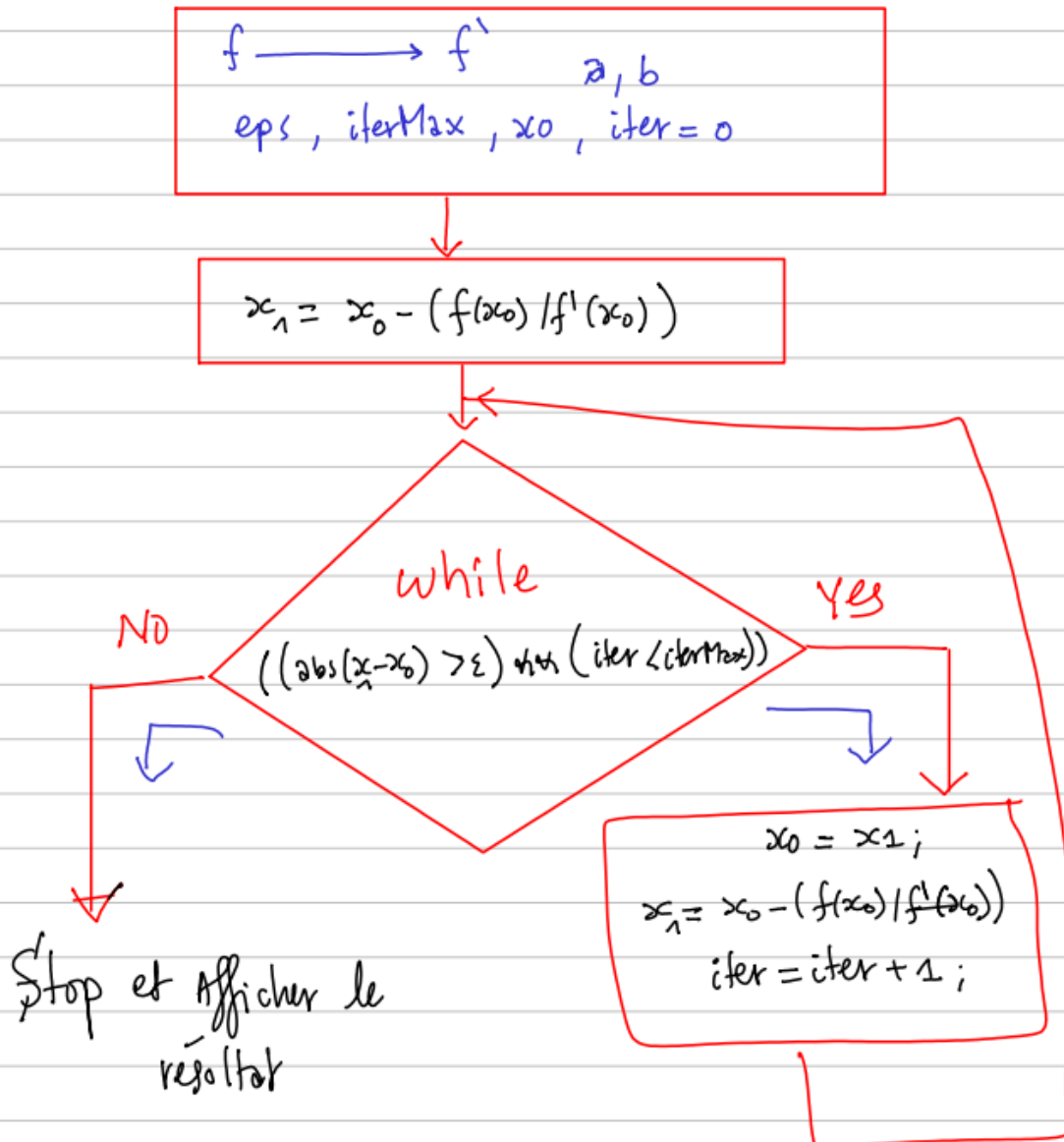
Criteria # 2. The number of iterations (MaxIter)

\rightarrow for iter = 1: MaxIter \downarrow

Criteria # 3. Both criteria combined \rightarrow

$\text{while}((\text{abs}(a-b) > \text{eps}) \text{ \&\& } (\text{iter} < \text{MaxIter})) \downarrow$

A flowchart for the method



Matlab script(s) for the method:

```
1
2 f = inline('x.^3 + 4*x.^2 - 10');
3 fp=inline('3*x.^2 + 8.*x');
4 a=1;
5 b=2;
6 x0 =1.5;
7 x1 = x0-(f(x0)/fp(x0));
8 eps=1.0e-6;
9 iter=0;
10 if ((f(a)*f(b))< 0)
11 while (abs(x1-x0) > eps)
12 x0=x1;
13 x1 = x0-(f(x0)/fp(x0));
14 iter=iter+1;
15 fprintf('For iteration =%d \t , the solution is x0=%f\n',iter,x0)
16 end
17 fprintf('The final solution is x0 = %f \n',x0) ;
18 else
19 disp('There is no solution in [a,b]')
20 end
```

```
1
2
3 f = inline('x.^3 + 4*x.^2 - 10');
4 %-----
5 fp=inline('3*x.^2 + 8.*x');
6 a=1;
7 b=2;
8 x0 =1.5;
9 x1 = x0-(f(x0)/fp(x0));
10
11 iterMax=10;
12 if ((f(a)*f(b))< 0)
13 for iter=1:iterMax
14 x0=x1;
15 x1 = x0-(f(x0)/fp(x0));
16
17 fprintf('For iteration =%d \t , the solution is x0=%f\n',iter,x0)
18 end
19 fprintf('The final solution is x0 = %f \n',x0) ;
20 else
21 disp('There is no solution in [a,b]')
22 end
```