## Final Exam

## Data Mining \& Information Retrieval

## Exercice 1 ( 5 pts) :

The similarity of two attributes with nominal values is evaluated using a generalization of binary variables. We propose to study two characteristics of plants: Leaf Color (yellow, green, red) and Leaf Size (small, large). Let's consider the data of plants as follows:

|  | Leaf Color | Leaf Size |
| :---: | :---: | :---: |
| $\mathbf{A}$ | red | small |
| $\mathbf{B}$ | yellow | large |
| $\mathbf{C}$ | green | small |
| $\mathbf{D}$ | yellow | large |

Convert the plant data into binary values and calculate the distances $d(A, B), d(B, C), d(A, C)$ and $d(B, D)$ using Jaccard index. Comment these distances.

## Exercice 2 (7 pts)

Given the dataset $D$ in the following table,
1- Consider the following measurement which calculates the distance between two points $a$ and $b$ in $D$ :

$$
d(a, b)=\max _{i}\left|x_{i}-y_{i}\right|
$$

Is this distance a measurement of similarity or dissimilarity?
2- Using the complete link as a measure of distance between 2 clusters, perform a bottom-up hierarchical clustering on $D$ and plot the corresponding dendrogram.

| Points | $\mathbf{x}_{\mathbf{i}}$ | $\mathbf{y}_{\mathbf{i}}$ |
| :---: | :---: | :---: |
| A | 1 | 1 |
| B | 1.5 | 0.5 |
| C | 0.8 | 1.2 |
| D | -1 | -0.8 |
| E | -0.2 | 0.5 |
| F | 0.2 | -1 |

## Exercice 3 ( 8 pts)

Consider the following data corresponding to 7 observations of three variables $\mathrm{X}, \mathrm{Y}$ and Z . The target class is the last column.

Apply the naive Bayesian classifier algorithm to this binary classification problem for predicting each of the following 3 new observations:

|  | $\mathbf{X}$ | $\mathbf{Y}$ | $\mathbf{Z}$ | Class |
| :---: | :---: | :---: | :---: | :---: |
| $\mathbf{x 8}$ | A | $\alpha$ | 2 | $?$ |
| $\mathbf{x 9}$ | C | $\beta$ | 1 | $?$ |
| $\mathbf{x 1 0}$ | B | $\beta$ | 1 | $?$ |

N.B. Do not forget to use Laplacian Correction if necessary.

|  | $\mathbf{X}$ | $\mathbf{Y}$ | $\mathbf{Z}$ | Class |
| :---: | :---: | :---: | :---: | :---: |
| $\mathbf{x} 1$ | A | $\alpha$ | 1 | C 1 |
| $\mathbf{x} 2$ | A | $\beta$ | 1 | C 1 |
| $\mathbf{x 3}$ | A | $\alpha$ | 1 | C 1 |
| $\mathbf{x 4}$ | B | $\alpha$ | 3 | C 1 |
| $\mathbf{x 5}$ | B | $\alpha$ | 1 | C 2 |
| $\mathbf{x 6}$ | C | $\beta$ | 2 | C 2 |
| $\mathbf{x} 7$ | C | $\beta$ | 2 | C 2 |

# Solution of Final Exam 

Data Mining \& Information Retrieval

## Exercice 1 (5 pts)

|  | LeafColorYellow | LeafColorGreen | LeafColorRed | LeafSizeSmall | LeafSizeLarge |
| :---: | :---: | :---: | :---: | :---: | :---: |
| A | 0 | 0 | 1 | 1 | 0 |
| B | 1 | 0 | 0 | 0 | 1 |
| C | 0 | 1 | 0 | 1 | 0 |
| D | 1 | 0 | 0 | 0 | 1 |


|  | B |  |  |  | C |  |  |  | C |  |  |  | D |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A |  | 1 | 0 | A |  | 1 | 0 | B |  | 1 | 0 | B |  | 1 | 0 |
|  | 1 | 0 | 2 |  | 1 | 1 | 1 |  | 1 | 0 | 2 |  | 1 | 2 | 0 |
|  | 0 | 2 | 1 |  | 0 | 1 | 2 |  | 0 | 2 | 1 |  | 0 | 0 | 3 |

$\mathrm{d}(\mathrm{A}, \mathrm{B})=(2+2) /(0+2+2)=1 \quad \mathrm{~d}(\mathrm{~A}, \mathrm{C})=(1+1) /(1+1+1)=2 / 3 \quad \mathrm{~d}(\mathrm{~B}, \mathrm{C})=(2+2) /(0+2+2)=1 \quad \mathrm{~d}(\mathrm{~B}, \mathrm{D})=(0+0) /(2+0+0)=0$
No similarity: A and B, B and C (distance=1)
Partial similarity; A and C (in Leaf Size property!) (distance=0.67)
Complete similarity: B and D (distance=0)
Jaccard Index is a measure of dissimilarity.

## Exercice 2 (7 pts)

1- Consider the following measurement which calculates the distance between two points $a$ and $b$ in $D$ :

$$
\begin{aligned}
& d(a, b)=\max _{i}\left|x_{i}-y_{i}\right| \\
& d(A, B)=\max _{1,2}\left(\left|x_{1}-y_{1}\right|,\left|x_{2}-y_{2}\right|\right)=\max (|1-1|,|1.5-0.5|)=1
\end{aligned}
$$ Since $d(a, b)$ take the maximum of absolute values, it may be a measurement of dissimilarity.

2- Bottom-up hierarchical clustering of $D$ and corresponding dendrogram.

| Points | $\mathbf{x}_{\mathbf{i}}$ | $\mathbf{y}_{\mathbf{i}}$ |
| :---: | :---: | :---: |
| A | 1 | 1 |
| B | 1.5 | 0.5 |
| C | 0.8 | 1.2 |
| D | -1 | -0.8 |
| E | -0.2 | 0.5 |
| F | 0.2 | -1 |


|  | A | B | C | D | E | F |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A | 0 | 1 | 0.4 | $\mathbf{0 . 2}$ | 0.7 | 1.2 |
| B |  | 0 | 1 | 1 | 1 | 1.2 |
| C |  |  | 0 | 0.4 | 0.7 | 1.2 |
| D |  |  |  | 0 | 0.7 | 1.2 |
| E |  |  |  |  | 0 | 1.2 |
| F |  |  |  |  |  | 0 |


|  | AD | $\mathbf{B}$ | $\mathbf{C}$ | $\mathbf{E}$ | $\mathbf{F}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{A D}$ | 0 | 1 | $\mathbf{0 . 4}$ | 0.7 | 1.2 |
| $\mathbf{B}$ |  | 0 | 1 | 1 | 1.2 |
| $\mathbf{C}$ |  |  | 0 | 0.7 | 1.2 |
| $\mathbf{E}$ |  |  |  | 0 | 1.2 |
| $\mathbf{F}$ |  |  |  |  | 0 |


|  | ADC | B | E | F |
| :---: | :---: | :---: | :---: | :---: |
| ADC | 0 | 1 | $\mathbf{0 . 7}$ | 1.2 |
| B |  | 0 | 1 | 1.2 |
| E |  |  | 0 | 1.2 |
| F |  |  |  | 0 |


|  | ADCE | B | F |
| :---: | :---: | :---: | :---: |
| ADCE | 0 | $\mathbf{1}$ | 1.2 |
| B |  | 0 | 1.2 |
| F |  |  | 0 |


|  | ADCEB | F |
| :---: | :---: | :---: |
| ADCEB | 0 | 1.2 |
| F |  | 0 |



## Exercice 3 ( 8 pts )

Consider the following data corresponding to 7 observations of three variables $\mathrm{X}, \mathrm{Y}$ and Z . The target class is the last column.

Apply the naive Bayesian classifier algorithm to this binary classification problem for predicting each of the following 3 new observations:

|  | $\mathbf{X}$ | $\mathbf{Y}$ | $\mathbf{Z}$ | Class |
| :---: | :---: | :---: | :---: | :---: |
| $\mathbf{x 8}$ | A | $\alpha$ | 2 | $?$ |
| $\mathbf{x 9}$ | C | $\beta$ | 1 | $?$ |
| $\mathbf{x 1 0}$ | B | $\beta$ | 1 | $?$ |


|  | $\mathbf{X}$ | $\mathbf{Y}$ | $\mathbf{Z}$ | Class |
| :---: | :---: | :---: | :---: | :---: |
| $\mathbf{x 1}$ | A | $\alpha$ | 1 | C 1 |
| $\mathbf{x} 2$ | A | $\beta$ | 1 | C 1 |
| $\mathbf{x 3}$ | A | $\alpha$ | 1 | C 1 |
| $\mathbf{x 4}$ | B | $\alpha$ | 3 | C 1 |
| $\mathbf{x 5}$ | B | $\alpha$ | 1 | C 2 |
| $\mathbf{x 6}$ | C | $\beta$ | 2 | C 2 |
| $\mathbf{x} 7$ | C | $\beta$ | 2 | C 2 |

$\mathrm{P}\left(\mathrm{C}_{1}\right):=4 / 7=0.571 \quad \mathrm{P}\left(\mathrm{C}_{2}\right):=3 / 7=0.428$

1) Classification of $\mathbf{x 8}(\mathbf{X}=\mathrm{A}, Y=\alpha, \mathbf{Z}=2)$

Compute $\mathrm{P}\left(\mathrm{x} 8 \mid \mathrm{C}_{\mathrm{i}}\right)$ for each class
$\mathrm{P}(\mathrm{X}=\mathrm{A} \mid \mathrm{C} 1)=3 / 4=0.75, \mathrm{P}(\mathrm{X}=\mathrm{A} \mid \mathrm{C} 2)=0 / 3=0$
Laplacian correction, $\mathrm{P}(\mathrm{X}=\mathrm{A} \mid \mathrm{C} 1)=4 / 5=0.8, \mathrm{P}(\mathrm{X}=\mathrm{A} \mid \mathrm{C} 2)=1 / 4=0.25$
$\mathrm{P}(\mathrm{Y}=\alpha \mid \mathrm{C} 1)=3 / 4=0.75, \mathrm{P}(\mathrm{Y}=\alpha \mid \mathrm{C} 2)=1 / 3=0.333$
$\mathrm{P}(\mathrm{Z}=2 \mid \mathrm{C} 1)=0 / 4=0, \mathrm{P}(\mathrm{Z}=2 \mid \mathrm{C} 2)=2 / 3=0.667$
Laplacian correction, $\quad \mathrm{P}(\mathrm{Z}=2 \mid \mathrm{C} 1)=1 / 5=0.2, \mathrm{P}(\mathrm{Z}=2 \mid \mathrm{C} 2)=3 / 4=0.75$
$\mathbf{P}\left(\mathbf{x} 8 \mid \mathbf{C}_{\mathbf{i}}\right): \mathrm{P}(\mathrm{x} 8 \mid \mathrm{C} 1)=0.8 \times 0.75 \times 0.2=0.12$

$$
\mathrm{P}(\mathrm{x} 8 \mid \mathrm{C} 2)=0.25 \times 0.333 \times 0.75=0.062
$$

$\mathbf{P}\left(\mathbf{x} 8 \mid \mathbf{C}_{\mathbf{i}}\right) * \mathbf{P}\left(\mathbf{C}_{\mathbf{i}}\right): \mathrm{P}(\mathrm{x} 8 \mid \mathrm{C} 1) \times \mathrm{P}(\mathrm{C} 1)=0.12 \times 0.571=0.0685$

$$
\mathrm{P}(\mathrm{x} 8 \mid \mathrm{C} 2) \times \mathrm{P}(\mathrm{C} 2)=0.062 \times 0.428=0.0265
$$

Therefore, $x 8$ belongs to class C 1
2) Classification of $\mathbf{x} 9(\mathbf{X}=\mathbf{C}, \mathbf{Y}=\boldsymbol{\beta}, \mathbf{Z}=1)$

Compute $\mathrm{P}\left(\mathrm{x} 9 \mid \mathrm{C}_{\mathrm{i}}\right)$ for each class
$\mathrm{P}(\mathrm{X}=\mathrm{C} \mid \mathrm{C} 1)=0 / 4=0, \mathrm{P}(\mathrm{X}=\mathrm{C} \mid \mathrm{C} 2)=2 / 3=0.666$
Laplacian correction, $\mathrm{P}(\mathrm{X}=\mathrm{C} \mid \mathrm{C} 1)=1 / 5=0.2, \mathrm{P}(\mathrm{X}=\mathrm{C} \mid \mathrm{C} 2)=3 / 4=0.75$
$P(Y=\beta \mid C 1)=1 / 4=0.25, P(Y=\beta \mid C 2)=2 / 3=0.666$
$\mathrm{P}(\mathrm{Z}=1 \mid \mathrm{C} 1)=3 / 4=0.75, \mathrm{P}(\mathrm{Z}=1 \mid \mathrm{C} 2)=1 / 3=0.333$
$\mathbf{P}\left(\mathbf{x} 9 \mid \mathbf{C}_{\mathbf{i}}\right): \mathbf{P}(\mathrm{x} 9 \mid \mathrm{C} 1)=0.2 \times 0.25 \times 0.75=0.0375$

$$
\mathrm{P}(\mathrm{x} 9 \mid \mathrm{C} 2)=0.75 \times 0.666 \times 0.333=0.1663
$$

$\mathbf{P}\left(\mathbf{x} 9 \mid \mathbf{C}_{\mathbf{i}}\right) * \mathbf{P}\left(\mathbf{C}_{\mathbf{i}}\right): \mathbf{P}(\mathrm{x} 9 \mid \mathrm{C} 1) \times \mathrm{P}(\mathrm{C} 1)=0.0375 \times 0.571=0.0214$

$$
\mathrm{P}(\mathrm{x} 9 \mid \mathrm{C} 2) \times \mathrm{P}(\mathrm{C} 2)=0.1663 \times 0.428=0.0711
$$

Therefore, $\mathbf{x} 9$ belongs to class $\mathbf{C} 2$
3) Classification of $\times 10(X=B, Y=\beta, Z=1)$

Compute $\mathrm{P}\left(\mathrm{x} 10 \mid \mathrm{C}_{\mathrm{i}}\right)$ for each class
$\mathrm{P}(\mathrm{X}=\mathrm{B} \mid \mathrm{C} 1)=1 / 4=0.25, \mathrm{P}(\mathrm{X}=\mathrm{B} \mid \mathrm{C} 2)=1 / 3=0.333$
$P(Y=\beta \mid \mathrm{C} 1)=1 / 4=0.25, \mathrm{P}(\mathrm{Y}=\beta \mid \mathrm{C} 2)=2 / 3=0.666$
$\mathrm{P}(\mathrm{Z}=1 \mid \mathrm{C} 1)=3 / 4=0.75, \mathrm{P}(\mathrm{Z}=1 \mid \mathrm{C} 2)=1 / 3=0.333$
$\mathbf{P}\left(\mathbf{x 1 0} \mid \mathbf{C}_{\mathbf{i}}\right): \mathrm{P}(\mathrm{x} 10 \mid \mathrm{C} 1)=0.25 \times 0.25 \times 0.75=0.0468$
$P(x 10 \mid C 2)=0.333 \times 0.666 \times 0.333=0.0738$
$\mathbf{P}\left(\mathbf{x 1 0} \mid \mathbf{C}_{\mathbf{i}}\right) * \mathbf{P}\left(\mathbf{C}_{\mathbf{i}}\right): \mathrm{P}(\times 10 \mid \mathrm{C} 1) \times \mathrm{P}(\mathrm{C} 1)=0.0468 \times 0.571=0.0267$
$\mathrm{P}(\mathrm{x} 10 \mid \mathrm{C} 2) \times \mathrm{P}(\mathrm{C} 2)=0.0738 \times 0.428=0.0315$
Therefore, $x 10$ belongs to class $\mathbf{C} 2$

