Tutorial one in Fuzzy sets First year master AMD 2023/2024

Sure! Here are a few exercises on fuzzy sets along with their solutions:

Exercise 1:

Consider a universe of discourse $X = \{1, 2, 3, 4, 5\}$ and a fuzzy set A defined by the membership function μA(x), where:

$\mu A(1) = 0.2, \mu A(2) = 0.6, \mu A(3) = 1.0, \mu A(4) = 0.4, \mu A(5) = 0.1$

a) What is the support of fuzzy set *A*?

b) What is the core of fuzzy set *A*?

c) Find the α-cut of fuzzy set *A* for $\alpha = 0.5$

Exercise 2:

Consider two fuzzy sets A and B defined on the universe of discourse $X = \{1, 2, 3, 4, 5\}$ with the following membership functions:

$\mu A(1) = 0.8, \mu A(2) = 0.5, \mu A(3) = 0.2, \mu A(4) = 0.4, \mu A(5) = 0.1$

$\mu B(1) = 0.3, \mu B(2) = 0.9, \mu B(3) = 0.6, \mu B(4) = 0.7, \mu B(5) = 0.2$

a) Calculate the complement of fuzzy set *A*.

b) Calculate the union of fuzzy sets A and *B*.

c) Calculate the intersection of fuzzy sets *A* and *B*.

Exercice3:

Consider a fuzzy set A defined on the universe of discourse $X = \{1, 2, 3, 4, 5\}$ with the following membership function:

$\mu A(1) = 0.2, \mu A(2) = 0.6, \mu A(3) = 1.0, \mu A(4) = 0.4, \mu A(5) = 0.1$

a) Calculate the complement of fuzzy set *A*.

b) Calculate the intersection of fuzzy set *A* with its complement.

Exercice4

Consider two fuzzy sets A and B defined on the universe of discourse $X = \{1, 2, 3, 4, 5\}$ with the following membership functions:

$\mu A(1) = 0.8, \mu A(2) = 0.5, \mu A(3) = 0.2, \mu A(4) = 0.4, \mu A(5) = 0.1$

$\mu B(1) = 0.3, \mu B(2) = 0.9, \mu B(3) = 0.6, \mu B(4) = 0.7, \mu B(5) = 0.2$

a) Calculate the algebraic sum of fuzzy sets A and B.

b) Calculate the bounded sum of fuzzy sets A and B.

Solution:

a) The algebraic sum of fuzzy sets A and B is obtained by adding their membership values at each element of the universe of discourse. Therefore, the algebraic sum is:

$\mu A+B(1) = 0.8 + 0.3 = 1.1$ (truncated to 1)

$\mu A+B(2) = 0.5 + 0.9 = 1.4$ (truncated to 1)

$\mu A+B(3) = 0.2 + 0.6 = 0.8$

$\mu A+B(4) = 0.4 + 0.7 = 1.1$ (truncated to 1)

$\mu A+B(5) = 0.1 + 0.2 = 0.3$

b) The bounded sum of fuzzy sets A and B is obtained by taking the minimum of the sum of their membership values and 1 at each element of the universe of discourse. Therefore, the bounded sum is:

$\mu A \oplus B(1) = \min(0.8 + 0.3, 1) = \min(1.1, 1) = 1$

$\mu A \oplus B(2) = \min(0.5 + 0.9, 1) = \min(1.4, 1) = 1$

$\mu A \oplus B(3) = \min(0.2 + 0.6, 1) = \min(0.8, 1) = 0.8$

$\mu A \oplus B(4) = \min(0.4 + 0.7, 1) = \min(1.1, 1) = 1$

$\mu A \oplus B(5) = \min(0.1 + 0.2, 1) = \min(0.3, 1) = 0.3$

These examples demonstrate the complement, intersection, algebraic sum, and bounded sum operations on fuzzy sets. These operationscan help in combining and manipulating fuzzy sets to represent uncertainty, imprecision, and vagueness in various applications such as decision-making, pattern recognition, and control systems.

Solution:

Eercice01

a) The support of fuzzy set A is the set of all elements in the universe of discourse X for which the membership value is non-zero. In this case, the support is {1, 2, 3, 4, 5}.

b) The core of fuzzy set A is the set of all elements in the universe of discourse X for which the membership value is equal to 1. In this case, the core is {3}.

c) The α-cut of fuzzy set A for $\alpha = 0.5$ is the set of all elements in the universe of discourse X for which the membership value is greater than or equal to 0.5. In this case, the α-cut is {2, 3, 4}.

Exercice02

Solution:

a) The complement of fuzzy set A is obtained by subtracting the membership values of A from 1. Therefore, the complement of A is:

$\mu A'(1) = 1 - 0.8 = 0.2$

$\mu A'(2) = 1 - 0.5 = 0.5$

$\mu A'(3) = 1 - 0.2 = 0.8$

$\mu A'(4) = 1 - 0.4 = 0.6$

$\mu A'(5) = 1 - 0.1 = 0.9$

b) The union of fuzzy sets A and B is obtained by taking the maximum membership value at each element of the universe of discourse. Therefore, the union of A and B is:

$\mu A \cup B(1) = \max(0.8, 0.3) = 0.8$

$\mu A \cup B(2) = \max(0.5, 0.9) = 0.9$

$\mu A \cup B(3) = \max(0.2, 0.6) = 0.6$

$\mu A \cup B(4) = \max(0.4, 0.7) = 0.7$

$\mu A \cup B(5) = \max(0.1, 0.2) = 0.2$

c) The intersection of fuzzy sets A and B is obtained by taking the minimum membership value at each element of the universe of discourse. Therefore, the intersection of A and B is:

$\mu A \cap B(1) = \min(0.8, 0.3) = 0.3$

$\mu A \cap B(2) = \min(0.5, 0.9) = 0.5$

$\mu A \cap B(3) = \min(0.2, 0.6) = 0.2$

$\mu A \cap B(4) = \min(0.4, 0.7) = 0.4$

$\mu A \cap B(5) = \min(0.1, 0.2) = 0.1$

Exo3

Solution:

a) The complement of fuzzy set A is obtained by subtracting the membership values of A from 1. Therefore, the complement of A is:

$\mu A'(1) = 1 - 0.2 = 0.8$

$\mu A'(2) = 1 - 0.6 = 0.4$

$\mu A'(3) = 1 - 1.0 = 0.0$

μA'(4) = 1 - 0.4 = 0.6

μA'(5) = 1 - 0.1 = 0.9

b) The intersection of fuzzy set A with its complement is obtained by taking the minimum membership value at each element of the universe of discourse. Therefore, the intersection is:

μA∩A'(1) = min(0.2, 0.8) = 0.2

μA∩A'(2) = min(0.6, 0.4) = 0.4

μA∩A'(3) = min(1.0, 0.0) = 0.0

μA∩A'(4) = min(0.4, 0.6) = 0.4

μA∩A'(5) = min(0.1, 0.9) = 0.1

.