First set of exercises
2^{nd} year Physics Feb-2024

A. Compton effect

I- Mawell's equations

1) Show that the both electric and magnetic field propagate in the vaccum in the form of waves.
2) Within the Jeans box taking as cube of edge length l, show that equation of propagation is described by standing waves.

II- Density of states

3) Find the density of states $N(f)$ in 1, 2 and 3 dimension
4) Show that density of states in 3d is given by

$$N(f)df = \frac{8\pi L^3}{c^3} f^2 df$$

III- Rayleigh-Jeans formula

5) Derive equipartition energy theorem starting from the Boltzmann distribution and show that $\overline{E} = k_B T$ and show that energy density ($\rho = \frac{\overline{E}}{V}$)

$$\rho(f)df = \frac{8\pi}{c^3} f^2 k_B T df$$

6) IF $\rho(f)df = -\rho(\lambda)d\lambda$, derive the Rayleigh-Jeans formula

$$\rho(\lambda)d\lambda = \frac{8\pi k_B T}{\lambda^4} d\lambda$$

IV- Planck Quantization

7) Assuming that the energy levels are discrete $E_n = n\varepsilon_0$ recalculation the average energy $\overline{E} = \sum_i E_i P_i = \sum_i E_i \frac{e^{-E_i/k_B T}}{Z}$ where $Z = \sum_i e^{-E_i/k_B T}$ is the partition function. Show that

$$\overline{E} = \frac{\varepsilon_0}{e^{\varepsilon_0/k_B T} - 1}$$

8) Deduce that if $\varepsilon_0 = hf$

$$\rho(f)df = \frac{8h\pi}{c^3} f^3 \frac{1}{e^{hf/k_B T} - 1} df$$

9) Knowning that $\rho(f)df = -\rho(\lambda)d\lambda$, show that

$$\rho(\lambda)d\lambda = \frac{8\pi hc}{\lambda^5} \frac{1}{e^{hc/\lambda k_B T} - 1} d\lambda$$

10) Plot for both approach (Rayleigh and Planck) the function $\rho(\lambda)$

V- Stefan-Boltzmann law

11) If the intensity I is given by

$$I \propto \int_0^\infty \rho(\lambda) d\lambda$$

Deduce the Stefan-Boltzmann law.

VI- Wien's displacement law

12) The maximum wavelength λ_{max} corresponds to $\frac{d\rho(\lambda)}{d\lambda} = 0$ show that

$$\lambda_{max} T = b$$

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13) The constante b is obtained by solving the equation $x = 5(1 - e^{-x})$. solve numerically such equation using the Newton-Raphson iterative method.

B. Photoelectric effect

1) How many photons would be emitted per second by a sodium lamp rated at 100 W which radiated all its energy with 100 per cent efficiency as yellow light of wavelength 589 nm?
2) Calculate the speed of an electron emitted from a clean potassium surface ($\Phi = 2.3eV$) by light of wavelength (a) 300 nm, (b) 600 nm.

C. Compton effect

Deduce the equation

$$\delta\lambda = 2\lambda_C \sin^2(\vartheta/2) \qquad \lambda_C = \frac{h}{m_e c}$$

for the Compton effect on the basis of the conservation of energy and linear momentum. Hint. Use the relativistic expressions. Initially the electron is at rest with energy $m_e c^2$. When it is travelling with momentum p its energy is $E = \sqrt{p^2c^2 + m^2c^4}$. The photon, with initial momentum h/λ and energy $h\nu$, strikes the stationary electron, is deflected through an angle ϑ, and emerges with momentum h/λ_f and energy $h\nu_f$. The electron is initially stationary ($p = 0$) but moves off with an angle ϑ' to the incident photon. Conserve energy and both components of linear momentum. Eliminate ϑ' then p, and so arrive at an expression for $\delta\lambda$

D. deBroglie wavelength :

Calculate the deBroglie wavelength of (a) a mass of 1.0 g travelling at 1.0 cm/s, (b) the same at 95 per cent of the speed of light, (c) a hydrogen atom at room temperature (300 K); estimate the mean speed from the equipartition principle (which implies that the mean kinetic energy of an atom is equal to $\frac{3}{2}k_B T$, where k_B is the Boltzmann constant), (d) an electron accelerated from rest through a potential difference of (i) 1.0 V, (ii) 10 kV. Hint. For the momentum in (b) use $p = \frac{m_e v}{\sqrt{1 - \frac{v^2}{c^2}}}$ and for the speed in (d) use $\frac{1}{2}m_e v^2 = eV$, where V is the potential difference.