PLANE GEOMETRY
(TWO-DIMENSIONAL GEOMETRY)

Plane Geometry deals with flat shapes which can be drawn on a piece of paper. These include lines, triangles & circles of two dimensions. Plane geometry is also known as two-dimensional geometry.

The important terminologies in plane geometry are :

- *Points and Lines*
- *Angles*
- *Circles*

1. POINTS AND LINES

1.1 Definitions

a) A point is an exact location in space, it has no size. Points are represented by dots, and named with capital letters.
b) A line is a straight arrangement of points. Lines extend forever in opposite directions. It could be horizontal, vertical, or oblique
c) A line segment is part of a line. Line segments consist of 2 endpoints and all the points in between. Line segments are named using their endpoints.
d) A ray is a part of a line that has one endpoint and extends forever in one direction. Rays are named by writing the endpoint first, then another point on the ray.

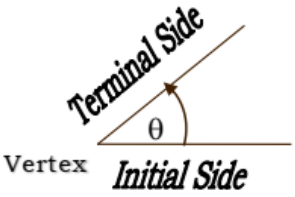
1.2 Position of lines in a plane

a) Line l is parallel to line m. (Lines l and m are parallel).
b) Line a cuts line b. Line a intersects line b.
c) Lines a and b are intersected lines. If two lines intersect, their intersection is a point called the point of intersection.
d) Line p is orthogonal to line q. (Line p is perpendicular to line q). Lines p and q are orthogonal
e) Line k is called a transversal line, which is a straight line drawn across a set of two or more parallel lines.

2. ANGLES

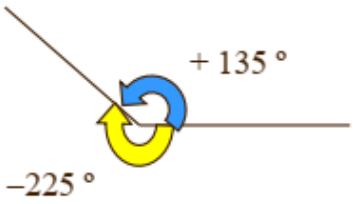
2.1 Definitions

a) An angle is the figure formed by rotating a ray around its end point.
b) One ray is fixed, and is called the initial side.
c) The second ray is called the terminal side.
d) The common end point is called the vertex.



2.2 Positive, Negative & Coterminal Angles

a) A positive angle results from a counter-clockwise rotation.
b) A negative angle results from a clockwise rotation



c) Two angles with the same initial and terminals side are coterminal. So + 135 º and - 225 º are coterminal!

2.3 Types of Angles

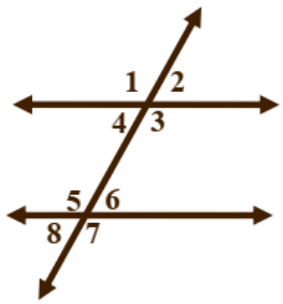
In the table below types of angles according to their measures

Measure of Angle θ	Name
$0° < \theta < 90°$	acute angle
$\theta = 90°$	right angle
$90° < \theta < 180°$	obtuse angle
$\theta = 180°$	straight angle
$180° < \theta < 360°$	reflex angle
$\theta = 360°$	full angle

2.4 Complementary & Supplementary Angles

a) Two angles that have a sum of 90º are complementary.
b) Two angles that have a sum of 180º are **s**upplementary**.**

Angles Formed by Transversal Line

	a) Angle 3 and angle 5 are alternate interior angles. b) Angle 1 and angle 7 are alternate exterior angles. c) Angle 1 and angle 2 are adjacent angles. d) Angle 2 and angle 6 are corresponding angles. e) Angle 5 and angle 7 are opposite angles.

3. Two Dimensional Figures

There are two basic groups of two-dimensional figures : Polygon and Circle.

3.1 Polygon

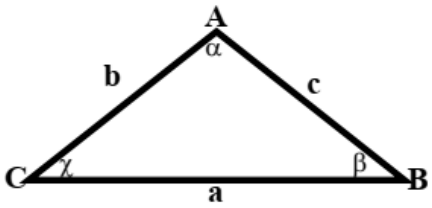
Polygon are two-dimensional figures which consists of n points and nearby points connected by straight lines which are called sides. They are named by the number of the sides.

Example:

- TRIANGLE – 3 sides
- QUADRILATERAL – 4 sides
- PENTAGON – 5 sides
- HEXAGON – 6 sides
- HEPTAGON – 7 sides
- OCTAGON – 8 sides
- NONAGON – 9 sides
- N-GON – N sides

3.1.1 Triangles

A triangle is a three – sided figure. The three sides of a triangle meet at points called vertices (singular: vertex)



A triangle has three vertices (A,B,C), three sides (a,b,c), and three interior angles (α,β,χ) whose sum is 180 degrees.

a. Special Types of Triangles

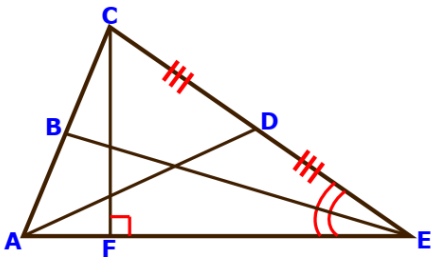
1. *Isosceles triangle* : is a triangle that has two equal sides. The top vertex is called apex and the bottom side is called base.
2. *Equilateral triangle* : is a triangle that has three equal sides.
3. *Right-angle triangle* : (usually called right triangle) is a triangle that has one right angle. The side opposite the right angle is called hypotenuse.
4. *Scalene triangle*: is a triangle that has three different sides or all of its sides have different length.
5. *Acute triangle*: is a triangle which all of its internal angles are acute angles.
6. *Obtuse triangle*: is a triangle that has an obtuse angle.

b. Important Lines of Triangles

Lines	Definitions
angle bisector	a segment which bisects an angle and connects a vertex and a point on the opposite side.
median (bisector)	a segment that connects a vertex of the triangle and the midpoint of the opposite side
altitude	a segment from the vertex of the triangle perpendicular to the line containing the opposite side

Practice

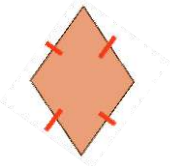
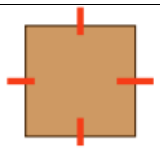
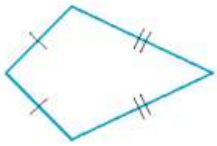
Which one is the angle bisector, median, or altitude of this triangle ?



3.1.2 Quadrilateral

A quadrilateral is any 4 sided shape. The sum of all interior angles of any quadrilateral is 360 degrees.

a. Types of Quadrilaterals.

Terms		Definitions	Illustrations
Parallelogram (has 2 pairs of parallel sides)	Rectangle	parallelogram with 4 right angles	
	Rhombus	parallelogram with 4 sides of equal length	
	Square	parallelogram with 4 right angles and 4 sides of equal length.	
Trapezoid		has exactly one pair of parallel sides	
Kite		has exactly two pairs of congruent adjacent sides	
Trapezium		has exactly one pair of parallel sides and two right angles	

b. Area and Perimeter

The formula of **area** of a triangle is the half of the product of length of altitude and length of base

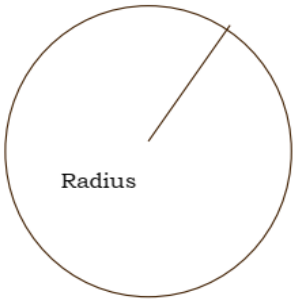
The **perimeter** of a n-sided figure is the sum of all its side‘s lengths.

c. Practice

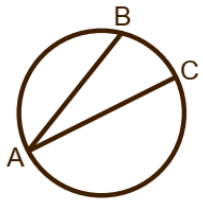
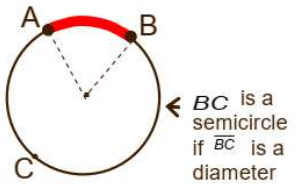
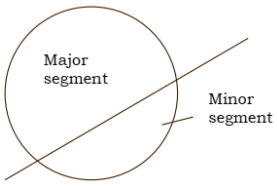
Find the formula for area of square, rectangle, rhombus, parallelogram, and trapezium.

3.2 Circle

a) **Circle** is the set of all points in a plane that are a given distance from the center.
b) **Radius** (plural: **radii**) is a segment line that joins the center to a point on the circle.



Parts of a Circle

Chord : a line joining two points on a circle. – AC and AB are chords. **Diameter:** a chord that passes through the circles center. – AC is a diameter	
Arc: two points on a circle and all the points needed to connect them. – minor arc AB or – major arc ACB or **Central Angle:** an angle whose vertex is at the center of the circle **Sector:** It is a region enclosed by two radii and arc of the circle.	
Segment: It is a region enclosed by a chord and the arc joining the chord. The segment made my minor arc is called minor segment and segment by major arc as major segment.	

Circumference :

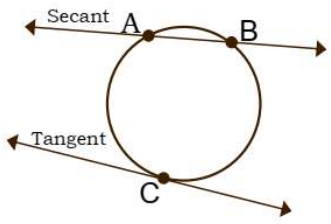
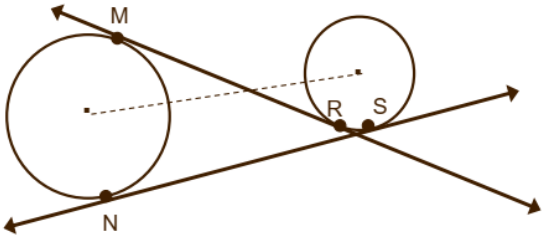
the perimeter of a circle, the distance around a circle.

$C = d\pi$ or $C=2\ \pi\ r$ (d = diameter,r =radius)

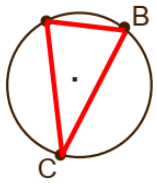
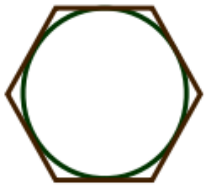
Area of a circle :

$$A = \pi r^2$$

c) Line that Cuts the Circle

Secant: a line that intersects a circle at exactly two points Tangent: a line that intersects a circle at exactly one point – The point of contact is called the point of tangency or point of contact.	
A common tangent is a line tangent to two circles (not necessarily at the same point)	 *MR is a common internal tangent* *NS is a common external tangent*

d) Inscribed and Circumscribed Polygons

Inscribed: A polygon is inscribed in a circle (or another polygon) if all of its vertices lie on the circle (or another polygon). – The circle center is the incenter of the polygon	
Circumscribed: A polygon is circumscribed about a circle if each of its sides is tangent to the circle. – The circle center is the circumcenter of the polygon	

e) Practice

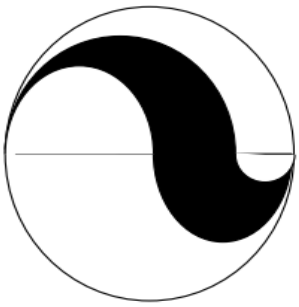
Fill the blank with the right words

1. _____ consist of 2 endpoints and all the points in between.
2. If each angle in a triangle is less than 90°, then the triangle is called _____.
3. A line which meets another _____ at 90° is called a _____ line.
4. If two angles of a triangle are equal to 45°, then the triangle is called _____
5. If we _____ a right angle, we will have two _____ angles of 45°.
6. If the measure of angle A is 130°, then the _____ is -230°.
7. A segment that perpendicular to a side of triangles and through a vertex is called _____.
8. Each triangle has 3 points, or _____
9. _____ is a rectangle with four congruent sides.
10. An octagon is _____ in a square if all of its _____ lie on the square.
11. A _____ with radius 10 m has _____of 20π m.
12. A quadrilateral which only has one pair of right angle can be called_____.
13. _____ has seven vertices and _____ sides.
14. If the _____of a sector is 60 degrees, then the area of the sector is _____ of the circle‘s area.

Answer these questions

1. A right triangle has a hypotenuse of 6 and a perimeter of 14. Find the area of the triangle.
2. A regular hexagon is inscribed in a circle of radius 4 meters. What is the area of the hexagon?
3. The total number of interior angles in two regular polygons is 17, and the total number of diagonals is 53. How many sides does each regular polygon have ?
4. A triangle has sides of length 30, 40, and 50 meters. What is the length of the shortest altitude of this triangle ?
5. A circle is inscribed in a triangle that has sides of lengths 60, 80, and 100 cm. Find the length of the radius of the circle.
6. We know that the vertices of a quadrilateral are 2, 3, 5, and 6 cm, respectively, from a point P. What is the largest possible area of this quadrilateral?
7. Five of the angles of an octagon have measures whose sum is 8450. Of the remaining three angles, two are complementary to each other and two are supplementary to each other. Find the measures of these three angles.
8. Gene wants to put a brick border around a tree. The border is to be placed 1.5m from the tree. If the circumference of the tree is 56.52cm, what is the inner circumference of the brick border?
9. A hexagon is inscribed in a circle, which is inscribed in a square of side 10 cm. What is the length of each side of the hexagon?
10. Find the dimension of a rectangle of maximum area with a given perimeter P.

11. In how many different ways can you divide a square into four congruent shapes ?
12. Find the area of the shaded region if the diameter of the circle is 24 cm and it is divided into four equal segments as shown.



Describe points, lines, and angles in these figures