M'sila University
Fcaulty of Mathematics and Computer
Department of Mathematics
Year 2023/2024
Algebra 4 course

TD Number 2

Exercise 1.

We consider the linear map $q : \mathbb{R}^3 \longrightarrow \mathbb{R}$, defined by

$$q(x, y, z) = x^2 - 2yz - xz$$

1. Is q a quadratic form?

2. Find the matrix A of q.

3. Determine the bilinear form φ associated to q.

4. Find the sum of squares, using the Gauss method.

5. Find the rank and signature of q.

6. Find an orthogonal basis B' for q.

Exercise 2.

Let the linear map $q : \mathbb{R}^3 \longrightarrow \mathbb{R}$, defined by

$$q(x, y, z) = xy + yz + zx$$

1. Find the matrix A of q.

2. Using Gauss method, deduce the sum of squares and an orthogonal basis B' for q.

3. Find the rank and signature of q.

4. Find whether q has isotropic vectors and in that case find them.