

M'sila University
Faculty of Mathematics and Computer
Department of Mathematics
Year 2023/2024
Algebra 4 course

TD Number 3

Exercise 1.

Let q be a quadratic form over \mathbb{R}^4 defined by $\forall u = (x, y, z, t)$ in \mathbb{R}^4 ,

$$q(u) = 9x^2 + 5y^2 + 8yz - 4yt + 5z^2 + 4zt + 8t^2$$

1. Find the matrix associated to q .
2. Using Gauss method, define the sum of squares of q .
3. Find the rank and signature of q and whether it is degenerate or not.
4. Find $\ker(q)$, the kernel of q .
5. Find the isotropic vectors of q .

Exercise 2.

Let q be a quadratic form over \mathbb{R}^4 defined by $\forall x = (x_1, x_2, x_3, x_4)$ in \mathbb{R}^4 ,

$$q(x) = x_1x_2 + x_1x_3 + x_1x_4 + x_2x_3 + x_2x_4 + x_3x_4$$

1. Find the matrix associated to q .
2. Using Gauss method, find the sum of squares of q .
3. Find the rank and signature of q and whether it is degenerate or not.
4. Find an orthogonal basis B' for q , i.e a basis such that $M_{B'}(q)$ is a diagonal matrix.