Tutorial 3

Exercise 1

1.7. Compute the scalar cardinalities for each of the following fuzzy sets:
(a) $A = .4/v + .2/w + .5/x + .4/y + 1/z$;
(b) $B = 1/x + 1/y + 1/z$;
(c) $C(x) = \frac{x}{x+1}$ for $x \in \{0, 1 \ldots, 10\} = X$;
(d) $D(x) = 1 - x/10$ for $x \in \{0, 1, \ldots, 10\} = X$.

1.8. Let A, B be fuzzy sets defined on a universal set X. Prove that

$$|A| + |B| = |A \cup B| + |A \cap B|,$$

where $\cap, \cup$ are the standard fuzzy intersection and union, respectively.

1.9. Order the fuzzy sets defined by the following membership grade functions (assuming $x \geq 0$) by the inclusion (subset) relation:

$$A(x) = \frac{1}{1+10x}, B(x) = \left(\frac{1}{1+10x}\right)^{1/2}, C(x) = \left(\frac{1}{1+10x}\right)^{2}.$$

.10. Consider the fuzzy sets A, B, and C defined on the interval $X = [0, 10]$ of real numbers by the membership grade functions

$$A(x) = \frac{x}{x+2}, B(x) = 2^{-x}, C(x) = \frac{1}{1+10(x-2)^2}.$$

Determine mathematical formulas and graphs of the membership grade functions of each of the following sets:
(a) $\overline{A}, \overline{B}, \overline{C}$;

(b) $A \cup B, A \cup C, B \cup C$;
(c) $A \cap B, A \cap C, B \cap C$;
(d) $A \cup B \cup C, A \cap B \cap C$;
(e) $A \cap \overline{C}, \overline{B \cap C}, \overline{A \cup C}$.

1.11. Calculate the α-cuts and strong α-cuts of the three fuzzy sets in Exercise 1.10 for some values of α, for example, $\alpha = 0.2, 0.5, 0.8, 1$.

1.12. Restricting the universal set to [0, 10], determine which fuzzy sets in Exercise 1.10 are convex.

1.13. Let A, B be two fuzzy sets of a universal set X. The *difference* of A and B is defined by

$$A - B = A \cap \overline{B};$$

and the *symmetric difference* of A and B is defined by

$$A \Delta B = (A - B) \cup (B - A).$$

Prove that:
(a) $(A \Delta B) \Delta C = A \Delta (B \Delta C)$;
(b) $A \Delta B \Delta C = (\overline{A} \cap \overline{B} \cap C) \cup (\overline{A} \cap B \cap \overline{C}) \cup (A \cap \overline{B} \cap \overline{C}) \cup (A \cap B \cap C)$.

Exercise 2

The standard fuzzy intersection is the only idempotent t-norm.

where i_{min} denotes the drastic intersection.

Let i_w denote the class of Yager t-norms . Then

$$i_{\min}(a, b) \leq i_w(a, b) \leq \min(a, b)$$

for all $a, b \in [0, 1]$.

Exercise 3

The fuzzy binary relation R is defined on sets $X = \{1, 2, \ldots, 100\}$ and $Y = \{50, 51, \ldots, 100\}$ and represents the relation "x is much smaller than y." It is defined by membership function

$$R(x, y) = \begin{cases} 1 - \dfrac{x}{y} & \text{for } x \leq y \\ 0 & \text{otherwise,} \end{cases}$$

where $x \in X$ and $y \in Y$.

(a) What is the domain of R?

(b) What is the range of R?

(c) What is the height of R?

(d) Calculate R^{-1}.

Exercise 4

Show that

The following membership matrix defines a fuzzy partial ordering R on the set $X = \{a, b, c, d, e\}$:

$$\mathbf{R} = \begin{array}{c} \\ a \\ b \\ c \\ d \\ e \end{array} \begin{array}{c} \begin{array}{ccccc} a & b & c & d & e \end{array} \\ \begin{bmatrix} 1 & .7 & 0 & 1 & .7 \\ 0 & 1 & 0 & .9 & 0 \\ .5 & .7 & 1 & 1 & .8 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & .1 & 0 & .9 & 1 \end{bmatrix} \end{array}$$