Properties of max-min composition

1. *Associativity* The *max-min* composition is associative, that is

$$(\tilde{R}_3 \circ \tilde{R}_2) \circ \tilde{R}_1 = \tilde{R}_3 \circ (\tilde{R}_2 \circ \tilde{R}_1)$$

   Consequence: the power of a fuzzy relation. If
   $\tilde{R}_1 = \tilde{R}_2 = \tilde{R}_3 = \tilde{R}$, we can write:
   $\tilde{R}^2 = \tilde{R} \circ \tilde{R}$,
   $\tilde{R}^3 = \tilde{R} \circ \tilde{R} \circ \tilde{R}$ and so on.
   Hence, for any n natural, we can define $\tilde{R}^n$
2. *Reflexivity Definition:* A fuzzy relation defined in $X \times X$ is reflexive iff (if and only if) $\mu_{\tilde{R}}(x,x) = 1, \forall x \in X$
   Property: If the fuzzy relations $\tilde{R}_1$ and $\tilde{R}_2$ are reflexive, then their max-min composition, $\tilde{R}_1 \circ \tilde{R}_2$ is also reflexive.
3. *Symmetry Definition:* A fuzzy relation is called symmetric if $\tilde{R}(x,y) = \tilde{R}(y,x)$
4. *Antisymmetry Definition:* A fuzzy relation is called antisymmetric if, $\forall x, y \in X$, if $x \neq y$ then either $\mu_{\tilde{R}}(x,y) \neq \mu_{\tilde{R}}(y,x)$, or $\mu_{\tilde{R}}(x,y) = \mu_{\tilde{R}}(y,x) = 0$
5. A fuzzy is called perfect antisymmetric if $\forall\ x \neq y$, whenever $\mu_{\tilde{R}}(x,y) > 0$, then $\mu_{\tilde{R}}(y,x) = 0$
6. *Transitivity Definition:* A fuzzy relation $\tilde{R}$ is called max-min transitive if $\tilde{R} \circ \tilde{R} \subseteq \tilde{R}$.

Similarity, de pre-order and order fuzzy relations

Definition

A *similarity relation* is a fuzzy relation that is reflexive, symmetrical and max-min transitive

The idea of similarity is analogous to the idea of equivalence, being possible to create similarity trees.

Definition

A fuzzy relation which is max-min transitive and reflexive is called *fuzzy preorder relation*.

Definition

A fuzzy relation which is max-min transitive, reflexive and antisymmetric is called *fuzzy order relation*.