# CHAPTER I: ELECTROSTATICS

## I- Introduction

The name electricity is coined from the Greek word ''elektron'' meaning ''amber''.

Electricity & Magnetism includes a vast variety of phenomena that can be divided in four groups:

- Electrostatics is a branch of physics that studies the phenomena created by electric charges at rest.
- Electric currents & circuits is a branch of physics that studies the phenomena created by moving electric charges.
- Magnetism (permanent magnets, magnetic effects due to electric currents).
- Electromagnetism & Electrodynamics (time-dependent phenomena in electrical machines, electromagnetic waves)

## II- Historical overview

Electricity (and less Magnetism) penetrates modern life, whereas in the past people could only observe a few electrostatic phenomena such as sparks. That was the beginning of the investigation of electricity.

- 2750 BC**, e**lectricity first recorded in the form of electric fish. A touching the fish with an iron rod causes electric shocks. The first attempt to use electrical energy may have been in medicine: Greeks, Romans, and Egyptians reportedly used **electric fish** as a treatment for epilepsy and gout.
- 500 BC, the next discovered type of electricity was static electricity. Thales of Miletus, a Greek philosopher, discovered that static electricity could be made by rubbing lightweight objects such as fur or feathers on amber. This static effect remained unknown for almost 2,000 years until around 1600 AD.
- 1600 AD, the term “electricity” didn’t exist at the time. An English scientist named William Gilbert came up with the word “electricus” to describe objects that attracted dust “like amber,” and this eventually led to the modern usage of the word “electricity.”

- Some of the biggest milestones in the history of electricity occurred in the $18^{th}$ and $19^{th}$ centuries when individual scientists performed experiments that furthered our knowledge of electrical energy.

Several of the scientists even lent their name to scientific laws and units of measurement:

**- Benjamin Franklin's** *(1706-1790) (American polymath: writer, scientist, inventor, statesman, diplomat, printer, publisher and political philosopher)*: In 1751, he published book Benjamin Franklin's discoveries made about the behaviour of electricity. He also introduced a host of new terms to the field including positive, negative, charge, battery and electric shock.

- **James Watt** *(1736-1819) (Scottish engineer and chemist)*: He transformed the Industrial Revolution with the invention of a modified Newcome engine, now known as the Watt steam engine. Boiling water could drive the pistons back and forth. Although Watt's engine didn't generate electricity, it created a foundation that would eventually lead to the steam turbine – still the basis of much of the globe's electricity generation today.

- **Alessandro Volta** *(1745-1827) (Italian physicist and chemist)*: He invented the first true battery in 1800. Volta realised that a current was created when zinc and silver were immersed in an electrolyte. The chemical batteries are still based today on this principle.

*Breakthroughs in electric motors and batteries in the early 1800 led to experimentation with electrically powered vehicles. The British inventor Robert Anderson is often credited with developing the first crude electric carriage at the beginning of the $19^{th}$ century, but it would not be until 1890 that American chemist William Morrison would invent the first practical electric car (though it closer resembled a motorised wagon), boasting a top speed of 14 miles per hour.*

**-** Michael Faraday *(1791-1867) (English scientist)***:** He developed the idea of the electromagnetic field. He also identified the laws of electrolysis, which are still used today to determine how much energy has to pass through a substance in order to create a chemical change. He developed the first electric motor and demonstrated it in 1821.

**- Thomas Alva Edison** *(1847-1931) (American inventor and businessman)*: He patented the first practical and accessible incandescent light bulb, using a carbonised bamboo filament which could burn for more than 1,200 hours. Edison made the first public demonstration of his incandescent lightbulb on 31st December 1879 where he stated that, "electricity would be so cheap that only the rich would burn candles." Although he was not the only inventor to experiment with incandescent light, his was the most enduring and practical. He would soon go on to develop not only the bulb, but an entire electrical lighting system.

*In 1882, the world's first public power station ''Holborn Viaduct power station'', also known as the Edison Electric Light Station, burnt coal to drive a steam turbine and generate electricity. The power was used for Holborn's newly electrified streetlighting, an idea which would quickly spread around London.*

- **Nikola Tesla** *(1856-1943) (Serbian-American inventor, electrical engineer, mechanical engineer, and futurist):* He proved that alternating current (AC) – as is generated at power stations – was safe for domestic use, going against the Edison Group's opinion that a direct current (DC) – as delivered from a battery – was safer and more reliable.

*This became known as the War of the Currents and led to electrical engineering feats such as powering Buffalo, New York, with energy from Niagara Falls. Tesla also developed several different types of transformers, including the Tesla coil, a high-voltage, low-current transformer that's often used for entertainment. In 1901, Great Britain's first industrial power station was opened. After that came the Leyden jar, a device that can store an electrical charge and paved the way for laboratory experiments with electricity and magnetism.*

## III- Electric charge

### III.1- Definition and types of electric charge

Electric charge is the property associated with matter due to which it produce and experience electric and magnetic effect.

There are two types of charges (convention derived from Benjamin Franklin's experiments.):

a- Positive charge: Lack of electron in a matter is called positive charge.

b- Negative charge: Excess of electron in a matter is called negative charge.

The SI unit of charge is called the Coulomb (C).

The smallest unit of "free" charge known in nature is the charge of an electron or proton, which has a magnitude of:

- $-1.6 \times 10^{-19}$ C for one electron
- $+1.6 \times 10^{-19}$ C for one proton

In a closed system, the total amount of charge is conserved. In the atom, the nucleus has a charge which is a multiple of +e while the orbiting electrons each have a charge of −e. The charge of the nucleus comes from the constituent protons, each of which has a charge of +e; the neutrons in the nucleus have no charge.

According to the fundamental law of electrostatics:

- Like charges repels
- Unlike charges attracts each other.

**III.2- Basic properties of Electric charge**

- Additivity of Electric charges: Charges can be added by simple rules of algebra. Addition of equal positive and negative charge makes Zero charge
- Quantization of Electric charge: Electric charge is not a continuous quantity, but is an integral multiple of minimum charge ( e). Charge on a body Q is given by **Q = + ne.** Where n is a whole number 1,2,3….. and e = 1.6 x $10^{-19}$
- Conservation of Electric Charge: Like conservation of energy, and Momentum, the electric charges also follow the rules of conservation.
- Isolated (Individual) Electric charge can neither be created nor destroyed, it can only be transferred.

**III.3- Types of matter**

According to flow of charges, there are three types of matters:

**a- Conductor**: In metals, the outer atomic electrons are only weakly bound to the nuclei. These outer electrons become detached from their parent nuclei and are free to wander about through the entire solid. The electrons can flow easily. Ex: iron, aluminum, copper, silver etc.

**b- Insulator:** The electrons in the insulator are all tightly bound to the positive nuclei and not free to move around. The electrons can't flow. Ex: plastic, wood etc.

**c- Semiconductor**: It is a material which under little stimulation (heat or Elect. Field) converts from insulator to a conductor. Ex: germanium, silicon etc.

**Remark:**

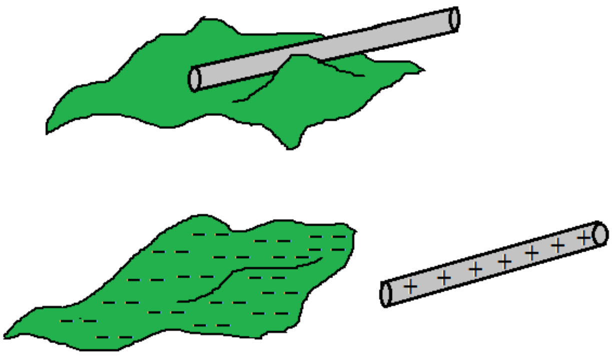
When some charges is given to surface of a conductor then due to property of conduction that charges distributed uniformly at the surface of that conductor. But if we give some charges to insulators then due to its property the charges does not distributed to its surface. They remain at the same point where we give that charges.

### III.4- Types of charging

The observations show that objects can be "charged'' by rubbing, by contact between charged and uncharged objects and by induction.

#### III.4.1- By rubbing method

When we rub two substances then due to friction, heat is produced. Then one of substance absorbs that heat and emit electron, and second one absorb that electron. The one which emit electron due to lack of electron it become positive charged substance and second one due to excess of electron become negative charged substance.

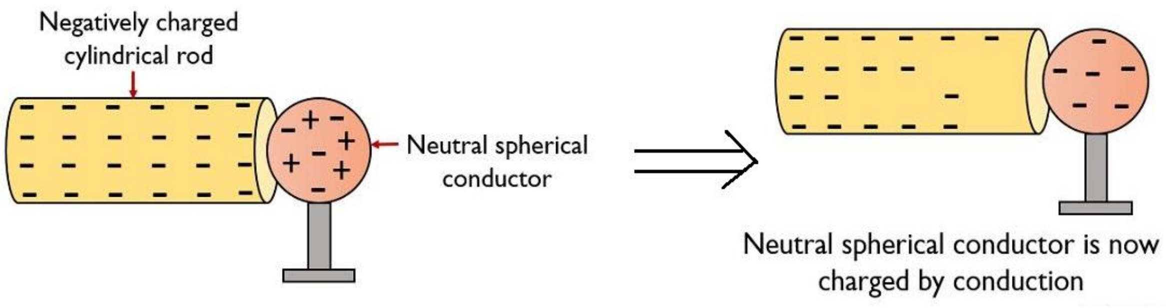


The following table represents negative and positive charge body when it is rubbed.

| Positive charge | Negative charge |
|---|---|
| Glass rod | Silk cloth |
| Flannel or cat skin | Ebonite rod |
| Woollen cloth | Amber rod |
| - | Plastic seat |

#### III.4.2- By contact method

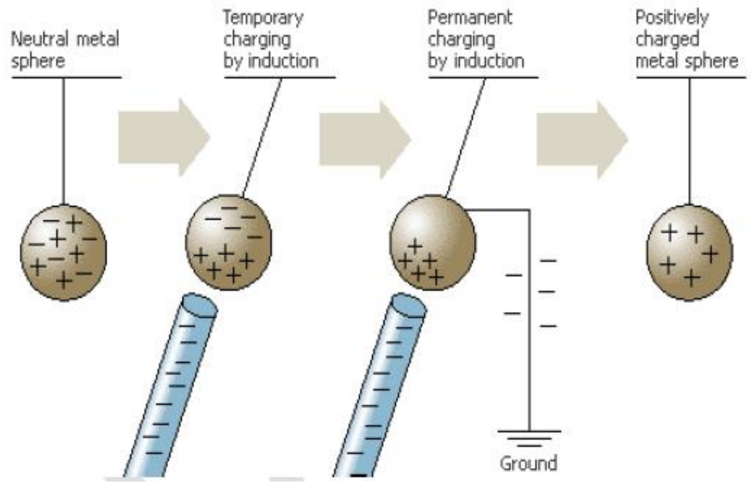
When two identical shape and size conductor touches each other. Where one of conductor is charged and second one is neutral. Then by touching charge will transfer form charged conductor to neutral conductor, and by this both will have same charge.




### III.4.3- By induction method

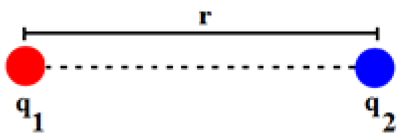
Induction charging is a charging method in which a neutral object in the presence of a negatively or positively charged body is charged without actually touching another charged object.

The metal sphere is mounted on an insulating stand. The sphere has a mixture of positive and negative charges. When a charged object is brought closer, the charges in it rearrange themselves (*like charges repel and unlike attract*) on the sides of the sphere. Then the sphere is earthed and the free charges (not attracted to those of the external object) either move to the earth or get neutralized by electrons that come from the earth. Once the earth connection is removed, the sphere is positively or negatively charged - without being in contact with the charged external object.




## IV- Electric Force - Coulumb's Law

Consider a system of two point charges, $q_1$ and $q_2$, separated by a distance r in vacuum.




Electric Force between two stationery point charges is directly proportional to the product of the charges, inversely proportional to the square of the distance between them and acts along the straight line joining the two charges.

$$F = k\frac{|q_1 . q_2|}{r^2}$$

In SI the unit of charge is Coulomb (C) and the Coulomb constant $k$ is given by

$$k = \frac{1}{4\pi\epsilon_0} = 9 \times 10^9\ Nm^2/C^2$$

$\varepsilon_0$: vacuum permittivity ($\varepsilon_0 = 8.85 \times 10^{-12}$ F/m).

Noted that the electrostatic force is a vector quantity as it has both magnitude and direction:

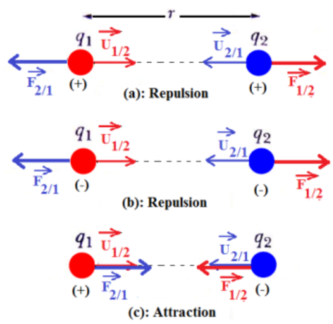
$$\vec{F} = k\frac{q_1 . q_2}{r^2}\ \vec{u}$$

$\vec{u}$ is the unit vector pointing from the source charge to the test charge. $\vec{u} = \frac{\vec{r}}{r}$

$$\Rightarrow \vec{F} = k\frac{q_1 . q_2}{r^3}\ \vec{r}$$

The force exerted by $q_1$ on $q_2$ is given by: $\overrightarrow{F_{1/2}} = k\frac{q_1 . q_2}{r^2}\ \overrightarrow{u_{1/2}}$

The force exerted by $q_2$ on $q_1$ is given by: $\overrightarrow{F_{2/1}} = k\frac{q_1 . q_2}{r^2}\ \overrightarrow{u_{2/1}}$




$$\overrightarrow{u_{1/2}} = -\overrightarrow{u_{2/1}} \quad \Rightarrow \quad \overrightarrow{F_{1/2}} = -\overrightarrow{F_{2/1}}$$

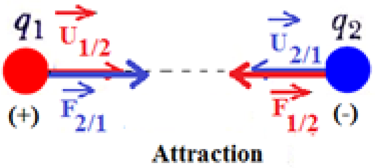
According to the fundamental **Coulomb's law**:

- Like charges $\Rightarrow$ the force is positive $\Rightarrow$ repulsion force.
- Unlike charges $\Rightarrow$ the force is negative $\Rightarrow$ attraction force.

**Example 1**

A point charge of + 3 μC is 12 cm distant from a second point charge of – 1.5 μC. Calculate the magnitude of the force on each charge.

Being of opposite signs, the two charges attract one another.




1- The electrostatic force between the two charged.

$$\boldsymbol{F_e = F_{1/2} = F_{2/1} = k\frac{|q_1.q_2|}{r^2} = 9 \times 10^9 \frac{|3 \times 10^{-6}.(-1.5 \times 10^{-6})|}{(12 \times 10^{-2})^2}}$$

$$\Rightarrow \boldsymbol{F_e = 2.81\ N}$$

$$\overrightarrow{F_{1/2}} = -2.81\,\vec{\imath} \qquad , \qquad \overrightarrow{F_{2/1}} = 2.81\,\vec{\imath}$$

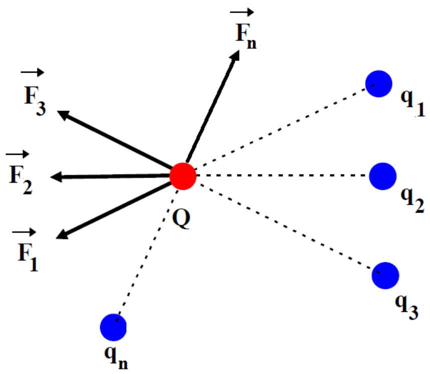
Each charge experiences a force of attraction of magnitude 2.81 N.

**- Vector superposition of electric forces**

If several point charges $q_1$, $q_2$, $q_3$, … $q_n$ simultaneously exert electric forces on a charge Q, the resultant force is:

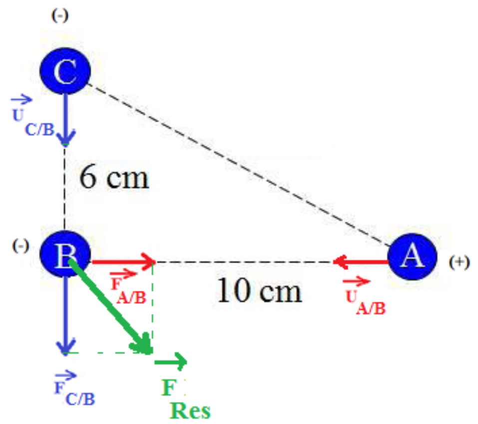
$$\overrightarrow{F_{Res}} = \overrightarrow{F_1} + \overrightarrow{F_2} + \overrightarrow{F_3} + \cdots\cdots\cdots\cdots + \overrightarrow{F_n}$$

$$\overrightarrow{F_{Res}} = \sum_{i=1}^{n} k\frac{Q.q_i}{r_i^2}\,\overrightarrow{u_i}$$




**Example 2**

Three charges A(4 µC ), B(-6 µC) and C(-2 µC) are placed at the vertices of a right angle triangle ABC. BA=a=10 cm, BC=b=6 cm. Find the net force on charge B due to C and A charges.




$$\overrightarrow{F_{Res}} = \overrightarrow{F_{A/B}} + \overrightarrow{F_{C/B}}$$

$$\overrightarrow{F_{A/B}} = k\frac{q_A \cdot q_B}{AB^3}\,\overrightarrow{AB}$$

$$\overrightarrow{F_{C/B}} = k\frac{q_C \cdot q_B}{CB^3}\,\overrightarrow{CB}$$

**A(a,0), B(0,0), C(0,b)** $\Rightarrow \overrightarrow{AB} = -a\,\vec{\imath}\;, \overrightarrow{CB} = -b\,\vec{\jmath}$ **, AB =** $a$ **, CB = b**

$$\overrightarrow{F_{A/B}} = k\frac{q_A \cdot q_B}{(a)^3}(-a\,\vec{\imath}) = -k\frac{q_A \cdot q_B}{a^2}\,\vec{\imath}$$

$$\overrightarrow{F_{C/B}} = k\frac{q_C \cdot q_B}{b^3}(-b\,\vec{\jmath}) = -k\frac{q_C \cdot q_B}{b^2}\,\vec{\jmath}$$

$$\overrightarrow{F_{Res}} = \overrightarrow{F_{A/B}} + \overrightarrow{F_{C/B}} = -k\frac{q_A \cdot q_B}{a^2}\,\vec{\imath} - k\frac{q_C \cdot q_B}{b^2}\,\vec{\jmath}$$

$$\overrightarrow{F_{Res}} = -k.q_B\left[\left(\frac{q_A}{a^2}\right)\vec{\imath} + \left(\frac{q_C}{b^2}\right)\vec{\jmath}\right]$$

$$\overrightarrow{F_{Res}} = -(9\times10^{9}).(-6\times10^{-6})\left[\left(\frac{4\times10^{-6}}{(10\times10^{-2})^2}\right)\vec{\imath} + \left(\frac{-2\times10^{-6}}{(6\times10^{-2})^2}\right)\vec{\jmath}\right]$$

$$\overrightarrow{F_{Res}} = 21.6\,\vec{\imath} - 29.7\,\vec{\jmath} \Rightarrow F_{Res} = 36.72\ N$$

## V- Electric Field due to a discrete charge distribution

The charge $Q$ produces an electric field everywhere in the surrounding.

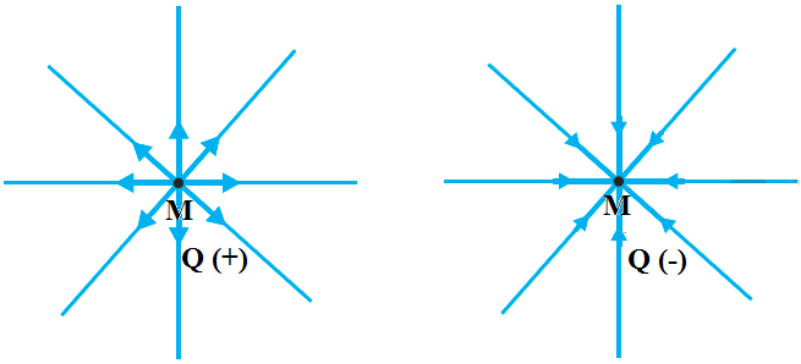
The electric field produced by the charge $Q$ at a point r is given as:

$$\vec{E} = k\frac{Q}{r^2}\,\vec{u}$$

$$\vec{E} = k\frac{Q}{r^3}\,\vec{r}$$

The SI unit of electric field is N/C or V/m.

For a positive charge, the electric field will be directed radially outwards from the charge. On the other hand, if the source charge is negative, the electric field vector, at each point, points radially inwards. Thus at equal distances from the charge $Q$, the magnitude of its electric field E is same.




(a) (b)

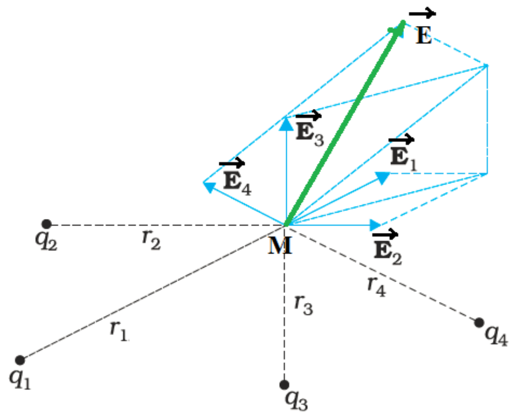
When another charge $q$ is brought at some point M, the field there acts on it and produces a force.

$$\vec{F} = k\frac{Q.q}{r^2}\,\vec{u}$$

$$\Rightarrow \vec{F} = q\,\vec{E}$$

**- Electric field due to a system of charges**

Consider a system of charges $q_1$, $q_2$, ..., $q_n$ with position vectors $r_1$, $r_2$, ..., $r_n$




$$\vec{E} = \vec{E_1} + \vec{E_2} + \vec{E_3} + \cdots\cdots\cdots\cdots + \vec{E_n}$$

$$\vec{E} = k\sum_{i=1}^{n}\frac{q_i}{r_i^2}\,\vec{u_i}$$

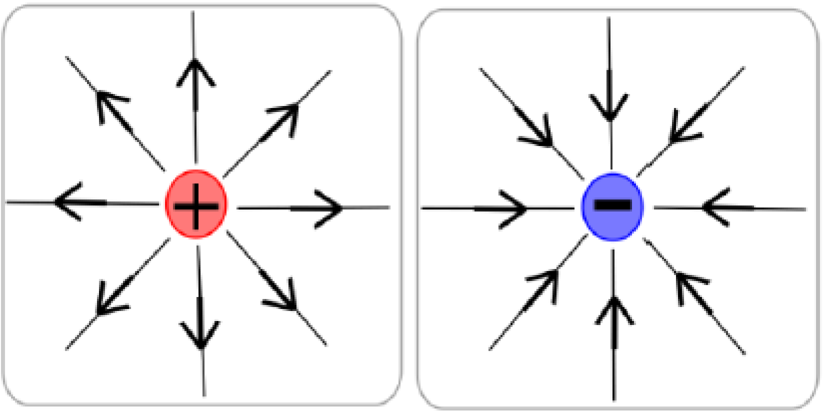
**- Electric field lines**

The electric field is represented by some imaginary and continues lines. These lines are called electric field lines. The properties of electric field lines are as below:

1- These lines are imaginary and continue lines without any break.

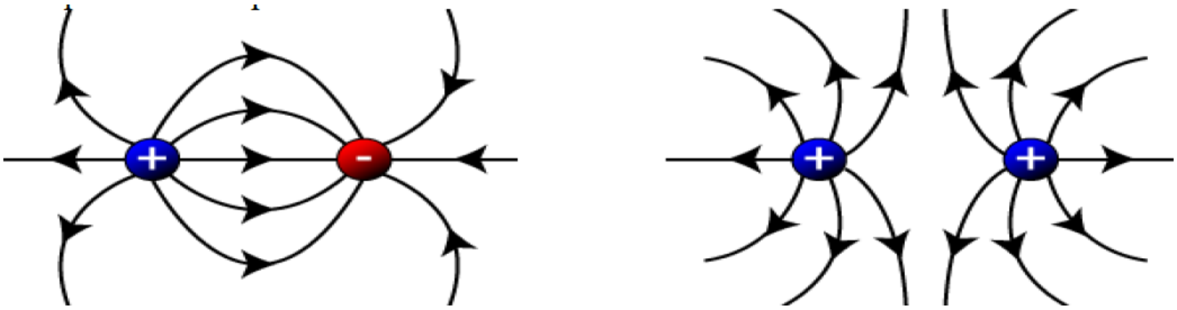
2- The electric field lines are start from the positive charge and end at negative charge.

2- For a point charge, the electric field lines is radially move outside or inside according to nature of charge.




4- The field lines between the unlike charge then have a nature to being compressed. It represents attraction.

5- The field lines between the like charges then have a nature to being expansion. It represents repulsion.




6- Two electric field lines cannot intersect each other.

7- Electric field lines does not form close loop.

**VI- Electrostatic potential due to a discrete charge distribution**

$$W_{r_1 \to r_2} = \int_{r_1}^{r_2} dW = \int_{r_1}^{r_2} \vec{F} . d\vec{r}$$

$$\vec{F} = q. \vec{E}$$

Noted that the electrostatic field ($\vec{E}$) is a conservative field. Therefore, the circulation $\zeta = \oint \vec{\mathbf{E}}.\boldsymbol{d\vec{r}}$ for any closed path is zero

$$W_{r_1 \to r_2} = q\int_{r_1}^{r_2} \vec{\mathbf{E}}.\,d\vec{\mathbf{r}} = -\Delta\, U$$

$$\Delta\, U = q.\Delta V$$

$$W_{r_1 \to r_2} = q\int_{r_1}^{r_2} \vec{\mathbf{E}}.\,d\vec{\mathbf{r}} = -\Delta\, U \Rightarrow W_{r_1 \to r_2} = q\int_{r_1}^{r_2} \vec{\mathbf{E}}.\,d\vec{\mathbf{r}} = -\,q.\Delta V$$

$$\Rightarrow \Delta V = -\int_{r_1}^{r_2} \vec{\mathbf{E}}.\,d\vec{\mathbf{r}}$$

$$\Rightarrow dV = -\vec{\mathbf{E}}.\,d\vec{\mathbf{r}}$$

$$\Rightarrow E_x = -\frac{\partial V}{\partial x}\ ,\ E_y = -\frac{\partial V}{\partial y}\ ,\ E_z = -\frac{\partial V}{\partial z}$$

$$\Rightarrow \vec{\mathbf{E}} = -\vec{\nabla}\,\mathbf{V} = -\overrightarrow{\mathbf{grad}}\,\mathbf{V}$$

The potential due to a point charge q is:

$E = -\frac{\partial V}{\partial r} \Rightarrow dV = -Edr \Rightarrow V = -\int k\frac{q}{r^2}\,dr \Rightarrow V = k\frac{q}{r} + \mathrm{C}$

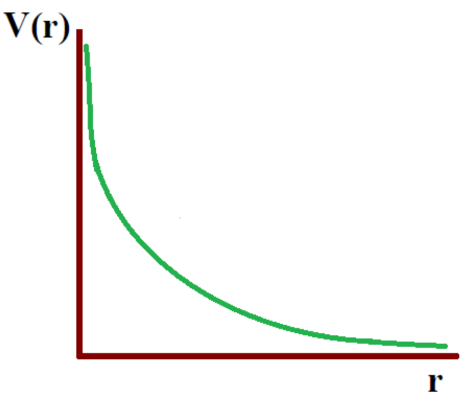
$V(\infty) = 0 \Rightarrow \mathrm{C} = 0 \Rightarrow V = k\frac{q}{r}$

r is the distance of the charge from the point of interest. The electrical potential is **zero** at "infinity".

Furthermore, for a set of point charges $q_1$, $q_2$, $q_3$, . . . the electrical potential is:

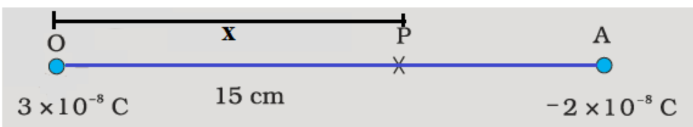
$$V = k\sum_{i=1}^{n}\frac{q_i}{r_i}$$

Where, $r_i$ is the distance of each charge from the point of interest.




**Example**

Two charges $3 \times 10^{-8}\ C$ and $2 \times 10^{-8}\ C$ are located 15 cm apart. At what point P on the line joining the two charges is the electric potential zero? Take the potential at infinity to be zero.



$$V = k\sum_{i=1}^{n}\frac{q_i}{r_i} = k(\frac{q_1}{r_1}+\frac{q_2}{r_2}) = 9\times10^9\,(\frac{3\times10^{-8}}{x}-\frac{2\times10^{-8}}{15\times10^{-2}-x}) = 0 \Rightarrow \frac{3\times10^{-8}}{x}-\frac{2\times10^{-8}}{15\times10^{-2}-x} = 0$$

$$\Rightarrow \frac{3}{x} = \frac{2}{15\times10^{-2}-x} \Rightarrow 5\,x = 45\times10^{-2} \Rightarrow x = 9\times10^{-2}\text{ m} = 9\text{ cm}$$

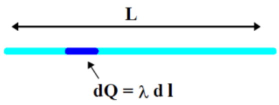
**VII- Electric Field due to a continuous charge distribution**

The electric field from a continuous distribution of charge is the superposition (i.e. integral) of all the (infinite) contributions from each infinitesimal dQ as follows:

$$d\vec{E} = k\frac{dQ}{r^2}\,\vec{u}$$

$$\vec{E} = \int d\vec{E} = k\int\frac{dQ}{r^2}\,\vec{u}$$

**- Charge Distributions:**




- Linear charge density $\lambda$

  $\lambda =$ **charge/unit length**

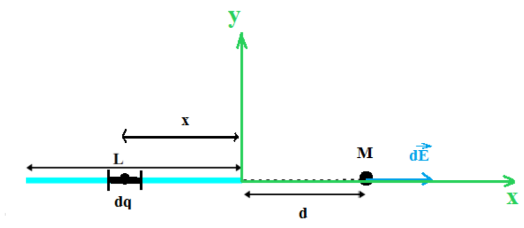
$$dQ = \lambda\,.dl$$

$$\vec{E} = \int d\vec{E} = k\int\frac{\lambda\,dl}{r^2}\,\vec{u}$$

If $\lambda = \lambda_0$ is constant $\Rightarrow$ dQ = $\lambda_0.dl \Rightarrow$ Q = $\lambda_0$ L, where L is the length.

**Example:**

A total amount of charge **Q** is uniformily distributed along a thin **straight rod** of length **L**. What is the electric field at a point **M** on the x-axis a distance **d** from the end of the rod?




$$dQ = \lambda\,.dx$$

$$\vec{E} = \int d\vec{E} = k\int\frac{\lambda\,dx}{r^2}\,\vec{u} = k\int\frac{\lambda\,dx}{(x+d)^2}\,\vec{u} \quad (r = x+d)$$

$$x + d = r \Rightarrow dx = dr$$

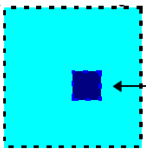
$$\vec{E} = k\int_{L+d}^{L} \frac{\lambda\, dr}{r^2}\,\vec{\imath}$$

$$\vec{E} = -k\left[\frac{\lambda}{r}\right]_{d}^{L+d}\vec{\imath} = -k\lambda\left[\frac{1}{L+d} - \frac{1}{d}\right]\vec{\imath} = k\lambda\left[\frac{L}{d(L+d)}\right]\vec{\imath}$$

- **Surface charge density σ:**

**σ =charge/unit area**

$$dQ = \sigma\,.\,dS \Rightarrow Q = \iint \sigma ds$$




$$\vec{E} = \int d\vec{E} = k\iint \frac{\sigma\,.\,dS}{r^2}\,\vec{u}$$

If $\boldsymbol{\sigma} = \boldsymbol{\sigma}_0$ is constant ⇒ dQ = $\boldsymbol{\sigma}_0 . dS$ ⇒ Q = $\boldsymbol{\sigma}_0$ S, where S is the surface (area).

**Example**

A total amount of charge Q is uniformily distributed on the surface of a disk of radius R. What is the electric field at point M on the z-axis a distance z from the center of the disk? Deduce the electric field in the case of infinite plane.

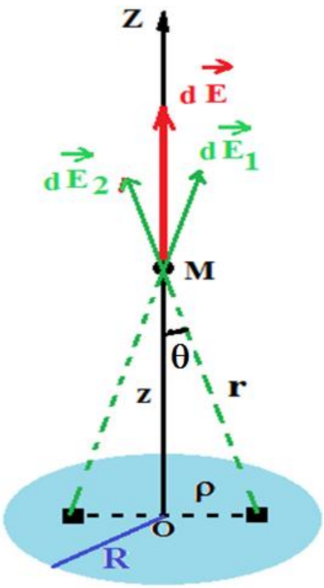
$$dQ = \sigma\,.\,dS = \sigma\,\rho d\rho d\varphi$$

$$\vec{E} = \int d\vec{E} = k\iint \frac{\sigma\,\rho d\rho d\varphi}{r^2}\,\vec{u}$$

$$r^2 = \rho^2 + z^2$$

$$\vec{u_1} = sin\theta\ \vec{u_\rho} + cos\theta\,\vec{k}$$

$$\vec{u_2} = -sin\theta\ \vec{u_\rho} + cos\theta\,\vec{k}$$




$$\vec{E_1} = k\iint \frac{\sigma\,\rho d\rho d\varphi}{\rho^2 + z^2}\left(sin\theta\ \vec{u_\rho} + cos\theta\,\vec{k}\right)$$

$$\vec{E_2} = k\iint \frac{\sigma\,\rho d\rho d\varphi}{\rho^2 + z^2}\left(-sin\theta\ \vec{u_\rho} + cos\theta\,\vec{k}\right)$$

$$\vec{E} = \vec{E_1} + \vec{E_2} = k\iint \frac{\sigma\,\rho d\rho d\varphi}{\rho^2 + z^2}\left(2\,cos\theta\,\vec{k}\right)$$

$$\cos\theta = \frac{z}{r} = \frac{z}{\sqrt{\rho^2 + z^2}}$$

$$\vec{E} = \vec{E_1} + \vec{E_2} = k \iint \frac{\sigma\, \rho d\rho d\varphi}{\rho^2 + z^2} \left(2\, \frac{z}{\sqrt{\rho^2 + z^2}}\, \vec{k}\right)$$

$$\vec{E} = \vec{E_1} + \vec{E_2} = 2kz \iint \frac{\sigma\, \rho d\rho d\varphi}{(\rho^2 + z^2)^{3/2}}\, \vec{k}$$

$$\vec{E} = \vec{E_1} + \vec{E_2} = 2k\sigma z \int_0^R \frac{\rho d\rho}{(\rho^2 + z^2)^{3/2}} \int_0^\pi d\varphi\, \vec{k}$$

$$\vec{E} = \vec{E_1} + \vec{E_2} = 2\pi k\sigma z \int_0^R \frac{\rho d\rho}{(\rho^2 + z^2)^{3/2}} \vec{k}$$

$$\vec{E} = \vec{E_1} + \vec{E_2} = 2\pi k\sigma z \left[\frac{-1}{\sqrt{\rho^2 + z^2}}\right]_0^R \vec{k}$$

$$\vec{E} = \vec{E_1} + \vec{E_2} = 2\pi k\sigma\, z\left(\frac{-1}{\sqrt{(R^2+z^2)}} + \frac{1}{z}\right) \vec{k} \Rightarrow \vec{E} = 2\pi k\sigma \left(1 - \frac{z}{\sqrt{(R^2+z^2)}}\right) \vec{k}$$

**2-** The electric field in the case of infinite plane.

Infinite plane $\Rightarrow R \to \infty \Rightarrow \vec{E} = 2\pi k\sigma \left(1 - \frac{z}{\sqrt{(\infty^2+z^2)}}\right) \vec{k}$

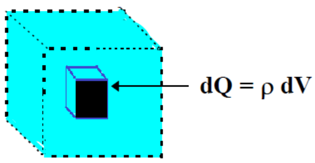
$$\Rightarrow \vec{E} = 2\pi k\sigma\, \vec{k} \Rightarrow \vec{E} = \frac{2\pi\sigma}{4\pi\varepsilon_0} \vec{k} \Rightarrow \vec{E} = \frac{\sigma}{2\varepsilon_0} \vec{k}$$

- **Volume charge density** $\rho$**:**

$\rho$ = **charge/unit volume**

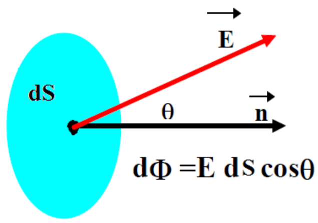
$$dQ = \rho \,.\, dV$$

$$\vec{E} = \int d\vec{E} = k \iiint \frac{\rho \,.\, dV}{r^2}\, \vec{u}$$




If $\rho = \rho_0$ is constant $\Rightarrow dQ = \rho_0 . dV \Rightarrow Q = \rho_0\, V$, where V is the volume.

**VIII- Electric field flow**




The total number of electric lines passing through the surface ds, is called electric flux $\emptyset$.
So, the electric flux through the infinitesimal area dS is equal to:

$$d\emptyset = \vec{E} . d\vec{S}$$

$$d\vec{S} = ds . \vec{n}$$

$$d\emptyset = \vec{E} . ds . \vec{n}$$

$$d\emptyset = E.ds.cos\theta$$

$\theta$: angle between surface area vector ($d\vec{S}$) and ($\vec{E}$).

So, the total electric flux through a closed surface is:

$$\emptyset = \oint_S \vec{E}.d\vec{S}$$

The net flux through the closed surface is positive or negative, depending on whether is predominantly outward or inward at the surface. At points on the surface where is inward, is negative.

**IX- Gauss's theorem**

The electric flux through any closed surface is proportional to the net charge enclosed.

$$\emptyset = \oint_S \vec{E}.d\vec{S} = \frac{Q_{enclosed}}{\varepsilon_0}$$

The units for the electric flux are **$Nm^2/C$.**

For the discrete case the total charge enclosed is the sum over all the enclosed charges:

$$Q_{enclosed} = \sum_{i=1}^{N} q_i$$

For the continuous case the total charge enclosed is the integral of the charge density enclosed by the surface S:

$$Q_{enclosed} = \int dq$$

**- Important facts about Gaussian surface**

1. Any hypothetical closed surface enclosing a charge is called the Gaussian surface of that charge.

2. Gauss theorem is valid for a closed surface of any shape and for any general charge distribution. But in our problem–solving we will have the most use for surfaces which have a high degree of symmetry, for example spheres and cylinders. Gauss's law is often useful towards a much easier calculation of the electrostatic field when the system has some symmetry.

3. For convenience, Gaussian surface shape should be consider, where magnitude of electric field is same at every individual surface of any closed surface.

4. The term q on the right of gauss law, includes the sum of all charges enclosed by the surface. The charges may be located anywhere inside the surface.

Basically, there are only three types of symmetry that allow Gauss's law to be used to deduce the electric field. They are:

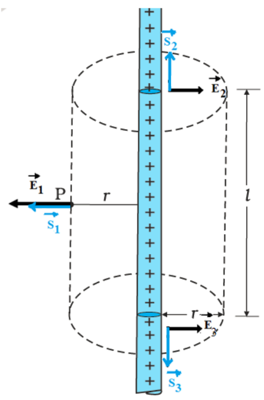
- A charge distribution with spherical symmetry
- A charge distribution with cylindrical symmetry
- A charge distribution with planar symmetry

## X- Application of Gauss's law

### X.1- Field due to an infinite line charge

Consider an infinity long straight wire having linear charge density λ.

- We have to consider a Gaussian surface: according to symmetry, electric field around wire is uniform in cylindrical shape. So, Gaussian surface should be cylindrical shaped (we consider cylinder of r radius).
- The electric field due to a long straight wire is always radially perpendicular to its length.
- For finding magnitude of electric field, we consider a Gaussian surface having three surface:
    - first surface: curved surface of cylinder,
    - second surface: upper circular disc of cylinder,
    - third surface: lower circular disc of cylinder.




According to Gauss law :

$$\emptyset = \oint_S \vec{E}.d\vec{S} = \frac{Q_{enclosed}}{\varepsilon_0}$$

$$\Rightarrow \emptyset = \int \overrightarrow{E_1}.\overrightarrow{dS_1} + \int \overrightarrow{E_2}.\overrightarrow{dS_2} + \int \overrightarrow{E_3}.\overrightarrow{dS_3}$$

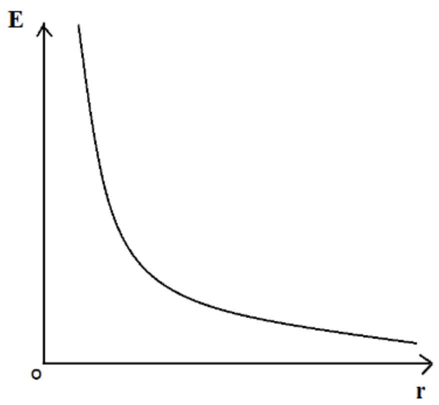
$$\emptyset = \int E_1.dS_1 \cos 0 + \int E_2.dS_2 \cos 90 + \int E_3.dS_3 \cos 90 = \frac{q}{\varepsilon_0}$$

$$\Rightarrow \int E_1.dS_1 = \frac{q}{\varepsilon_0} \Rightarrow E_1 \int dS_1 = \frac{q}{\varepsilon_0} \Rightarrow E_1 \times 2\pi r L = \frac{q}{\varepsilon_0}$$

$$\Rightarrow E_1 = \frac{q}{2\pi\varepsilon_0 L r} = \frac{\lambda L}{2\pi\varepsilon_0 L r} \Rightarrow E_1 = \frac{\lambda}{2\pi\varepsilon_0 r} \Rightarrow E_1 = \frac{2\,k\,\lambda}{r}$$

$$\Rightarrow \vec{E} = \frac{2\,k\,\lambda}{r}\ \overrightarrow{u_\rho}$$

Where $\overrightarrow{u_\rho}$ is the radial unit vector in the plane normal to the wire parallel through the point (**E** is directed outward if λ is positive and inward if λ is negative.



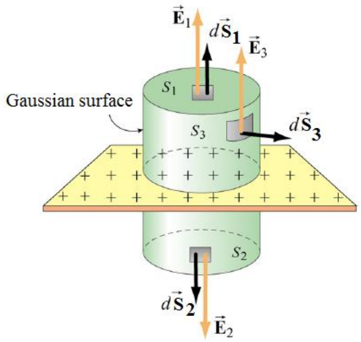

**X.2- Field due to a uniformly charged infinite plane**

Consider an infinitely large non-conducting plane in the xy-plane with uniform surface charge density σ. Determine the electric field everywhere in space.

1- An infinitely large plane possesses a planar symmetry.

2- Since the charge is uniformly distributed on the surface, the electric field E must point perpendicularly away from the plane, $\vec{E} = E\ \vec{k}$. The magnitude of the electric field is constant on planes parallel to the non-conducting plane.

We choose our Gaussian surface to be a cylinder, which consists of three parts: two end-caps, and a curved side.




3- Since the surface charge distribution on is uniform, the charge enclosed by the Gaussian surface is $Q_{enclosed} = \sigma S$, where S= $S_1$=$S_2$ is the area of the end-caps.

4- The total flux through the Gaussian surface flux is:

$$\emptyset = \oint_S \vec{E}.d\vec{S} = \frac{Q_{enclosed}}{\varepsilon_0}$$

$\Rightarrow \emptyset = \int \overrightarrow{E_1}.d\overrightarrow{S_1} + \int \overrightarrow{E_2}.d\overrightarrow{S_2} + \int \overrightarrow{E_3}.d\overrightarrow{S_3}$

$\emptyset = \int E_1.dS_1 \cos 0 + \int E_2.dS_2 \cos 0 + \int E_3.dS_3 \cos 90 = \frac{q}{\varepsilon_0}$

$\Rightarrow \int E_1.dS_1 + \int E_2.dS_2 = \frac{q}{\varepsilon_0} \Rightarrow E_1 \int dS_1 + E_2 \int dS_2 = \frac{q}{\varepsilon_0}$

Since the two ends are at the same distance from the plane, by symmetry, the magnitude of the electric field must be the same: $E_1 = E_2 = E$. Hence, the total flux can be rewritten as:

$$2\,E \times \pi r^2 = \frac{q}{\varepsilon_0}$$

$$\Rightarrow E = \frac{q}{2\pi\varepsilon_0 r^2} = \frac{\sigma\pi r^2}{2\pi\varepsilon_0 r^2} \Rightarrow E = \frac{\sigma}{2\varepsilon_0}$$

In unit-vector notation, we have:

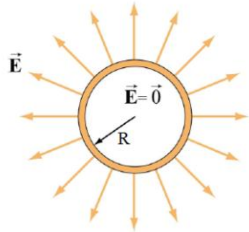
$$\vec{E} = \begin{cases} \dfrac{\sigma}{2\varepsilon_0}\vec{k}, & z > 0 \\ -\dfrac{\sigma}{2\varepsilon_0}\vec{k}, & z < 0 \end{cases}$$

Thus, we see that the electric field due to an infinite large non-conducting plane is uniform in space. The result is the same as that obtained using Coulomb's law.

### X.3- Field due to a uniformly charged spherical Shell

A thin spherical shell of radius *R* has a charge $Q^+$ evenly distributed over its surface. Find the electric field both inside and outside the shell.

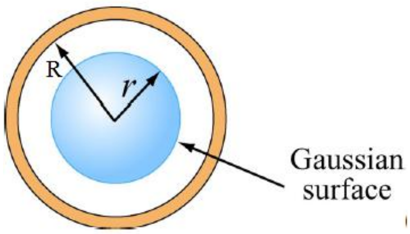
The charge distribution is spherically symmetric, with a surface charge density $\sigma = \frac{Q}{S} = \frac{Q}{4\pi R^2}$, where $S = 4\pi R^2$ is the surface area of the sphere. The electric field must be radially symmetric and directed outward. We treat the regions and separately.




***Case 1: r < R***

We choose our Gaussian surface to be a sphere of radius *r* < R.

The charge enclosed by the Gaussian surface is $Q_{enclosed} = 0$ since all the charge is located on the surface of the shell. Thus, from Gauss's law:




$$\emptyset = \oiint \vec{E}.d\vec{S} = \frac{Q_{enclosed}}{\varepsilon_0} = 0$$

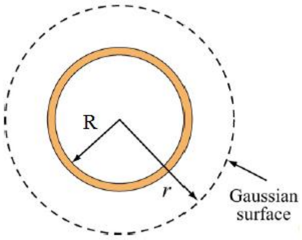
$$\mathbf{dS} = \boldsymbol{r^2}\ \mathbf{sin}\boldsymbol{\theta}\ \mathbf{d}\boldsymbol{\theta}\ \mathbf{d}\boldsymbol{\Phi} \Rightarrow \mathrm{S} = \boldsymbol{r^2} \int_0^{\pi} \mathbf{sin}\boldsymbol{\theta}\ \mathbf{d}\boldsymbol{\theta} \int_0^{2\pi} \mathbf{d}\boldsymbol{\Phi}$$

$$\Rightarrow \mathrm{S} = 4\pi r^2$$

$$\Rightarrow E \times 4\pi r^2 = \frac{q}{\varepsilon_0} = 0 \Rightarrow \boldsymbol{E} = \mathbf{0}$$

***Case 2: r > R***

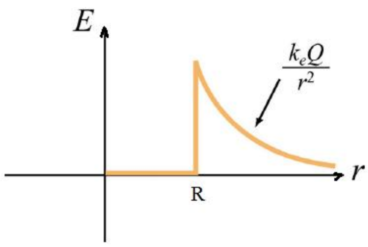
In this case, the Gaussian surface is a sphere of radius r. Since the radius of the "Gaussian sphere" is greater than the radius of the spherical shell, all the charge is enclosed:




$$\emptyset = \oiint \vec{E}.d\vec{S} = \frac{Q_{enclosed}}{\varepsilon_0}$$

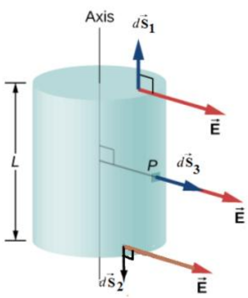
$$\Rightarrow E \times S = \frac{Q_{enclosed}}{\varepsilon_0} \Rightarrow E \times 4\pi r^2 = \frac{Q}{\varepsilon_0}$$

$$\Rightarrow \boldsymbol{E} = \frac{\boldsymbol{Q}}{\boldsymbol{4\pi\varepsilon_0}} \frac{\mathbf{1}}{\boldsymbol{r^2}} = \boldsymbol{kQ}\frac{\mathbf{1}}{\boldsymbol{r^2}}$$



### X.4- Field due to a infinite cylinder of uniform volume charge density

A cylinder of radius *R* has a charge $Q^+$ evenly distributed over its volume. Find the electric field both inside and outside the cylinder.




We consider a Gaussian surface in the form of a cylinder of radius r.

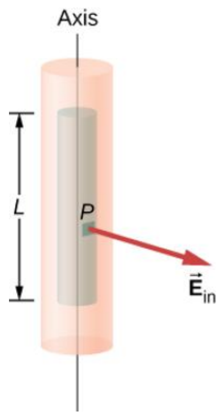
The electric field has the same magnitude at every point of the cylinder and is directed outward.

$$\emptyset = \oint_S \vec{E}.d\vec{S} = \frac{Q_{enclosed}}{\varepsilon_0}$$

$$\Rightarrow \emptyset = \int \overrightarrow{E_1}.\overrightarrow{dS_1} + \int \overrightarrow{E_2}.\overrightarrow{dS_2} + \int \overrightarrow{E_3}.\overrightarrow{dS_3}$$

$$\emptyset = \int E_1.dS_1 \cos 90 + \int E_2.dS_2 90 + \int E_3.dS_3 \cos 0 = E_3 \int dS_3 = E_3 .S_3$$

$$\Rightarrow \emptyset = \boldsymbol{E_3.2\pi r L = E.2\pi r L}$$



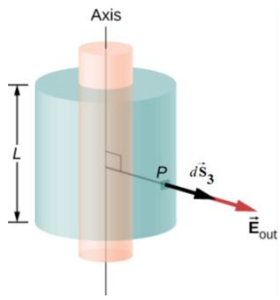

***Case 1: r<R***

$$\emptyset = \oint_S \vec{E}.d\vec{S} = \frac{Q_{enclosed}}{\varepsilon_0}$$

$$\Rightarrow E.2\pi r L = \frac{Q_{enclosed}}{\varepsilon_0} \Rightarrow E.2\pi r L = \frac{\iiint \rho \, .dV}{\varepsilon_0}$$

$$\Rightarrow E.2\pi r L = \frac{\rho \int_0^r r dr \int_0^{2\pi} \theta \, d\theta \int_0^L dz}{\varepsilon_0} = \frac{\rho(\pi r^2 L)}{\varepsilon_0}$$

$$\Rightarrow E.2\pi r L = \frac{\rho(\pi r^2 L)}{\varepsilon_0} \Rightarrow E = \frac{\rho}{2\varepsilon_0} \mathbf{r}$$




***Case 2: r>R***

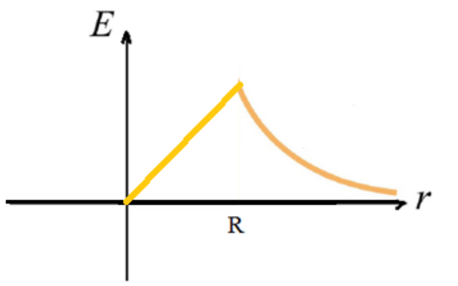
$$\emptyset = \oint_S \vec{E}.d\vec{S} = \frac{Q_{enclosed}}{\varepsilon_0}$$

$$\Rightarrow E.2\pi r L = \frac{Q_{enclosed}}{\varepsilon_0} \Rightarrow E.2\pi r L = \frac{\iiint \rho \, .dV}{\varepsilon_0}$$

$$\Rightarrow E.2\pi r L = \frac{\rho \int_0^R r dr \int_0^{2\pi} \theta \, d\theta \int_0^L dz}{\varepsilon_0} = \frac{\rho(\pi R^2 L)}{\varepsilon_0}$$

$$\Rightarrow E.2\pi r L = \frac{\rho(\pi R^2 L)}{\varepsilon_0} \Rightarrow E = \frac{\rho R^2}{2\varepsilon_0} \frac{1}{r}$$

Q= $\rho(\pi R^2 L) \Rightarrow E = \frac{2kQ}{L}\frac{1}{r}$

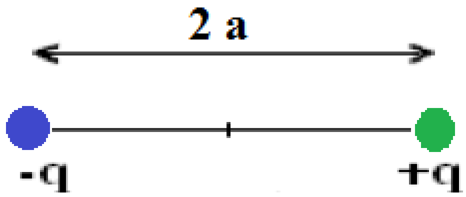


## XI- Electric dipole

### XI.1- Definition

An electric dipole is a pair of equal and opposite charges (+ q) and (– q) separated by a distance 2a .

The midpoint of location of –q and q is called the centre of the dipole. Example - HCl, H2O

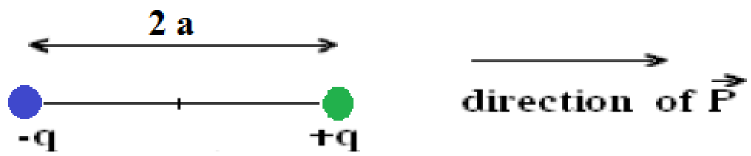



### XI.2- Electric dipole moment

Product of electric charge and distance between two charges is called electric dipole moment . It is denoted by $\vec{P}$ . It is a vector quantity, and the direction of electric dipole is from -q to +q .

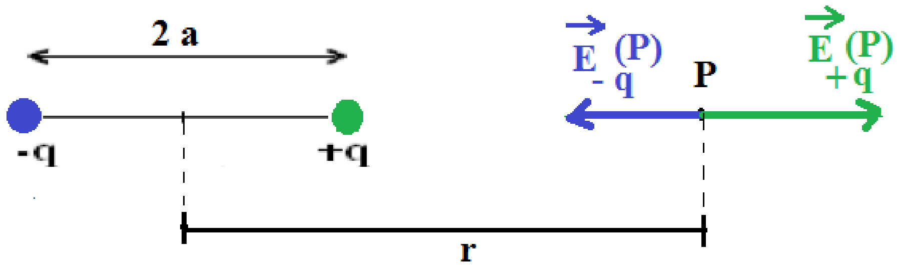
$$\vec{P} = q \times 2\,\vec{a}$$

Its unit is C.m




### XI.3- Electric field at axis due to electric dipole

Consider, two charges +q and -q is located at 2a distance, and a point P is located at r distance from centre of dipole and axis of dipole.




Electric field on P point due to +q charge $\vec{E}_{+q}(P)$ (towards + q to P) is :

$$\vec{E}_{+q}(P) = k\left[\frac{q}{(r-a)^2}\right]\vec{\imath}$$

Electric field on P point due to -q charge $\vec{E}_{-q}(P)$(towards P to − q) is:

$$\vec{E}_{-q}(P) = -\,k\left[\frac{q}{(r+a)^2}\right]\vec{\imath}$$

Then total electric field is:

$$\vec{E}(P) = \vec{E}_{+q}(P) + \vec{E}_{-q}(P)$$

$$\Rightarrow \vec{E}(P) = k\left[\frac{q}{(r-a)^2}\right]\vec{\imath} - k\left[\frac{q}{(r+a)^2}\right]\vec{\imath} \Rightarrow \vec{E}(P) = k\,q\left[\frac{1}{(r-a)^2} - \frac{1}{(r+a)^2}\right]\vec{\imath}$$

$$\Rightarrow \vec{E}(P) = k\,q\left[\frac{4a\,r}{(r^2-a^2)^2}\right]\vec{\imath}$$

$$\vec{P} = q \times 2\,\vec{a} = q \times 2a\,\vec{\imath} \text{ (from } -\text{q to } +\text{q)} \Rightarrow \Rightarrow \vec{E}(P) = \frac{2kr}{(r^2-a^2)^2}\vec{P}$$

If r >>>>> a then : $\Rightarrow \vec{E}(P) = \frac{2kr}{r^4}\vec{P} \Rightarrow \vec{E}(P) = \frac{2k}{r^3}\vec{P}$

$\vec{E}(P)$ is in the direction of $\vec{P}$ ($\vec{E}_{-q}(P) < \vec{E}_{+q}(P)$)

**XI.4- Electric field at equatorial due to electric dipole**

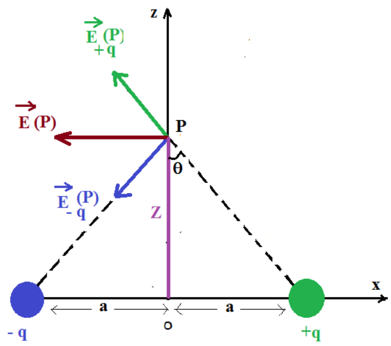
Consider two charges +q and -q is situated at 2a distance, and a point P is located at Z distance from centre point of dipole on distance equatorial axis.

Electric field on P point due to +q charge $\vec{E}_{+q}(P)$ (towards + q to P) is :

$$\vec{E}_{+q}(P) = k\left[\frac{q}{(r^2}\right]\overrightarrow{u_{+q}}$$

Electric field on P point due to -q charge $\vec{E}_{-q}(P)$(towards P to − q) is:

$$\vec{E}_{-q}(P) = -k\left[\frac{q}{r^2}\right]\overrightarrow{u_{-q}}$$




$$\overrightarrow{u_{+q}} = -\sin\theta\ \vec{\imath} + \cos\theta\,\vec{k}$$

$$\overrightarrow{u_{-q}} = \sin\theta\ \vec{\imath} + \cos\theta\,\vec{k}$$

r = $\sqrt{a^2 + z^2}$ , $\sin\theta = \frac{a}{r} = \frac{a}{\sqrt{a^2+z^2}}$

Then total electric field is:

$$\vec{E}(P) = \vec{E}_{+q}(P) + \vec{E}_{-q}(P)$$

$$\Rightarrow \vec{E}(P) = k\frac{q}{r^2}\left[\overrightarrow{u_{+q}} - \overrightarrow{u_{-q}}\right] \Rightarrow \vec{E}(P) = -2\,k\frac{q}{r^2}\sin\theta\ \vec{\imath} = -2\,k\frac{q}{(\sqrt{a^2+z^2})^2}\frac{a}{\sqrt{a^2+z^2}}\ \vec{\imath}$$

$$\Rightarrow \vec{\boldsymbol{E}}(\boldsymbol{P}) = -2\ \boldsymbol{k}\frac{\boldsymbol{q\ a}}{(\boldsymbol{a}^2+\boldsymbol{z}^2)^{\frac{3}{2}}}\quad \vec{\boldsymbol{\imath}}$$

**we have :** $\vec{P} = q\ \times 2a\ \vec{\imath} \Rightarrow \vec{\boldsymbol{E}}(\boldsymbol{P}) = -\ \boldsymbol{k}\frac{\vec{\boldsymbol{P}}}{(\boldsymbol{a}^2+\boldsymbol{z}^2)^{\frac{3}{2}}}$

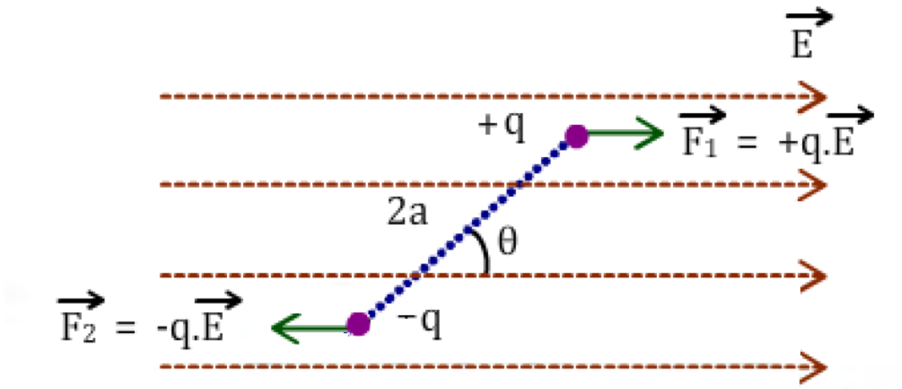
The direction of electric field and dipole moment is opposite to each other.

If Z >>> a

$$\vec{\boldsymbol{E}}(\boldsymbol{P}) = -\ \boldsymbol{k}\frac{\vec{\boldsymbol{P}}}{\boldsymbol{Z}^3}$$

**XI.5- Torque on an electric dipole**

Let an electric dipole be placed in a uniform (external) electric field ($\vec{\boldsymbol{E}}$) with its axis making an angle θ with the direction of field as shown in figure:



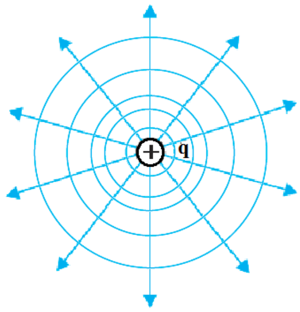

There is a force *q*E on *q* and a force –*q*E on –*q*. The net force on the dipole is zero, since E is uniform. However, the charges are separated, so the forces act at different points, resulting in a torque on the dipole.

$$\vec{\boldsymbol{\tau}} = \vec{\boldsymbol{P}} \wedge \vec{\boldsymbol{E}}$$

$$\boldsymbol{\tau} = \boldsymbol{P}.\boldsymbol{E}.\sin\boldsymbol{\theta}$$

This torque will tend to align the dipole with the field E. When p is aligned with E, the torque is zero.

## XII- Equipotential surfaces




An equipotential surface is a surface with a constant value of potential at all points on the surface.

For a single charge $q$, the potential is given by:

$$\boldsymbol{V = k\frac{q}{r}}$$

This shows that $V$ is a constant if $r$ is constant. Thus, equipotential surfaces of a single point charge are concentric spherical surfaces centered at the charge.

Now the electric field lines for a single charge $q$ are radial lines starting from or ending at the charge, depending on whether $q$ is positive or negative.

Clearly, the electric field at every point is normal to the equipotential surface passing through that point.

## XIII- Divergence Theorem

The theorem states that the surface integral of a flux vector is equivalent to the volume integral of the divergence of this flux vector.

$\emptyset = \oiint \ \vec{E}.d\vec{S} = \iiint \vec{\nabla}.\vec{E}$ dv (Theorem of Michael V. Ostrogradskii, or theorem of Gauss)

$\vec{\nabla}.\vec{E}$ = div $\vec{E} \Rightarrow \emptyset = \oiint \ \vec{E}.d\vec{S} = \iiint div\,\vec{E}.$ dv

$$\emptyset = \oint_S \ \vec{E}.d\vec{S} = \frac{Q_{enclosed}}{\varepsilon_0}$$

For volume charge density: $Q_{enclosed} = \iiint \boldsymbol{\rho\,.dV} \Rightarrow \emptyset = \frac{\iiint \boldsymbol{\rho\,.dV}}{\varepsilon_0}$

$\Rightarrow \iiint div\,\vec{E}.$ dv $= \frac{\iiint \boldsymbol{\rho\,.dV}}{\varepsilon_0} \Rightarrow \boldsymbol{div\vec{E} = \frac{\rho}{\varepsilon_0}}$