University of M 'sila

Faculty of: Technology **Common Base**

First Series Of Exercises - Phys 02

Exercise 01:

In an orthonormal base $(\vec{i},\vec{j},\vec{k})$, *we give the scalar function* $F(x,y,z) = 3yx^2 - y^3z^2$

1/ *Determine the partial derivatives* $\frac{\partial F}{\partial x}; \frac{\partial F}{\partial y}; \frac{\partial F}{\partial z}$ *of* F.

2/ *Deduce the gradient of the function* $F(x,y,z)$ $(\vec{\nabla}F)$ *at the point* $P(1,-2,-1)$.

3°/ *Determine the normal to the surface* $F(x,y,z) = 0$ *at the point* $Q(1,1,1)$.

4°/ *Determine the directional derivative at the point* $Q(1,1,1)$ *in the direction* $\vec{u} = \vec{i} - \vec{j} - \vec{k}$.

Exercise 02:

Let the vector function $\vec{G}(x,y,z) = xyz\,\vec{i} + 3x^2y\,\vec{j} + (xz^2 - y^2z)\,\vec{k}$

1/ *Calculate the divergence of the function* $\vec{G}(x,y,z)$ $(\vec{\nabla} \circ \vec{G})$ *at the point* $P(2,-1,1)$.

2°/ *Calculate the rotational of the function* $\vec{G}(x,y,z)$ $(\vec{\nabla} \wedge \vec{G})$ *at the point* $P(2,-1,1)$.

3°/ *Give the value of* " a " *so that the vector function* $\vec{A} = (2x - y)\vec{i} + (z + y)\vec{j} + (1 - a)z\vec{k}$ *will be solenoidal field.*

4/ *What are the values* (a,b,c) *for the vector field* $\vec{B}$ *to be irrotational*

We give: $\vec{B} = (2z + ay)\,\vec{i} + (x - bz)\,\vec{j} + (y + cx)\,\vec{k}$

Exercise 03:

Let the vector function $\vec{r}(x,y,z) = x\,\vec{i} + y\,\vec{j} + z\,\vec{k}$, *which is the vector position*

1/ *Show that* $\vec{\nabla}(r) = \frac{\vec{r}}{r} = \vec{u}_r$ ($\vec{u}_r$ *is the unit vector of* $\vec{r}$).

2°/ *Show that* $\vec{\nabla}(r^n) = n.r^{(n-2)}\vec{r}$. *From this, deduce the gradient of* $\frac{1}{r}$.

3°/ *Show that* $\vec{\nabla}\left(\frac{\vec{A}\wedge\vec{r}}{r^n}\right) = \frac{2-n}{r^n}\vec{A} + \frac{n}{r^{(n+2)}}(\vec{A} \circ \vec{r})\vec{A}$. *With* $\vec{A}$ *is a constant vector.*

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***Exercise 04**: (Additional)*

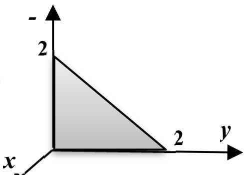

Fig. 5.1

1°/ *Check STOKES's theorem for the vector field* $\vec{A}$ *for the area of the triangle in Figure 5.1.:* $\vec{A} = xy\,\vec{i} + 2yz\,\vec{j} + 3xz\,\vec{k}$

2°/ *Check GAUSS's theorem the vector field* $\vec{B}$ *for the northern hemisphere of the sphere of radius* R *delimited by the equatorial plane. (figure 5.2)*

$$\vec{B} = r.\cos(\theta)\vec{u}_r + r.\sin(\theta)\vec{u}_\theta + r.\sin(\theta).\cos(\varphi)\vec{u}_\varphi$$

(We give: $\nabla \circ \vec{A} = \frac{1}{r^2}.\frac{\partial(r^2 A_r)}{\partial r}\vec{u}_r + \frac{1}{r\sin\theta}.\frac{\partial(\sin\theta A_\theta)}{\partial\theta}\vec{u}_\theta + \frac{1}{r\sin\theta}\frac{\partial A_\varphi}{\partial\varphi}\vec{u}_\varphi$*)*


Fig. 5.2

***Exercise 05**: (Additional)*

Bearing in mind that
$$\begin{cases} div(\vec{a}f) = f div(\vec{a}) + \vec{a} \circ \overrightarrow{grad}(f) = \nabla \circ (\vec{a}f(r)) = f\nabla \circ \vec{a} + \vec{a} \circ \nabla f \\ \overrightarrow{rot}(\vec{a}f) = f\overrightarrow{rot}(\vec{a}) + \vec{a} \wedge \overrightarrow{grad}(f) = \nabla \wedge (\vec{a}f(r)) = f\nabla \wedge \vec{a} + \vec{a} \wedge \nabla f \end{cases}$$

1°- *Show that* $\nabla \wedge (\vec{r}f(r)) = 0$

2°- *Show the following equalities:* $(\overrightarrow{rot} \equiv \vec{\nabla} \wedge;\ \overrightarrow{grad} \equiv \vec{\nabla};\ div \equiv \vec{\nabla} \circ;\ \Delta \equiv \vec{\nabla} \circ \vec{\nabla})$

- $\overrightarrow{rot}(\overrightarrow{grad}) = 0$; $div(\overrightarrow{grad}) = \Delta$; $div(\overrightarrow{rot}) = 0$
- $\vec{\nabla} \wedge (\vec{\nabla} \wedge \vec{A}) = \vec{\nabla}(\vec{\nabla} \circ \vec{A}) - \Delta\vec{A}$, $\vec{\nabla} \circ (\vec{A} \wedge \vec{B}) = \vec{B} \circ \vec{\nabla} \wedge \vec{A} - \vec{A} \circ \vec{\nabla} \wedge \vec{B}$, $\vec{\nabla}(UV) = V\vec{\nabla}U + U\vec{\nabla}V$
- $\vec{\nabla}(\vec{A} \circ \vec{B}) = \vec{A} \wedge (\vec{\nabla} \wedge \vec{B}) + \vec{B} \wedge (\vec{\nabla} \wedge \vec{A}) + (\vec{B} \circ \vec{\nabla})\vec{A} + (\vec{A} \circ \vec{\nabla})\vec{B}$
- $\vec{\nabla} \wedge (\vec{A} \wedge \vec{B}) = \vec{A}(\vec{\nabla} \circ \vec{B}) - (\vec{A} \circ \vec{\nabla})\vec{B} + \vec{B}(\vec{\nabla} \circ \vec{A}) - (\vec{B} \circ \vec{\nabla})\vec{A}$

***Exercise 06** : (MD)*

- *Let the function* $f(x, y) = xy$,

1°- *Calculate the surface integral in the domain* $x = 0, x = a$ *and* $y = 0, y = x$.

- *If* $f(x, y) = 1$,

2°- *Calculate its integral on the surface of a sphere of radius "* R *"*

3°- *Calculate its integral on the volume of a sphere of radius "* R *"*

- *Given the vector* $\vec{A} = a\vec{r}$ *of spherical symmetry.*

1/ *What is its flow through a sphere of radius '*R*' ? (Use GAUSS' theorem).*

2°/ *What is the path (curvilinear) integral along a circular curve of equation* $x^2 + y^2 = a^2$; $z = 0$ *if the field is a function* $\vec{A}(x, y, z) = \sin(y)\vec{i} + x(1 + \cos(y))\vec{j}$.

(Use STOKES's theorem).

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