University of M'sila 2023/2024
Faculty of Technology
1^{st}year. Cycle Engineer

Practice Sheets N°**01 (Analysis1)**

Exercise 01

A) Find the first 4 terms of the Taylor series for the following functions:

1. $\ln x$ centered at (near) $a = 1$
2. $\dfrac{1}{x}$ centered at $a = 1$
3. $\sin x$ centered at $a = \dfrac{\pi}{4}$.

B)

1. Find the first 3 terms of the Taylor series for the function $\sin(\pi x)$ centered at $a = 0.5$.
2. Use your answer to find an approximate value to $\sin(\dfrac{\pi}{2} + \dfrac{\pi}{10})$

Exercise 02

Find the first 3 terms in the Maclaurin series for

1. $\sin 2x$
2. xe^{-x}
3. $\dfrac{x}{\sqrt{1-x^2}}$

Exercise 03

Find the Maclaurin series for the following functions

1. $(1 + \arctan x)(e^x + 2\sin x)$ (order 3)
2. $(1 + 2\cos(2x))(x - \ln(1 + x)$ (order 5)
3. $\dfrac{1 + \arctan x}{x^{\cos x}}$ (order 4)
4. $\dfrac{}{e^x - 1}$ (order 5)
5. $\dfrac{\ln(1 + x^3)}{x - \sin x}$ (order3)
6. $\sqrt{1 + 2\cos x}$ (order 2)
7. $e^{\sqrt{1+2\cos x}}$ (order 2)
8. $(1 + x)^{\frac{1}{x}}$ (order 2)
9. $\ln \dfrac{\sin x}{x}$ (order 4)
10. $\sqrt[3]{1 + \ln(1 + x)}$ (order 3)
11. $\cos(e^{\frac{x}{\cos x}})$ (order 4)

Exercise 04

Construct the third order Taylor polynomial at $x = 0$ for the function

$$f(x) = (1+x)^{\frac{1}{\sin x}}$$

Exercise 05

Construct Taylor polynomial of order 2 at $x = 0$ for the function

$$f(x) = \frac{e^{e^x} - e^{e^{-x}}}{\ln(x+x)}$$

Deduce $\lim\limits_{x \longrightarrow 0} f(x)$

Exercise 06 ($*$ hom$ework$)

The same questions of exercises 05 with the function $f(x) = \dfrac{e^{\left(\frac{1}{\cos x} + \frac{x}{\sin x}\right)} - e}{\ln(x+x)}$

Exercise 07

Using Taylor expansion, evaluate the limits

$$\lim_{x \longrightarrow 0} f(x) \frac{1-\cos x}{x \ln(1+x)}, \quad \lim_{x \longrightarrow 0} \frac{x - \arcsin x}{\sin^3 x}, \quad \lim_{x \longrightarrow 0} \left(\frac{\sin x}{\sinh x}\right)^{\frac{1}{x^2}}$$

Exercise 08

Using the Taylor expansion, study the position of the graphe of the function in relation of its tangent at $x_0 = 0$ in the following cases

1. $f(x) = \cos(2x) - 2\sin x$
2. $g(x) = \dfrac{x}{1+x^2} - xe^{-x}$
3. $h(x) = \ln\left(\dfrac{1+x}{1-x}\right)$

Exercise 09($*$ hom$ework$)

Two electrical charges of equal magnitude and opposite signs located near one another are called an electrical dipole. The charges Q and $-Q$ are a distance d apart. The electric field, E, at the point P is given by

$$E = \frac{Q}{R^2} - \frac{Q}{(R+d)^2}$$

Use series to investigate the behavior of the electric field far away from the dipole. Show that when R is large in comparison to d, the electric field is approximately proportional to $\dfrac{1}{R^3}$