University of M 'sila

Faculty of: Technology **Common Base**

Third Series Of Exercises - Phys 02

Exercise 01: Fig.01

Three-point charges are placed at the vertices of an equilateral triangle of side '$\boldsymbol{a}$'. $\boldsymbol{Q_1 = q}$ *at point* **A (0,0,0)**, $\boldsymbol{Q_2 = q}$ *at point* **B (0, a,0)** *and* $\boldsymbol{Q_3 = 2q}$ *at point* **C** $\left(0,\frac{1}{2}a,\frac{\sqrt{3}}{2}a\right)$

1/ *Find the field created at the centroid* '$\boldsymbol{G}$' *of this triangle*

2/ *Draw the field line of this system*

If we place a negative charged particle with masse '$\boldsymbol{m}$' $-\boldsymbol{Q_0}$ *at that centroid,*

3/ *What is the ratio* '$\frac{Q_0}{m}$' *of the particle to be in equilibrium.*

4/ *What is the energy required to form this system configuration?*

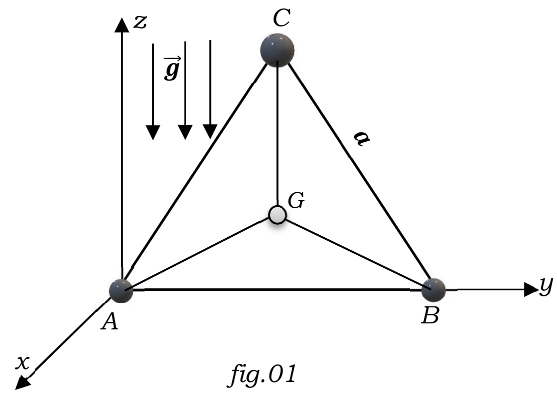


fig.01

Exercise 02: Fig.02

A uniform distributed charge over a surface of cylinder, of radius $\boldsymbol{R}$ *and Hight* $\boldsymbol{H}$, *with a charge density* $\sigma(\boldsymbol{R = H})$.

1/ *Find the electric field* $\vec{E}(P)$ *at a point* $\boldsymbol{P}$ *on its axis and located at a distance 2H from its upper end.*

2/ *Find the electric potential* $\boldsymbol{V}(\boldsymbol{P})$ *at that point*

3/ Additional question

Find the electric field and potential at point P in plan of symmetry perpendicular to the axis of cylinder at distance $\boldsymbol{x}$ *from the axis*

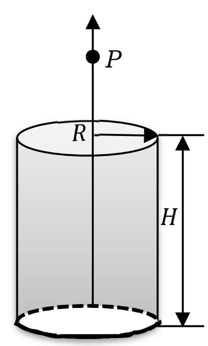


fig.02

Exercise 03: Fig.03

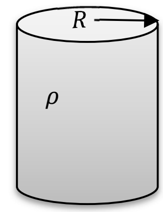


fig.03 - a

A very long cylinder of radius $\boldsymbol{R}$ *has a charge distributed in volume with a charge density positive* $\boldsymbol{\rho}$*. Using GAUSS law*

1/ *Find the electric field* $\vec{\boldsymbol{E}}$ *at every point in space.*

2/ *Deduce the electric potential* $\boldsymbol{V}$ *created at every point in space (Taking* $\boldsymbol{V}(\mathbf{0}) = \mathbf{0}$*).*

By creating in this cylinder, a cylindrical cavity such that the two axes are parallel and at distance $\boldsymbol{d}$

3/ *Find the field inside this cavity. What do you notice about this field?*

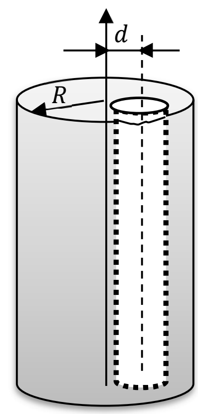


fig.03 - b

Exercise: 04

A spherical conductor of radius R1and charge $\boldsymbol{Q}$*, is surrounded by a neutral conducting shell with inner radius* $\boldsymbol{R_2}$ *and outer radius* $\boldsymbol{R_3}$*.*

1/ *Find the charge on each surface?*

2/ *Find the electric field at all points in space?*

3/ *Determine the potential at all points in space.*

If the outer surface is connected to ground,

4/ *Determine the potential difference between the two conductors? What is the capacitance of the formed capacitor? (**Additional**)*

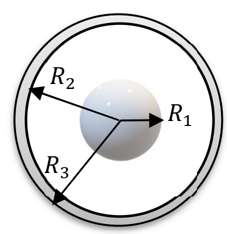


fig.04

Exercise: 05 (Homework)

Two identical charges $\boldsymbol{Q_1} = \boldsymbol{q}$ *located at point* $\boldsymbol{A}\ (\mathbf{0}, \boldsymbol{d}, \mathbf{0})$ *and* $\boldsymbol{Q_2} = \boldsymbol{q}$ *located at* $\boldsymbol{B}\ (\mathbf{0}, \boldsymbol{d}, \mathbf{0})$*.*

1/ *Find the electric field created, at point* $\boldsymbol{P}\ (\mathbf{0}, \mathbf{0}, \boldsymbol{z})$*, by these two charges*

2/ *Verify the limit case for* $\boldsymbol{z} >> \boldsymbol{d}$*. What do you observe?*

3/ *What will be the expression of the field if the charges are opposites* $\boldsymbol{Q_1} = \boldsymbol{q}$ *and* $\boldsymbol{Q_2} = -\boldsymbol{q}$*?*

4/ *Verify the limit case for* $\boldsymbol{z} >> \boldsymbol{d}$*. What do you observe?*

What does this configuration represent?

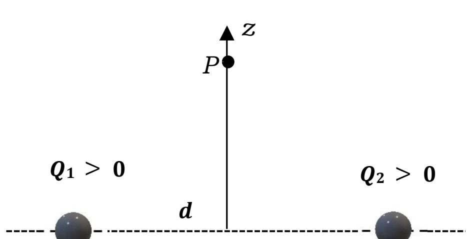


fig.02 - a

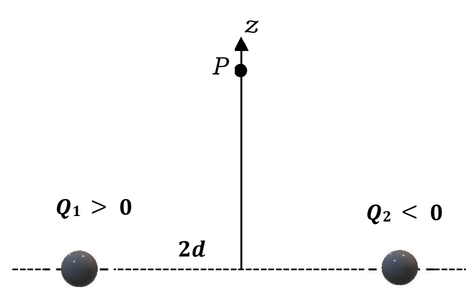
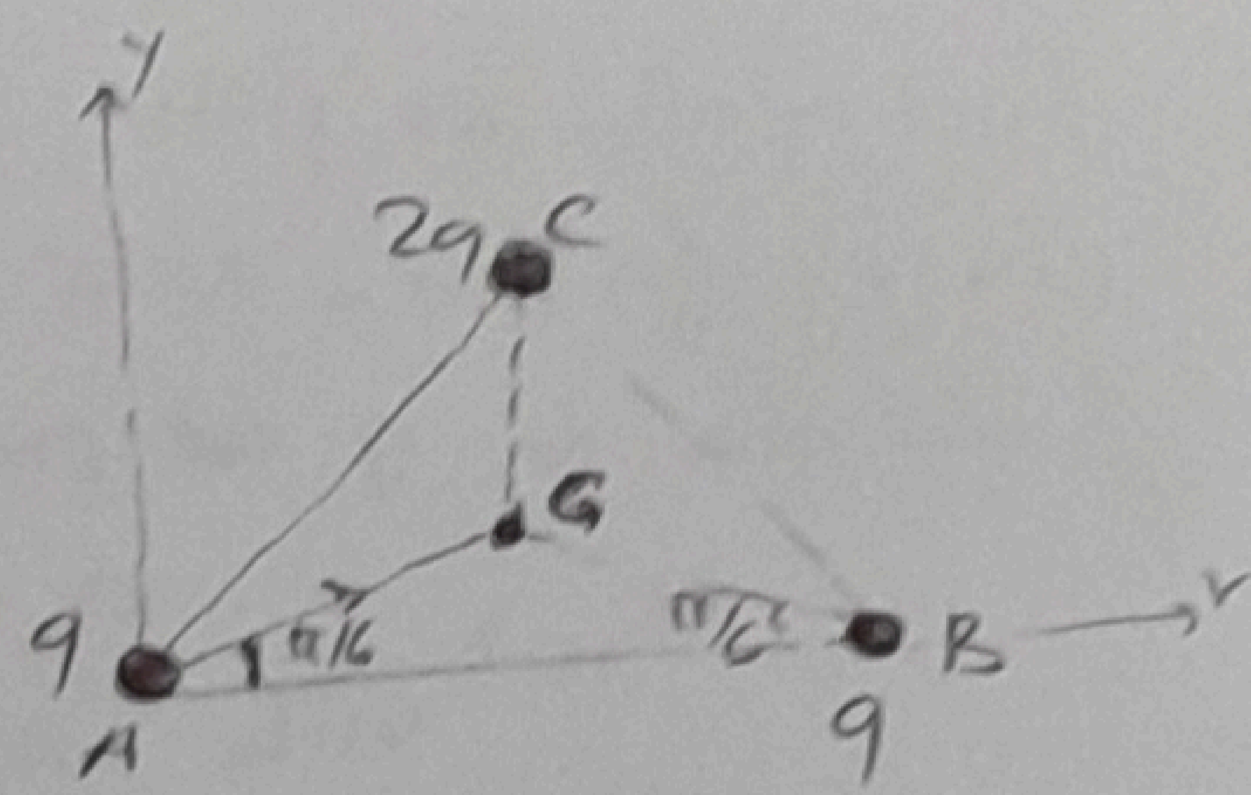


fig.05 - b

- G: The centroid ⇒ $AG = BG = CG$



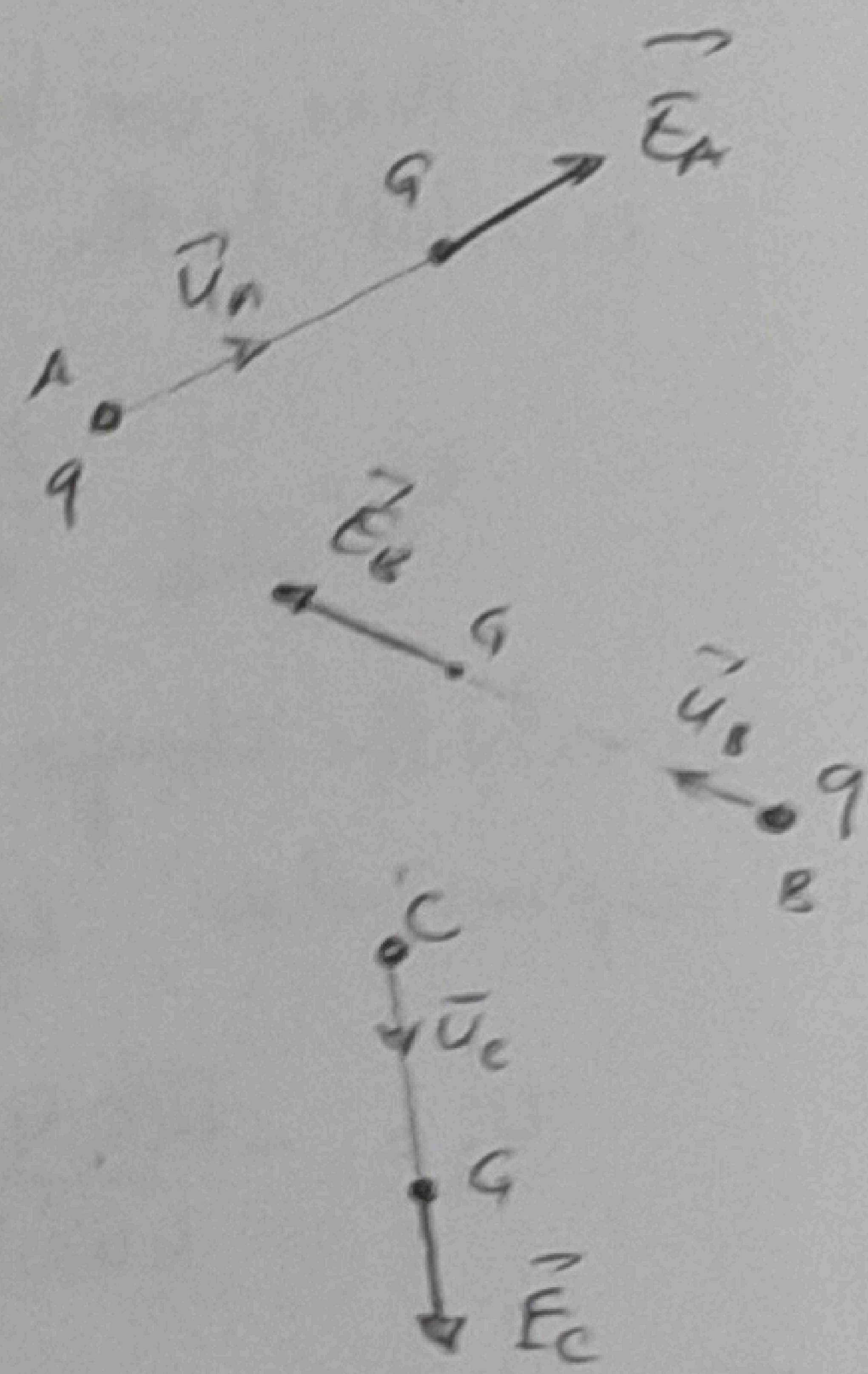
1-/ The electric field at G:

$$\vec{E}(G) = \vec{E}_A(G) + \vec{E}_B(G) + \vec{E}_C(G)$$

- $\vec{E}_A$: field due to the charge at point A:

$$\vec{E}_A = \frac{1}{4\pi\varepsilon_0}\frac{q_A}{|\vec{AG}|^2}\vec{u}_A = \frac{3q}{4\pi\varepsilon_0 a^2}\left(\frac{\sqrt{3}}{2}\vec{i} + \frac{1}{2}\vec{j}\right)$$

$$\vec{u}_A = \frac{\sqrt{3}}{2}\vec{i} + \frac{1}{2}\vec{j}$$



- $\vec{E}_B$: field due to the charge at point B

$$\vec{E}_B = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_B}{|\vec{BG}|^2}\vec{u}_B = \frac{3q}{4\pi\varepsilon_0 a^2}\left(-\frac{\sqrt{3}}{2}\vec{i} + \frac{1}{2}\vec{j}\right)$$

- $\vec{E}_C$: field due to the charge at point C

$$\vec{E}_C = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_C}{|\vec{CG}|^2}\vec{u}_C = \frac{6q}{4\pi\varepsilon_0 a^2}\left(-\vec{j}\right)$$

So, the total field created by the three charges at centroid G:

$$\vec{E} = \left(\frac{3q}{4\pi\varepsilon_0 a^2}\right)\left(\left(\cancel{\frac{\sqrt{3}}{2}\vec{i}} + \frac{1}{2}\vec{j}\right) + \left(\cancel{-\frac{\sqrt{3}}{2}\vec{i}} + \frac{1}{2}\vec{j}\right) - 2\vec{j}\right) = -\frac{3q}{4\pi\varepsilon_0 a^2}\vec{j}$$

$$\boxed{\vec{E} = -\frac{3kq}{a^2}\vec{j}}$$

2-/ The charge $-Q_0$ is subjected to the gravitational force and the electric force. These forces are opposite. To be in equilibrium, the net force acting on the charge $-Q$ must be equal in magnitudes:

$$F_g = F_e \qquad F_g = m\vec{g}, \quad F_e = Q_0 E$$

$$\Rightarrow \quad mg = Q_0 E \Rightarrow \frac{Q_0}{m} = \frac{g}{E} = \frac{g a^2}{3kq}$$

~~IExo2~~ 2°/ Energy required to form this system.

First of all, we place the charge $q_A = q$ at point "A", There is no energy for this. Now bring the second charge at point B in the field due to q at point A. The required energy is: $U_1 = U_{AB}$

$$U_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_A q_B}{AB} = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{a}$$

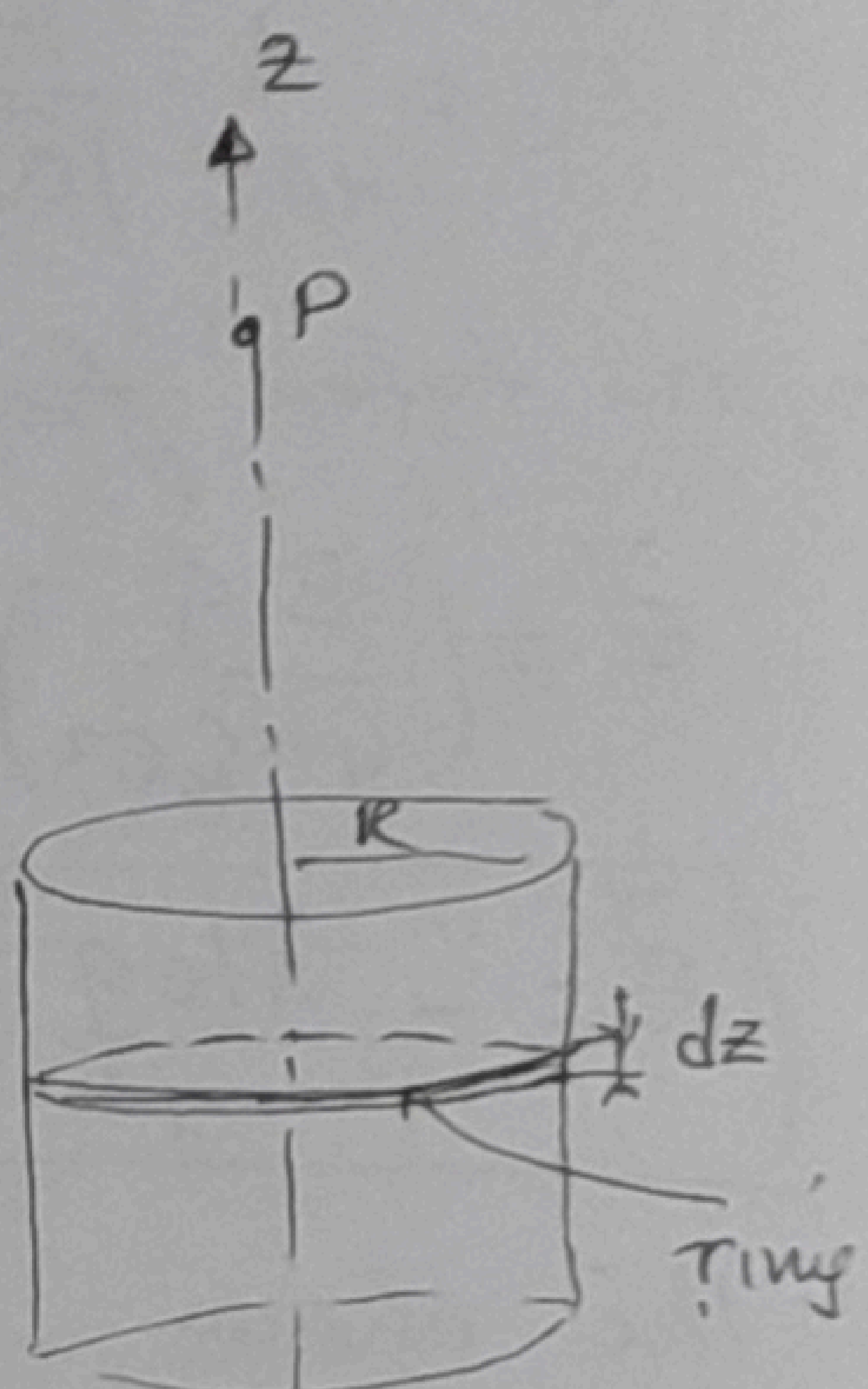
After this, we bring the third charge to point C in the field created by the charge in A and B $\Rightarrow$ $U_2 = \frac{1}{4\pi\varepsilon_0}\frac{q_A q_C}{AC} + \frac{1}{4\pi\varepsilon_0}\frac{q_B q_C}{BC}$

$$U_2 = \frac{1}{4\pi\varepsilon_0}\frac{2q^2}{a} + \frac{1}{4\pi\varepsilon_0}\frac{2q^2}{a} = \frac{4q^2}{4\pi\varepsilon_0 a}$$

finally the charge $-Q$ is brought to point G. the energy is required is $U_3 = -\frac{Q_0}{4\pi\varepsilon_0}\cdot\frac{q_A}{AG} + \frac{-Q_0}{4\pi\varepsilon_0}\frac{q_B}{BG} + \frac{-Q_0 q_C}{4\pi\varepsilon_0 CG}$

$$U_3 = -\frac{\sqrt{3}(Q_0 q + Q_0 q + 2Q_0 q}{4\pi\varepsilon_0 a} = \frac{-4\sqrt{\ }\, q_0 q}{4\pi\varepsilon_0 a}$$

The total energy is: $U = U_1 + U_2 + U_3 = \ldots$



IEx02

Let take a ring with width "dz", it contains a charge $dq = \frac{Q}{h}dz$. Q: total charge
h: Height of cylinder

for convenience, let take the point "P" as origin. The ring create a field given by.

$$d\vec{E} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{dq\, z}{\sqrt{(R^2+z^2)^3}}\vec{k}$$

The electric field ~~generated~~ created ~~at~~ by the whole charge is

$$\vec{E} = \int d\vec{E} = \int_{2h}^{3h} \frac{Q}{4\pi\varepsilon_0 h}\frac{z\,dz}{(z^2+R^2)^{3/2}}\vec{k} = -\frac{Q}{4\pi\varepsilon_0 h}\left[\frac{1}{\sqrt{4h^2+R^2}} - \frac{1}{\sqrt{9h^2+R^2}}\right.$$

2°/ $\vec{E} = -\overrightarrow{grad}\, V = -\frac{dV}{dz}\vec{k} \Rightarrow V = \int E\, dz$

$$V = \frac{Q}{4\pi\varepsilon_0 h}\ln\left(z+\sqrt{z^2+R^2}\right)+C_1$$

or by the direct computing of V.

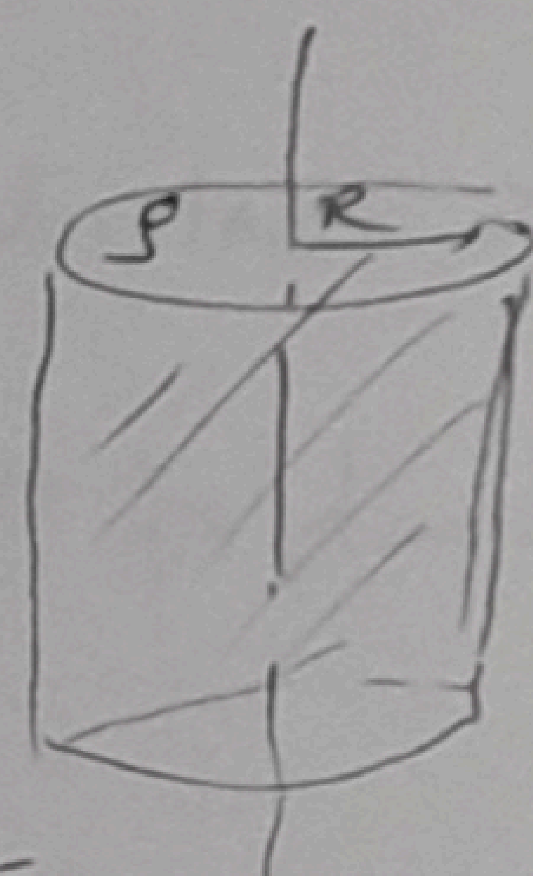
$$dV = \frac{dq}{4\pi\varepsilon_0} = \frac{Q/h\, dz}{4\pi\varepsilon_0\sqrt{z^2+R^2}}$$

$$\Rightarrow V = \int_{2h}^{3h}\frac{Q}{4\pi\varepsilon_0 h}\frac{dz}{(z^2+R^2)^{1/2}} \text{ —— }$$

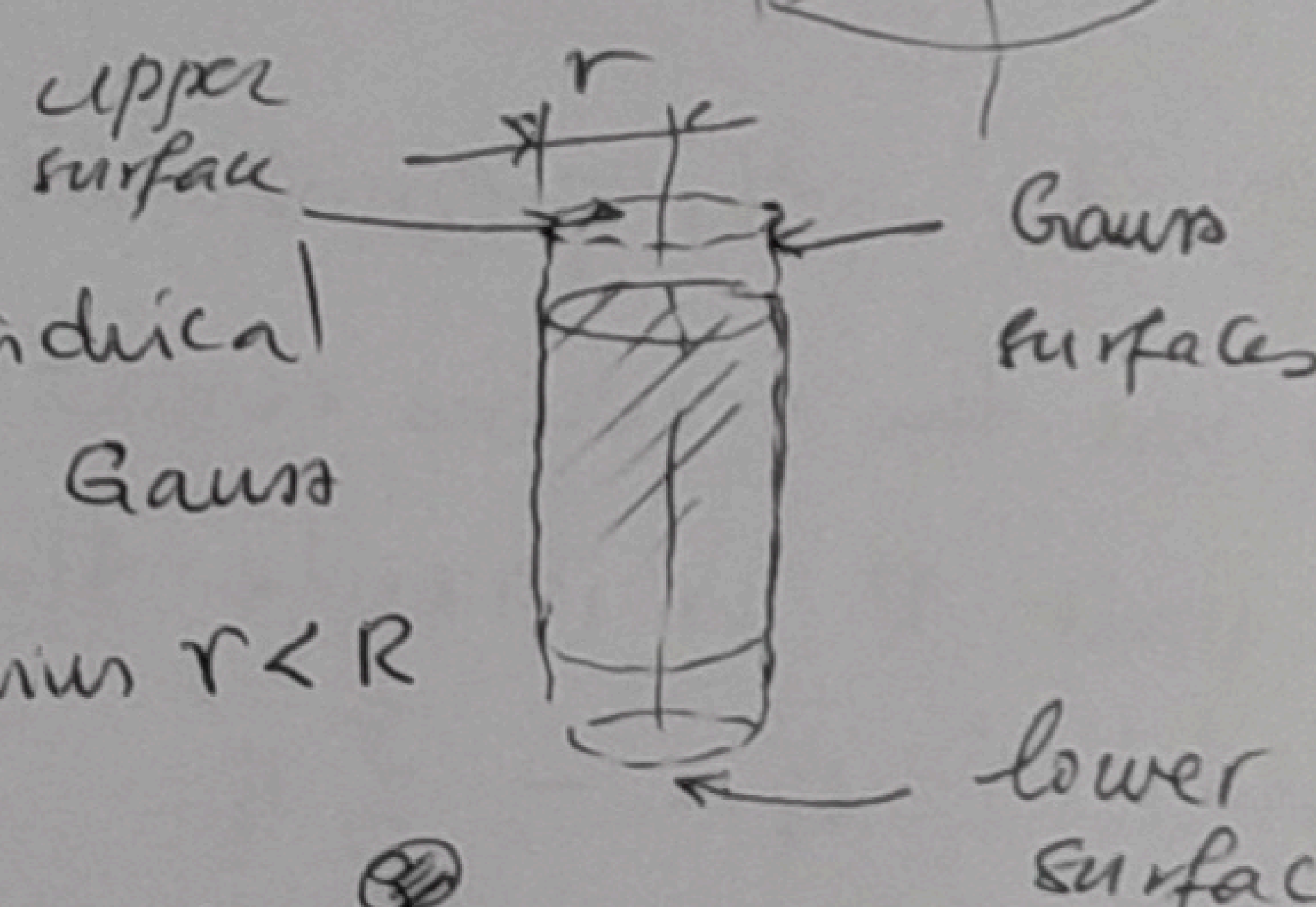
Ex 03.

1°/ Electric field at every point ~~…~~ in space

we must find $\vec{E}$ within the cylinder and ~~out of~~ (inside) and outside:



Inside: $r < R$



The distribution of charge has a cylindrical symmetry, then, the convenient Gauss surface is a cylinder with a radius $r<R$

The flux $\Phi = \oiint \vec{E}\cdot\vec{dS} = \frac{Q_{enc}}{\varepsilon_0}$.

In this symmetry the field is oriented in the radial direction

The flux is trought the closed surface formed by cylinder is

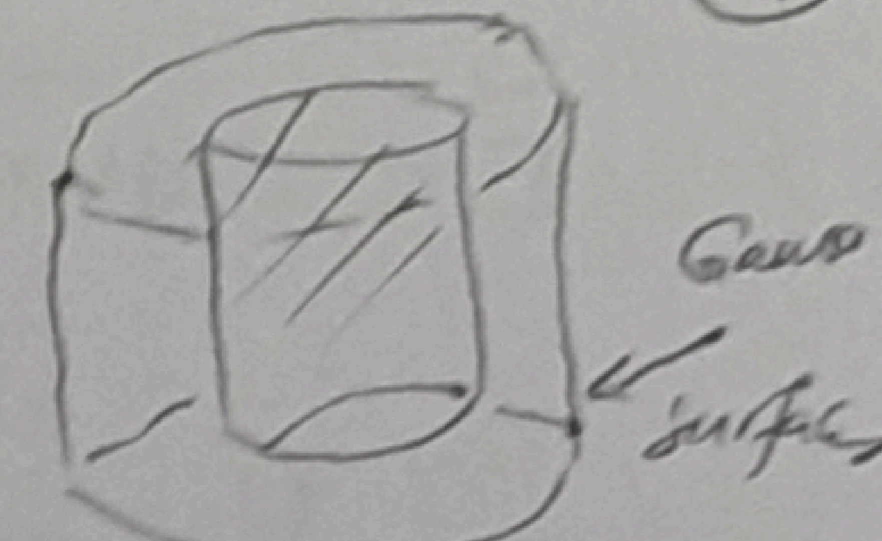
$$\Phi = \iint_{lateral}\vec{E}\cdot\vec{dS} + \underbrace{\iint_{upper\ surf}\vec{E}\cdot\vec{dS}}_{=0} + \underbrace{\iint_{lower\ surf}\vec{E}\cdot\vec{dS}}_{=0} = \frac{Q_{enc}}{\varepsilon_0}$$

$$\Rightarrow \Phi = \oiint \vec{E}\cdot\vec{dS} = \iint_{lat}\vec{E}\cdot\vec{dS} = E\iint dS = ES = Q_{enc}/\varepsilon_0$$

$$E(2\pi r h) = \frac{\rho v}{\varepsilon_0} = \rho\frac{\pi r^2 h}{\varepsilon_0} \Rightarrow \boxed{E = \frac{\rho}{2\varepsilon_0} r}$$

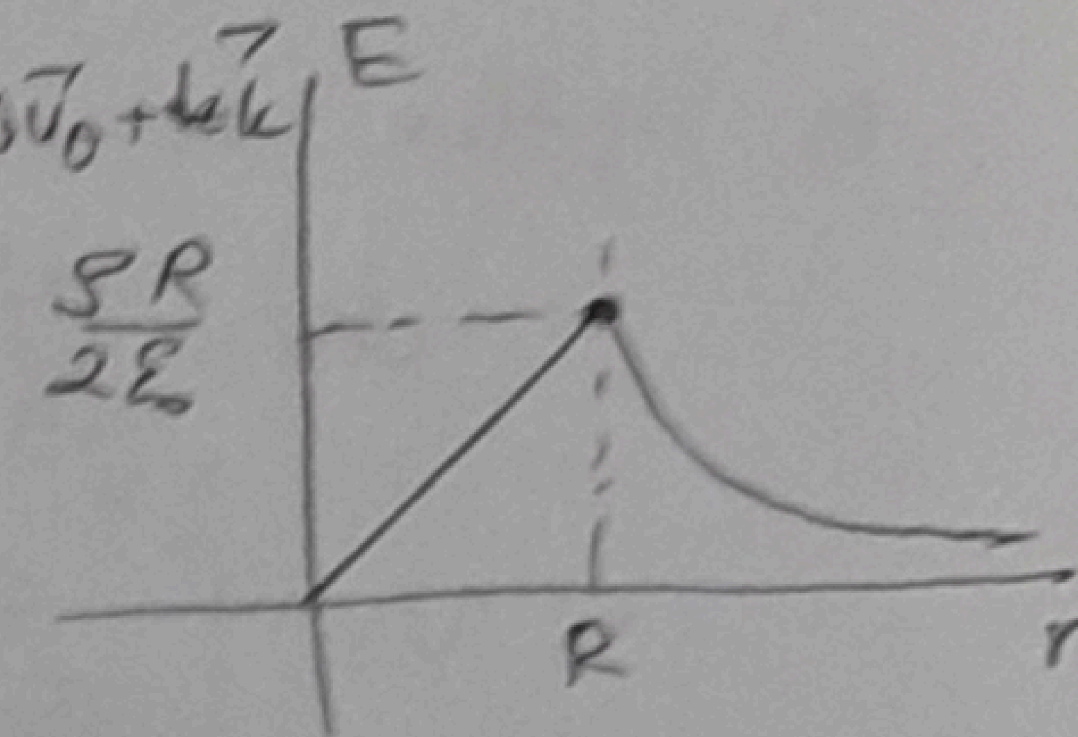
Outside $r > R$ $\vec{E}$, radial

$$\oiint \vec{E} \cdot \vec{ds} = ES = \frac{\rho v}{\varepsilon_0} =$$



$$E(2\pi r h) = \frac{\rho \pi R^2 h}{\varepsilon_0} \Rightarrow \boxed{E = \frac{\rho R^2}{2\varepsilon_0} \cdot \frac{1}{r}}$$

2°/ Potential $\quad V = -\int \vec{E} \cdot \vec{dl}$ $\quad \vec{dl} = dr\,\vec{u}_r + r d\theta\,\vec{u}_\theta + dz\,\vec{k}$



$r < R \Rightarrow V_1 = -\int_0^0 \frac{\rho}{2\varepsilon_0} r\, dr$

$$V_1 = \frac{-\rho}{4\varepsilon_0} r^2 + C_1$$

$V(0) = 0 \Rightarrow C_1 = 0 \Rightarrow V_1 = -\frac{\rho r^2}{4\varepsilon_0}$

$r > R$ (outside) : $V_2 = -\int \frac{\rho R^2}{2\varepsilon_0} \frac{dr}{r} = -\frac{\rho R^2}{2\varepsilon_0} \ln(r) + C_2$

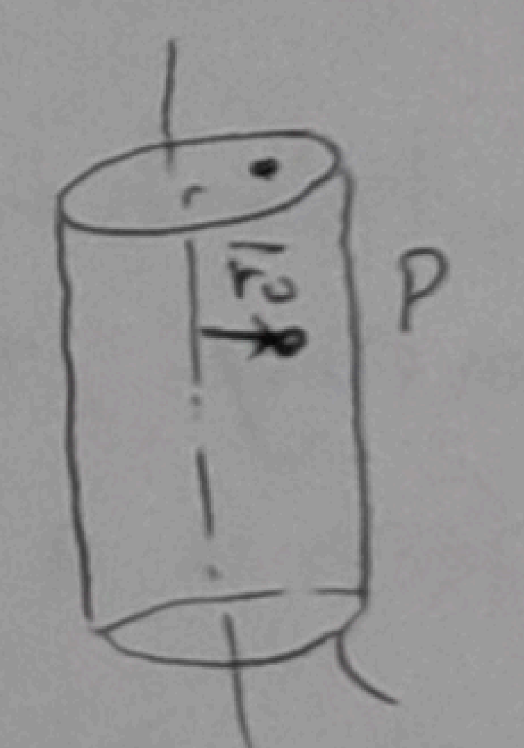
Continuity of the potential.

$$V_1(R) = V_2(R) \Rightarrow C_2 = -$$

~~Exercise~~. 3°/ the cylinder with a cavity.

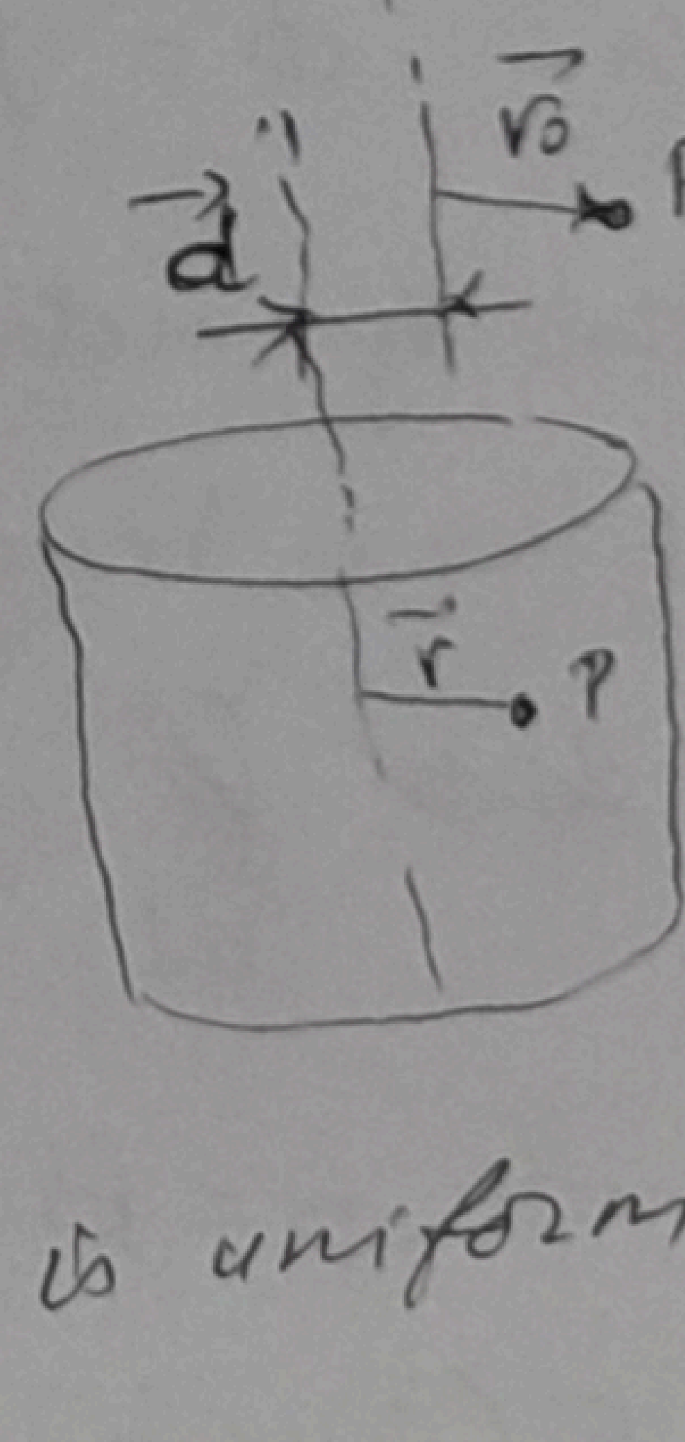
- We use the previous result:

The point P is inside the cavity so the field is $\vec{E}_1 = -\frac{\rho}{2\varepsilon}\vec{r}_0$



for the cylinder P also is inside

$$\vec{E}_2 = +\frac{\rho}{2\varepsilon}\vec{r}$$



the total field created at point "P" in the cavity is $\vec{E} = \vec{E}_1 + \vec{E}_2 = \frac{\rho}{2\varepsilon_0}(\vec{r} - \vec{r}_0)$

$$\vec{E} = \frac{\rho}{2\varepsilon}\vec{d}$$

since ρ, ε_0 d : are constants the field is uniform in the cavity.