4. The Electric Field

*In physics, the concept of a **"field"** describes how objects affect each other at a distance. For instance, the Earth's gravitational field influences objects through gravity. Similarly, electric charges influence one another, creating an electric field around them. This field, denoted by the vector E, measures the influence of the source charge and is expressed in N/C or V/m. The electric field extends outward from positive charges and inward towards negative charges, causing positive charges to experience a force in the same direction and negative charges to experience a force in the opposite direction.*

The electric field generated (induced) by an electric charge is a spatial electrical property related to the source charge and the distance from the source.

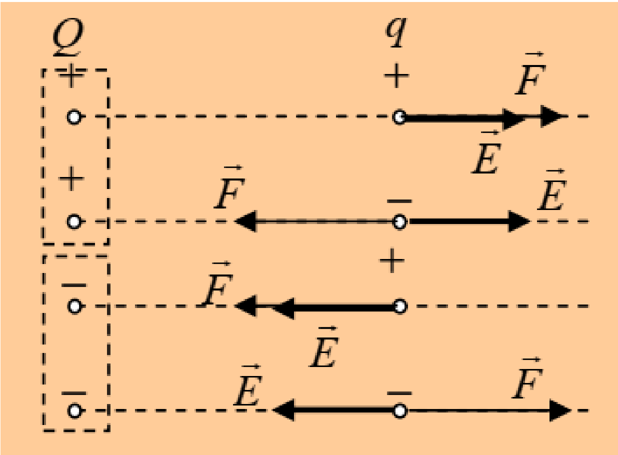


Figure 07: the direction of both the electric field and the electric force

The electric field generated by a point charge Q (located at the point P and considered as the source) at a specific point in space M is characterized by three main features:

- *Support: It is the line connecting the source charge to the point in question.*
- *Direction: The direction of the field depends on the polarity of the source charge, pointing outward for positive charges and inward for negative charges.*
- *Magnitude (Length): The length of the field's vector is influenced by both the magnitude of the source charge and the distance between the source charge and the point in question.*

The mathematical expression for the electric field vector produced by a point charge Q (placed at P and considered as the source) at a point in space is as follows:

$$\overrightarrow{E_Q}(M) = KQ\frac{\overrightarrow{PM}}{\left\|\overrightarrow{PM}\right\|^3}$$

$\overrightarrow{PM}$ *The vector connecting the source charge location P to the point M in space where the field is being calculated.*

The relationship between the electric field induced by the electric charge Q (located at P point) and the electric force applied by Q on q charge located at M point is given as follows:

$$\overrightarrow{F_{Q/q}} = \frac{KqQ}{\left\|\overrightarrow{PM}\right\|^3}\overrightarrow{PM}$$

$$\overrightarrow{F_{Q/q}}(M) = q\,\overrightarrow{E_Q}$$

The electric field at a specific point in space, generated by a collection of charges, is the vector sum of the individual partial fields

$$\vec{E}(M) = \sum_i \vec{E_\iota}(M)$$

4. The electric potential generated by a point charge

In order to move a charge q within an electric field E, work must be done by applying a force that opposes the electric force (overcoming the electric field).

$$\overrightarrow{F_{applied}} = -q\,\vec{E}$$

$$W_{AB}^{ext} = \int_A^B \overrightarrow{F_{applied}}\,d\vec{r} = -q\int_A^B \vec{E}\,d\vec{r}$$

The difference in potential between two points A and B within the field is defined as the result of dividing the applied work by the value of the electric charge

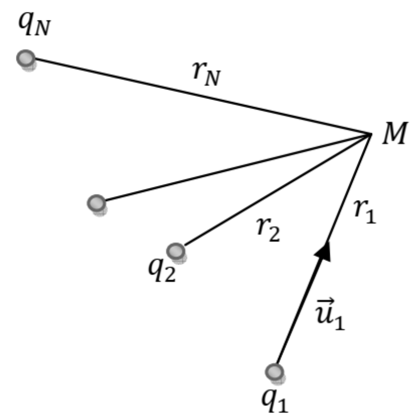
$$V_{BA} = \frac{W_{AB}^{ext}}{q} = -\int_A^B \vec{E}.d\vec{r}$$

The relationship between electric potential and electric field is expressed as follows:

$$dV = -\vec{E}.d\vec{r}$$

$$V = \int dV = -\vec{E}.d\vec{r} = \frac{Kq}{r} + c$$

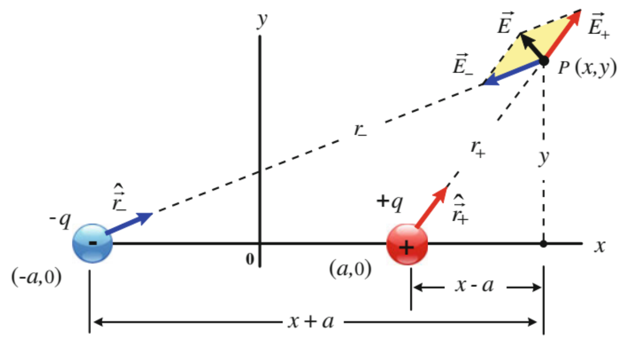
The total electric potential V arising from a collection of point charges q_i at a specific point M in space is equal to the sum of the potentials arising from each individual point charge at that point.



$$V(M) = \sum_{i=1}^{N} V_i\,(M) = k\sum_{i=1}^{N}\frac{q_i}{r_i} + C$$

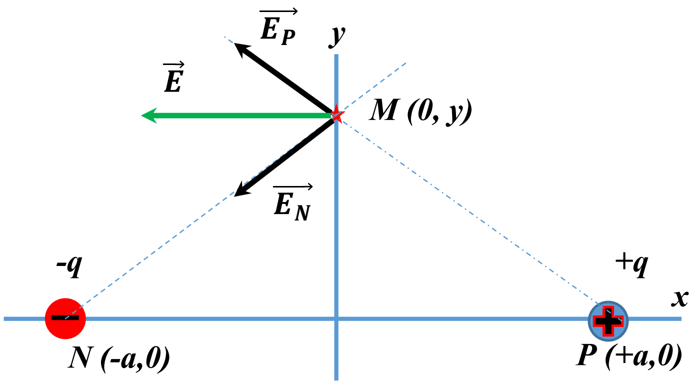
5. The electric field of an electric dipole

The electric dipole introduced in the figure below consists of a positive charge $q+ = +q$ and a negative charge $q- = -q$ separated by a distance $2a$



Example

The electric field produced by these two charges at point M, which lies on the midpoint between the ends of NP, can be calculated as follows:



$$\vec{E}\,(M) = \overrightarrow{E_P}(M) + \overrightarrow{E_N}(M)$$

$$\overrightarrow{E_P}(M) = Kq\frac{\overrightarrow{PM}}{\left\|\overrightarrow{PM}\right\|^3}$$

$$\overrightarrow{E_N}(M) = -Kq\frac{\overrightarrow{NM}}{\left\|\overrightarrow{NM}\right\|^3}$$

$$\overrightarrow{PM} = (x_M - x_P)\vec{\imath} + (y_M - y_P) = (-a)\vec{\imath} + (y)\vec{\jmath}$$

$$\overrightarrow{NM} = (x_M - x_N)\vec{\imath} + (y_M - y_N)\vec{\jmath} = (a)\vec{\imath} + (y)\vec{\jmath}$$

$$\left\|\overrightarrow{PM}\right\| = \sqrt{a^2 + y^2}$$

$$\left\|\overrightarrow{NM}\right\| = \sqrt{a^2 + y^2}$$

$$\overrightarrow{E_P}(M) = Kq\frac{(-a)\vec{\imath} + (y)\vec{\jmath}}{(\sqrt{a^2 + y^2})^3}$$

$$\overrightarrow{E_N}(M) = -Kq\frac{(+a)\vec{\imath} + (y)\vec{\jmath}}{(\sqrt{a^2 + y^2})^3}$$

$$\vec{E}\,(M) = Kq\frac{(-a)\vec{\imath} + (y)\vec{\jmath}}{\left(\sqrt{a^2 + y^2}\right)^3} - Kq\frac{(a)\vec{\imath} + (y)\vec{\jmath}}{(\sqrt{a^2 + y^2})^3}$$

$$\vec{E}\,(M) = \frac{-2Kq\,a\vec{\imath}}{\left(\sqrt{a^2 + y^2}\right)^3}$$