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Physics 02: Electricity and magnetism **University Year 2023-2024**

Series N° 01: ELECTROSTATICS

Part 1: Electrostatic charges and fields

EXERCISE 01

1- The number of electrons removed from a penny to leave it with a charge of $1 \times 10^{-7}\ C$?

Each electron is removed the penny picks up a charge of $1.6 \times 10^{-19}\ C$. So to be left with the given charge we need to remove N electrons, where N is:

$$N\ (e^-) = \frac{Q_T}{q_e} = \frac{1 \times 10^{-7}}{1.6 \times 10^{-19}} \Rightarrow \boldsymbol{N\ (e^-) = 6.2 \times 10^{11}}$$

2- The fraction of the electrons in the penny correspondent

mass (penny=Cu) = 3.11 g, M(Cu)= 63.54 g/mol, Z=29.

$$f = \frac{number\ of\ removed (e^-)}{total\ number\ of\ (e^-)\ in\ the\ penny} = \frac{N\ (e^-)}{N_T\ (e^-)}$$

To find N_T, we need to find the number of copper atoms in the penny and use the fact that each (neutral) atom contains 29 electrons.

- The moles of copper atoms in the penny is:

$$n_{Cu} = \frac{m\ (penny)}{M\ (Cu)} = \frac{3.11}{63.54} \Rightarrow n_{Cu} = \boldsymbol{4.89 \times 10^{-2}\ mol}$$

- The number of copper atoms is:

$$N_{atom}\ (Cu) = n_{Cu}\ N_A = (4.89 \times 10^{-2})(6.022 \times 10^{23}) \Rightarrow \boldsymbol{N_{atom}\ (Cu) = 2.95 \times 10^{22}}$$

- The number of electrons in the penny is:

$$N_T\ (e^-) = 29 \times N_{atom}\ (Cu) = (29)(2.95 \times 10^{22}) \Rightarrow \boldsymbol{N_T\ (e^-) = 8.55 \times 10^{23}}$$

- The fraction of electrons removed in giving the penny the given electric charge is

$$f = \frac{number\ of\ removed (e^-)}{total\ number\ of\ (e^-)\ in\ the\ penny} = \frac{\boldsymbol{6.2 \times 10^{11}}}{\boldsymbol{8.55 \times 10^{23}}}$$

$$\Rightarrow \boldsymbol{f = 7.3 \times 10^{-13}}$$

A very small fraction!!

EXERCISE 02

1- Does your hair or plastic comb have a greater affinity for electrons? Why?

The plastic comb has a greater affinity for electrons. This is because it gained electrons after the friction of combing your hair knocked electrons out of place.

2- The charge of the comb afterwards is:

$$Q_{com} = n_{e^-}\ (gained) \times q_{e^-} = (2.3 \times 10^4).(-1.6 \times 10^{-19}) \Rightarrow Q_{com} = -3.68 \times 10^{-19}$$

3- The charge of the hair afterwards is:

$$Q_{hair} = n_{e^+}\ (P) \times q_{e^+} = (2.3 \times 10^4).(1.6 \times 10^{-19}) \Rightarrow Q_{Hair} = 3.68 \times 10^{-19}$$

The hair became equally positive. It loses the same amount of electrons and now has that many more protons than electrons. This makes is the same charge but positive.

EXERCISE 03

1- The electrostatic force between the two particles.

$$\boldsymbol{F_e = k\frac{|e^+.e^-|}{r^2} = 9 \times 10^9 \frac{|1.6 \times 10^{-19}.(-1.6 \times 10^{-19})|}{(5.3 \times 10^{-11})^2}}$$

$$\boldsymbol{\Rightarrow F_e = 8.2 \times 10^{-8}\ N}$$

2- The gravitational force.

$$\boldsymbol{F_g = G\frac{m_{e^+}.m_{e^-}}{r^2} = 6.67 \times 10^{-11} \frac{(1.67 \times 10^{-27}).(9.1 \times 10^{-31})}{(5.3 \times 10^{-11})^2}}$$

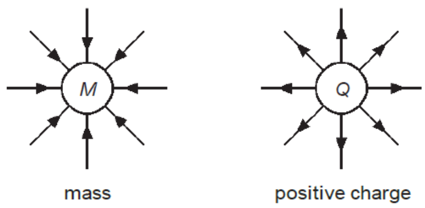
$$\boldsymbol{\Rightarrow F_G = 3.6 \times 10^{-47}\ N}$$

3- The ratio of the electric force to the gravitational force.

$$\boldsymbol{\frac{F_e}{F_G} = \frac{8.2 \times 10^{-8}}{3.6 \times 10^{-47}} = 2.29 \times 10^{39}}$$

4- The similarities and differences between the electrical force and the gravitational force.

Similarities

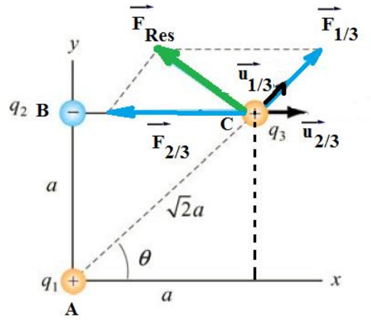


- Both produce **radial** fields
- Both obey an **inverse square law** with distance: $F \propto \frac{1}{r^2}$
- The field magnitudes are defined as force **per** unit (positive) charge or mass.
- Both produce **action at a distance**

Differences

- Electrical forces can be either **attractive** or **repulsive**, whereas gravitational forces are always **attractive**.
- Gravitational forces act between **masses**, whereas electrical forces act between **charges**.

EXERCISE 05



$$\overrightarrow{F_{Res}} = \overrightarrow{F_{1/3}} + \overrightarrow{F_{12/3}}$$

$$\overrightarrow{F_{1/3}} = k\frac{q_1.q_3}{AC^3}\overrightarrow{AC}$$

$$\overrightarrow{F_{2/3}} = k\frac{q_2.q_3}{BC^3}\overrightarrow{BC}$$

A(0,0), B(0,a), C(a,a) $\Rightarrow \overrightarrow{AC} = a\,\vec{\imath} + a\,\vec{\jmath}$, $\overrightarrow{BC} = a\,\vec{\imath}$, **AC =** $\sqrt{2}\,a$ **, BC = a**

$$\overrightarrow{F_{1/3}} = k\frac{q_1.q_3}{(\sqrt{2}\,a)^3}(a\,\vec{\imath} + a\,\vec{\jmath}) = k\frac{q_1.q_3}{2\sqrt{2}\,a^2}(\vec{\imath} + \vec{\jmath})$$

$$\overrightarrow{F_{2/3}} = k\frac{q_2.q_3}{a^3}\,a\,\vec{\imath} = -k\frac{q_1.q_3}{a^2}\,\vec{\imath}$$

$$\overrightarrow{F_{Res}} = \overrightarrow{F_{1/3}} + \overrightarrow{F_{12/3}} = k\frac{q_1.q_3}{2\sqrt{2}\,a^2}(\vec{\imath} + \vec{\jmath}) - k\frac{q_1.q_3}{a^2}\,\vec{\imath}$$

$$\overrightarrow{F_{Res}} = k\frac{q_1.q_3}{a^2}\left[\left(\frac{1}{2\sqrt{2}} - 1\right)\vec{\imath} + \frac{1}{2\sqrt{2}}\vec{\jmath}\right]$$

$$\overrightarrow{F_{Res}} = 9\times10^9\frac{(6\times10^{-6}).(3\times10^{-6})}{(2\times10^{-2})^2}\left[\left(\frac{1}{2\sqrt{2}} - 1\right)\vec{\imath} + \frac{1}{2\sqrt{2}}\vec{\jmath}\right]$$

$$\overrightarrow{F_{Res}} = -261{,}81\,\vec{\imath} + 143.18\,\vec{\jmath} \Rightarrow F_{Res} = 300\ N$$

EXERCISE 06

We have: $$q_1 + q_2 = 5\times10^{-5}\ C$$

$$F = k\frac{q_1.q_2}{r^2} = 1\ N$$

$$\Rightarrow q_1.q_2 = \frac{1\times r^2}{k} = \frac{1\times2^2}{9\times10^9} \Rightarrow q_1.q_2 = 4.44\times10^{-10}\ C^2$$

$$q_1 + q_2 = 5\times10^{-5} \quad \Rightarrow q_2 = 5\times10^{-5} - q_1$$

$$q_1.q_2 = 4.44\times10^{-10} \Rightarrow q_1.(5\times10^{-5} - q_1) = 4.44\times10^{-10}$$

$$\Rightarrow q_1^2 - (5\times10^{-5})q_1 + 4.44\times10^{-10} = 0$$

$$\Delta = (-5\times10^{-5})^2 - 4\times1\times 4.44\times10^{-10} = 7.24\times10^{-10}$$

$$q_1 = \frac{(5\times10^{-5}) \pm \sqrt{7.24\times10^{-10}}}{2}$$

$$\Rightarrow q_1' = 3.84\times10^{-5}\ C \qquad or \qquad q_1'' = 1.16\times10^{-5}\ C$$

$$q_1' = 3.84\times10^{-5}\ C \Rightarrow q_2 = 5\times10^{-5} - q_1' \quad \Rightarrow q_2 = 1.16\times10^{-5}\ C$$

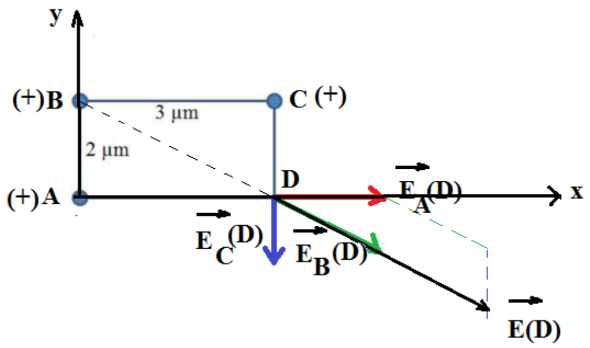
$$q_1'' = 1.16\times10^{-5}\ C \Rightarrow q_2 = 5\times10^{-5} - q_1'' \quad \Rightarrow q_2 = 3.84\times10^{-5}\ C$$

Actually, these are both the same answer, because our numbering of the charges was arbitrary. The answer is that one of the charges is $\mathbf{1.16\times10^{-5}\ C}$ and the other is $\mathbf{3.84\times10^{-5}\ C}$.

EXERCISE 07

q= 1 μC

1- The electric field at the forth corner.



$$\vec{E}_A(D) = k\frac{q}{AD^3}\,\overrightarrow{AD}$$

$$\vec{E}_B(D) = k\frac{q}{BD^3}\,\overrightarrow{BD}$$

$$\vec{E}_C(D) = k\frac{q}{CD^3}\,\overrightarrow{CD}$$

$\overrightarrow{AD} = AD\,\vec{\imath}$, $\overrightarrow{BD} = AD\vec{\imath} - BA\,\vec{\jmath}$, BD= $\sqrt{AD^2+BA^2} = 3.6 \times 10^{-6}$, $\overrightarrow{CD} = -CD\vec{\jmath} = -BA\,\vec{\jmath}$

$$\vec{E}(D) = \vec{E}_A(D) + \vec{E}_B(D) + \vec{E}_C(D)$$

$$\vec{E}_A(D) = k\frac{q}{AD^3}\,\overrightarrow{AD} = k\frac{q}{AD^3}(AD\,\vec{\imath}) = k\frac{q}{AD^2}(\vec{\imath}) = 9\times 10^9\frac{10^{-6}}{(3\times10^{-6})^2}\,\vec{\imath} \Rightarrow \vec{E}_A(D) = 10^{15}\,\vec{\imath}$$

$$\vec{E}_B(D) = k\frac{q}{BD^3}\,\overrightarrow{BD} = k\frac{q}{BD^3}\,(AD\vec{\imath} - BA\,\vec{\jmath})$$

$$= 9\times10^9\frac{10^{-6}}{(3.6\times10^{-6})^3}\,(3\times10^{-6}\,\vec{\imath} - 2\times10^{-6}\,\vec{\jmath})$$

$$\Rightarrow \vec{E}_B(D) = 9\times10^{15}\frac{1}{(3.6\;)^3}\,(3\,\vec{\imath} - 2\,\vec{\jmath}) = 0.57\times10^{15}\vec{\imath} - 0.38\times10^{15}\vec{\jmath}$$

$$\vec{E}_C(D) = k\frac{q}{CD^3}\,\overrightarrow{CD} = k\frac{q}{BA^3}(-BA\,\vec{\jmath}) = k\frac{q}{BA^2}(-\vec{\jmath}) = -9\times10^9\frac{10^{-6}}{(2\times10^{-6})^2}\,\vec{\jmath} \Rightarrow \vec{E}_A(D) =$$

$$-2.25\times10^{15}\vec{\jmath}$$

$$\vec{E}(D) = \vec{E}_A(D) + \vec{E}_B(D) + \vec{E}_C(D)$$

$$\vec{E}(D) = 10^{15}\,\vec{\imath} + 0.57\times10^{15}\vec{\imath} - 0.38\times10^{15}\vec{\jmath} - 2.25\times10^{15}\vec{\jmath}$$

$$\vec{E}(D) = 1.57\times10^{15}\,\vec{\imath} - 2.63\times10^{15}\vec{\jmath} \Rightarrow \text{E(D)}= 3.06\times10^{15}\ \text{N/C}$$

2- The force acted on the charge of +2.0 μC placed at the forth corner object

$$\vec{F}(D) = q_D\,\vec{E}(D) \Rightarrow \vec{F}(D) = 2\times10^{-6}\,(1.57\times10^{15}\,\vec{\imath} - 2.63\times10^{15}\vec{\jmath})$$

$$\Rightarrow \vec{F}(D) = 10^9(3.14\,\vec{\imath} - 5.26\,\vec{\jmath}) \Rightarrow \text{F(D)}= 6.12\times10^9\ \text{N}$$

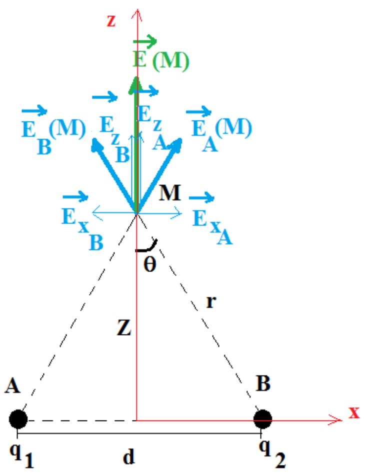
EXERCISE 09

$$\vec{E}(M) = \vec{E}_A(M) + \vec{E}_B(M)$$

$$\vec{E}_A(M) = k\frac{q}{AM^2}\,\overrightarrow{U_{AM}}$$

$$\vec{E}_B(M) = k\frac{q}{BM^2}\,\overrightarrow{U_{BM}}$$

AM= BM =r



$$\overrightarrow{U_{AM}} = \boldsymbol{sin\theta\, \vec{\imath} + cos\, \theta\, \vec{k}}$$

$$\overrightarrow{U_{AM}} = \boldsymbol{-\, sin\theta\, \vec{\imath} + cos\, \theta\, \vec{k}}$$

$$\overrightarrow{E_A}(M) = k\frac{q}{r^2}\ (sin\theta\, \vec{\imath} + cos\, \theta\, \vec{k})$$

$$\overrightarrow{E_B}(M) = k\frac{q}{r^2}\ (-\, sin\theta\, \vec{\imath} + cos\, \theta\, \vec{k})$$

$$\vec{E}(M) = \overrightarrow{E_A}(M) + \overrightarrow{E_B}(M) = k\frac{q}{r^2}(sin\theta\, \vec{\imath} + cos\, \theta\, \vec{k} -\ sin\theta\, \vec{\imath} + cos\, \theta\, \vec{k})$$

$$\vec{E}(M) = k\frac{q}{r^2}(2cos\, \theta\, \vec{k})$$

$$r = \sqrt{(\frac{d}{2})^2 + z^2}\quad , cos\, \theta = \frac{z}{r} = \frac{z}{\sqrt{(\frac{d}{2})^2+z^2}}$$

$$\vec{E}(M) = 2k\frac{q}{((\frac{d}{2})^2 + z^2)^{\frac{3}{2}}}(z\, \vec{k})$$

2- The electric field when $z >> d$.

The field of two identical charges, from far away($z >> d$), the two source charges should "merge" and we should then "see" the field of just one charge, of size 2**q**. So, let $z >> d$; then we can neglect $\left(\frac{d}{2}\right)^2$ in Equation to obtain:

$$\lim_{d\to 0} \vec{E}(M) = \lim_{d\to 0} 2k\frac{q}{\left(\left(\frac{d}{2}\right)^2 + z^2\right)^{\frac{3}{2}}}\left(z\, \vec{k}\right) = 2k\frac{q}{(z^2)^{\frac{3}{2}}}(z\, \vec{k}) = 2k\frac{q}{z^2}(\, \vec{k})$$

which is the correct expression for a field at a distance **z** away from a charge 2q.

EXERCISE 010

The point (other than infinity) at which the total electric field is zero.

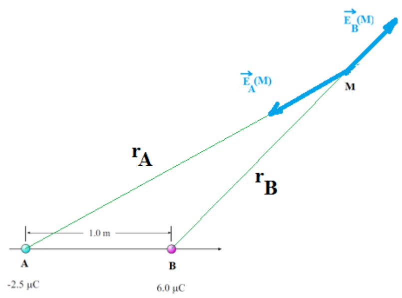
$$\vec{E}(M) = \overrightarrow{E_A}(M) + \overrightarrow{E_B}(M) = \vec{0} \Rightarrow \overrightarrow{E_A}(M) = -\, \overrightarrow{E_B}(M)$$

Case 1

This case is impossible. Because at this point,

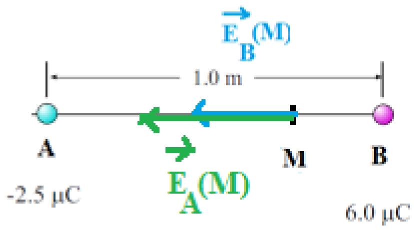
the two field vectors will point toward or away from the

individual charges and those vectors can't be parallel and so can't cancel.



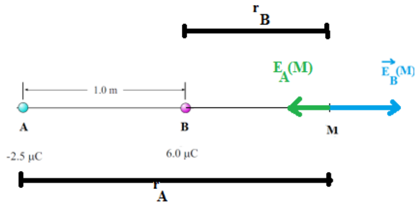
Case 2

This case is impossible. In that region, the field due to the −2.5 μC charge will point toward that charge (to the left) and that due to the +6.0 μC charge will point away from that charge (also to the left). Those vectors can't cancel regardless of their magnitudes.



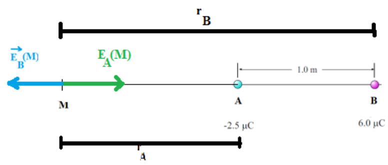
Case 3

This case is impossible. In that region, the +6.0 μC charge is always closer than the −2.5 μC charge ($r_B<r_A$). That being the case, the field from the +6.0 μC charge must always have the larger magnitude (charge is bigger and distance is smaller) so again the vectors can't cancel.



Case 4

This case is possible. In that region, the field due to the −2.5 μC charge points to the right and that due to the +6.0 μC charge points to the left. Note that the −2.5 μC charge is always closer and since it also has a smaller charge, there could be some point where the fields cancel.



$$\overrightarrow{E_{\mathrm{A}}}(M) = k\frac{q_A}{{r_a}^2}\,\vec{\imath}$$

$$\overrightarrow{E_{\mathrm{B}}}(M) = -k\frac{q_B}{{r_B}^2}\,\vec{\imath}$$

$$r_B = r_a + 1 \Rightarrow \overrightarrow{E_{\mathrm{B}}}(M) = -k\frac{q_B}{(r_a+1)^2}\,\vec{\imath}$$

$$\overrightarrow{E_{\mathrm{A}}}(M) = -\overrightarrow{E_{\mathrm{B}}}(M) \Rightarrow k\frac{q_A}{{r_a}^2}\,\vec{\imath} = -k\frac{q_B}{(r_a+1)^2}\,\vec{\imath} \Rightarrow \frac{q_A}{{r_a}^2} = -\frac{q_B}{(r_a+1)^2} \Rightarrow \frac{-2.5}{{r_a}^2} = -\frac{6}{(r_a+1)^2}$$

$$\Rightarrow 2.5(r_a+1)^2 - 6\,{r_a}^2 = 0 \Rightarrow -3.5\,{r_a}^2 + 5r_a + 2.5 = 0$$

$$\Delta = 5^2 - 4\times(-3.5)\times 2.5 = 60$$

$$\Rightarrow r_a = \frac{-5\pm\sqrt{60}}{2\times(-3.5)} \Rightarrow r_a = 1.82\ m \ \ or\ \ r_a = -0.39\ m$$

We take: $r_a = 1.82\ m$

Here we've considered only the "+" result since the other would give a negative value. we assumed that r_a ***is*** positive. So the point we want is 1.8 m to the left of the q_A=−2.5 μC charge.